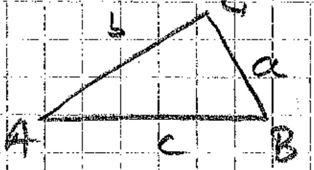
Førelesing 25 okt.

① Repetere sinus-setninga:



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

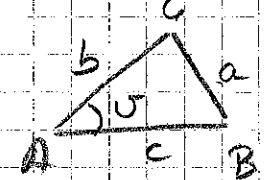
[?] Må $\triangle ABC$ vere rettvinkla?
– Nei

② Gå gjennom eksempel frå forrige notat.

③ Cosinus-setninga (8.6)

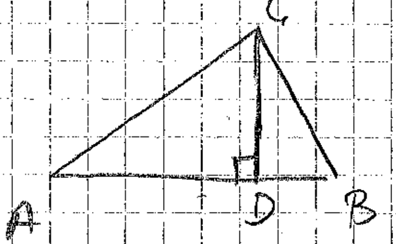
Ser slik ut:

$$a^2 = b^2 + c^2 - 2bc\cos v$$



[?] Kalla „den utvida pytagorassetninga"
– Kvifor?

Bevis Antar $\angle A \in [0°, 90°\rangle$



Pytagoras:

[?] $(BC)^2 = (CD)^2 + (BD)^2$

– Vil få punktet D „ut av likninga"

$$(BC)^2 = (CD)^2 + (AB - AD)^2 = (CD)^2 + (AB)^2 + (AD)^2 - 2(AB)\cdot(AD)$$

[?] Spør om 2. kvadrat-setning

$$(BC)^2 = \underbrace{(AD)^2 + (CD)^2}_{\text{Pytagoras: } (AC)^2} + (AB)^2 - 2\cdot(AB)\cdot(AD)$$

AD: $\frac{AD}{AC} = \cos A$, $AD = AC \cdot \cos A$

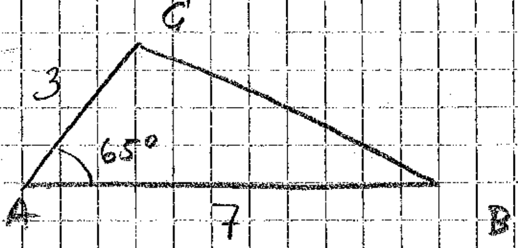
$$(BC)^2 = (AC)^2 + (AB)^2 - 2(AB)\cdot(AC)\cdot\cos A$$

– ingen D lenger

Innfører a, b, c, v:

$$\boxed{a^2 = b^2 + c^2 - 2ab\cos v}$$

Eksempel



Finn BC

Cosinus-setningen:

$$(BC)^2 = (AB)^2 + (AC)^2 - 2(AB)\cdot(AC)\cdot\cos A$$

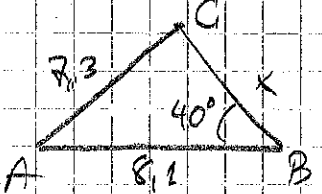
$$= 7^2 + 3^2 - 2\cdot 7\cdot 3\cdot\cos 65^\circ = 40{,}25$$

$$BC = \sqrt{40{,}25} = \underline{\underline{6{,}34}}$$

Nytt eksempel (noko vanskelegare)

I ΔABC er $\angle B = 40°$, $AB = 8{,}1$ og $AC = 7{,}3$

Finn sida BC



Cosinussetninga $(AC)^2 = (AB)^2 + (BC)^2 - 2(AB)\cdot(BC)\cdot\cos B$

$$7{,}3^2 = 8{,}1^2 + x^2 - 2\cdot 8{,}1\cdot x\cdot \cos 40°$$

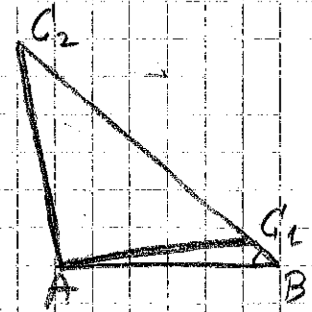
$$53{,}29 = 65{,}61 + x^2 - 12{,}41x$$

$$x^2 - 12{,}41x + 12{,}32 = 0$$

$$x = \frac{-(-12{,}41) \pm \sqrt{(-12{,}41)^2 - 4\cdot 1\cdot 12{,}32}}{2\cdot 1}$$

$$x = 1{,}09 \quad \text{eller} \quad x = 11{,}3$$

Sida BC har lengda <u>1,09 eller 11,3</u>.



[?] Alternative metodar?

Sinussetninga?

Ja (hoppe over?)

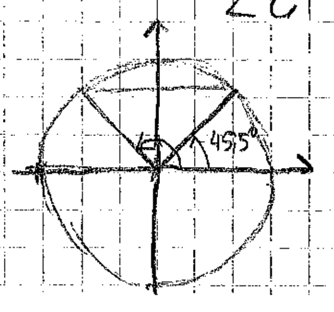
Finn $\angle A$ ved fyrst å finne $\angle C$

$$\frac{\sin C}{AB} = \frac{\sin B}{AC}$$

$$\sin C = \frac{AB}{AC}\sin B = \frac{8{,}1}{7{,}3}\cdot \sin 40° = 0{,}713$$

$$\sin^{-1} 0{,}713 = 45{,}5°$$

$$\angle C = 45{,}5° \quad \text{eller} \quad \angle C = 180° - 45{,}5° = 134{,}5°$$



$$\angle A = 180° - \angle C - \angle B$$

$$\angle A = 180° - 45{,}5° - 40° = 94{,}5° \quad \text{eller}$$

$\angle A = 180° - 134{,}5° - 40° = 5{,}5°$

$\underline{\angle A = 94{,}5°}$

$$\frac{\sin A}{BC} = \frac{\sin B}{AC}$$

$$\frac{BC}{\sin A} = \frac{AC}{\sin B}$$

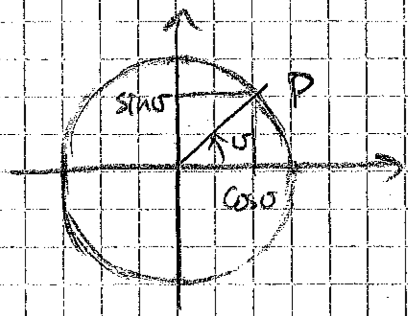
$$BC = AC \cdot \frac{\sin A}{\sin B} = 7{,}3 \cdot \frac{\sin 94{,}5°}{\sin 40°} = \underline{11{,}3}$$

$\underline{\angle A = 5{,}5°}$

$$BC = AC\,\frac{\sin A}{\sin B} = 7{,}3 \cdot \frac{\sin 5{,}5°}{\sin 40°} = \underline{1{,}09}$$

(4) Trigonometriske identitetar vi ser ut frå einings-sirkelen (7.7)

[?] – Dugnad!



- $\cos^2 v + \sin^2 v = 1$
- $\cos(-v) = \cos v$
- $\sin(-v) = -\sin v$
- $\cos(180° - v) = -\cos v$
- $\sin(180° - v) = \sin v$
- $\cos(v + n \cdot 360°) = \cos v, \quad n \in \mathbb{Z}$
- $\sin(v + n \cdot 360°) = \sin v$

[?] • $\tan(v + n\cdot 180^\circ) = \tan v$

Ser $\cos(v+180^\circ) = -\cos v$

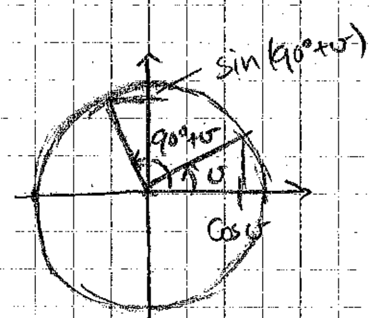
$\sin(v+180^\circ) = -\sin v$

$$\tan(v+180^\circ) = \frac{\sin(v+180^\circ)}{\cos(v+180^\circ)} = \frac{-\sin v}{-\cos v} = \frac{\sin v}{\cos v} = \tan v$$

$n\cdot 180^\circ$: n partal: Legg til eit heilt omløp

n oddetal: Legg til eit halvt omløp

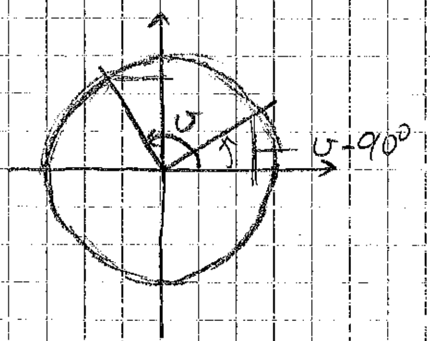
[?] $\sin(90^\circ + v)$



$\sin(90^\circ+v) = \cos v$

[?] $\cos(v - 90^\circ)$

$= \sin v$



− Gjeld også for negativ v

$\sin(90^\circ - v) = \sin(90^\circ + (-v)) = \cos(-v) = \cos v$

$\cos(90^\circ - v) = \cos(-(v - 90^\circ)) = \cos(v - 90^\circ) = \sin v$

- $\sin(90^\circ - v) = \cos v$
- $\cos(90^\circ - v) = \sin v$

I boka: Ser det ut frå rettvinkla trekantar (les dette!).

Moral: Håpar at de ser desse samanhengane <u>sjølve</u> – også utan formelsamling.

⑤ Sum og differanse av vinklar (7.8, 7.9)

Regel: $\sin(u+v) = \sin u \cdot \cos v + \cos u \cdot \sin v$

<u>Eksempel</u>

Finn ein eksakt verdi for $\sin 75^\circ$

[?] Kva vinklar har vi sett eksakte verdiar for? – $0^\circ, 30^\circ, 45^\circ, 60^\circ$ og 90°

$75^\circ = 30^\circ + 45^\circ$

$\sin 75^\circ = \sin(30^\circ + 45^\circ) = \sin 30^\circ \cdot \cos 45^\circ + \cos 30^\circ \cdot \sin 45^\circ$

Hugsar (?): $\sin 30^\circ = \frac{1}{2}$ $\cos 30^\circ = \frac{\sqrt{3}}{2}$

$\sin 45^\circ = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$ $\cos 45^\circ = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\sin 75^\circ = \frac{1}{2} \cdot \frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} = \underline{\underline{\frac{\sqrt{2}+\sqrt{6}}{4}}}$

Bevis?