Vedlegg G

Formelsamling for algoritmer og datastrukturer

G.1 Summen av endelige rekker

(G.1.1) $$\sum_{k=1}^{n} 1 = n$$

(G.1.2) $$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

(G.1.3) $$\sum_{k=1}^{n} k(k-1) = \frac{n(n^2-1)}{3}$$

(G.1.4) $$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

(G.1.5) $$\sum_{k=1}^{n} k(n-k) = \frac{n(n^2-1)}{6}$$

(G.1.6) $$\sum_{k=0}^{n} x^k = \frac{x^{n+1}-1}{x-1}$$

(G.1.7) $$\sum_{k=0}^{n} 2^k = 2^{n+1}-1$$

(G.1.8) $$\sum_{k=1}^{n} k\,x^k = \frac{nx^{n+2}-(n+1)x^{n+1}+x}{(x-1)^2}$$

(G.1.9) $$\sum_{k=1}^{n} k2^k = (n-1)2^{n+1}+2$$

(G.1.10) $$\sum_{k=1}^{n} \frac{k}{2^k} = 2-\frac{n+2}{2^n}$$

(G.1.11) $$\sum_{k=0}^{n} (k+1)2^k = n2^{n+1}+1$$

(G.1.12) $$\sum_{k=0}^{n} (2k+1)2^k = (2n-1)2^{n+1}+3$$

(G.1.13) $$\sum_{k=1}^{n} \frac{1}{k(k+1)} = \frac{n}{n+1}$$

G.2 Rekker med binomialkoeffisienter

(G.2.1) $$\sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^{k} = (a+b)^{n}$$

(G.2.2) $$\sum_{k=0}^{n} \binom{n}{k} = \binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \ldots\ldots + \binom{n}{n} = 2^{n}$$

(G.2.3) $$\sum_{k=0}^{n} (-1)^{k} \binom{n}{k} = 0$$

(G.2.4) $$\sum_{k=0}^{n} 2^{k} \binom{n}{k} = 3^{k}$$

(G.2.5) $$\binom{n}{k} = \binom{n-1}{k} + \binom{n-1}{k-1}$$ Pascals identitet

(G.2.6) $$\sum_{k=0}^{r} \binom{m}{r-k} \binom{n}{k} = \binom{m+n}{r}$$ Vandermondes identitet

(G.2.7) $$\sum_{k=0}^{n} \binom{n}{k}^{2} = \binom{2n}{n}$$

(G.2.8) $$\sum_{k} k \binom{m}{k} \binom{n}{k} = n \binom{m+n-1}{n}, \quad 1 \le k \le \min(m,n)$$

(G.2.9) $$\sum_{k=0}^{m} \binom{n+k}{k} = \binom{n+m+1}{m} \qquad \sum_{k=0}^{n} \binom{n+k}{k} = \binom{2n+1}{n+1} = \binom{2n+1}{n}$$

(G.2.10) $$\sum_{k=0}^{m} \binom{n+k}{n} = \binom{n+m+1}{n+1} \qquad \sum_{k=0}^{n} \binom{n+k}{n} = \binom{2n+1}{n} = \binom{2n+1}{n+1}$$

G.3 Summen av uendelige rekker

(G.3.1) $$\sum_{k=0}^{\infty} x^k = \frac{1}{1-x} \ , \ |x| < 1$$

(G.3.2) $$\sum_{k=0}^{\infty} \frac{1}{2^k} = 2$$

(G.3.3) $$\sum_{k=1}^{\infty} k\, x^k = \frac{x}{(x-1)^2} \ , \ |x| < 1$$

(G.3.4) $$\sum_{k=1}^{\infty} \frac{k}{2^k} = 2$$

(G.3.5) $$\sum_{k=1}^{\infty} \frac{1}{k(k+1)} = 1$$

G.4 Harmoniske tall

(G.4.1) Tallet $H_n = \sum_{k=1}^{n} \frac{1}{k} = 1 + \frac{1}{2} + \frac{1}{3} + \ . \ . \ . + \frac{1}{n}$ der $n \geq 1$, kalles det n-te harmoniske tallet.

(G.4.2) Grensen $\lim_{n \to \infty} (H_n - \log n) = \gamma$ kalles Eulers konstant. På desimalform er tallet gitt ved $\gamma = 0{,}5772156649 \ . \ . \ . \ .$

(G.4.3) $$H_n = \log n + \gamma + \frac{1}{2n} + \frac{1}{12n^2} + \frac{\varepsilon_n}{120n^4} \ , \quad 0 < \varepsilon_n < 1$$

(G.4.4) $$\sum_{k=1}^{n} H_k = (n+1)H_n - n$$

(G.4.5) $$\sum_{k=1}^{n-1} \frac{k}{n-k} = nH_{n-1} - (n-1)$$

(G.4.6) $$\sum_{k=1}^{n} kH_k = n(n+1)\frac{2H_{n+1} - 1}{4}$$