G.4 Harmoniske tall

(G.4.1) Tallet $H_n = \sum_{k=1}^{n} \frac{1}{k} = 1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{n}$ der $n \geq 1$, kalles det n-te harmoniske tallet.

(G.4.2) Grensen $\lim_{n \to \infty} (H_n - \log n) = \gamma$ kalles Eulers konstant.
På desimalform er tallet gitt ved $\gamma = 0{,}5772156649\ldots$.

(G.4.3) $$H_n = \log n + \gamma + \frac{1}{2n} + \frac{1}{12n^2} + \frac{\varepsilon_n}{120n^4}, \quad 0 < \varepsilon_n < 1$$

(G.4.4) $$\sum_{k=1}^{n} H_k = (n+1)H_n - n$$

(G.4.5) $$\sum_{k=1}^{n-1} \frac{k}{n-k} = nH_{n-1} - (n-1)$$

(G.4.6) $$\sum_{k=1}^{n} kH_k = n(n+1)\frac{2H_{n+1} - 1}{4}$$