Førelesing 21/3

① Fullføre eksempel frå sist

② Repetere integrasjonsreglar:

1) $\int (f(x)+g(x))dx = \int f(x)dx + \int g(x)dx$
2) $\int k\cdot f(x)\,dx = k\cdot \int f(x)dx$

$\Big\}$ linearitet

3) $\int x^r dx = \frac{1}{r+1} x^{r+1} + C \, , \quad r \neq -1$
4) $\int x^{-1} dx = \ln|x| + C$
5) $\int \sin x \, dx = -\cos x + C$
6) $\int \cos x\, dx = \sin x + C$
7) $\int \frac{1}{\cos^2 x} dx = \tan x + C$
8) $\int e^x dx = e^x + C$

③ Eksempel

Finn desse ubestemte integrala:

a) $\int (2x+3)^2 dx$

b) $\int (\sin x + e^x)\, dx$

c) $\int e^{3x} dx$

d) $\int \sin(\pi x - 3)\, dx$

a) $\int (2x+3)^2 dx = \int (4x^2+12x+9)dx = 4\cdot\int x^2 dx + 12\cdot\int x dx + \int 9 dx$

$= 4\cdot \frac{1}{3}x^3 + 12\cdot\frac{1}{2}x^2 + 9x + C = \frac{4}{3}x^3 + 6x^2 + 9x + C$

b) $\int (\sin x + e^x)dx = \int \sin x \, dx + \int e^x dx =$

$\underline{-\cos x + e^x + C}$

c) $\int e^{3x} dx$

$(e^{3x})' = e^{3x}\cdot 3$

$(\frac{1}{3}e^{3x})' = \frac{1}{3}\cdot e^{3x}\cdot 3 = e^{3x}$

$\int e^{3x}dx = \underline{\frac{1}{3}e^{3x} + C}$

d) $\int \sin(\pi x - 3)\, dx$

$(-\cos x)' = -(-\sin x) = \sin x$

$u = \pi x - 3$:

$(-\cos(\pi x - 3))' = -(-\sin u)\cdot u'(x) =$

$+\sin(\pi x - 3)\cdot \pi$

$(-\frac{1}{\pi}\cos(\pi x - 3))' = \frac{1}{\pi}(-\cos(\pi x-3)) = \frac{1}{\pi}\sin(\pi x - 3)\cdot\pi$

$= \sin(\pi x - 3)$

Altså:

$\int \sin(\pi x - 3)dx = \underline{-\frac{1}{\pi}\cos(\pi x - 3) + C}$

Generelt: Dersom $F'(x) = f(x)$ har vi at

$\int f(ax+b)dx = \frac{1}{a}F(ax+b) + C$

④ Viser dette (15.5):

Har at $F'(x) = f(x)$

Er då $\frac{1}{a}F(ax+b)$ ein anti-derivert til $f(ax+b)$?

$$\left(\frac{1}{a}F(ax+b)\right)' = \frac{1}{a}F'(ax+b)$$

$$u(x) = ax+b$$

$$\left(\frac{1}{a}F(ax+b)\right)' = \frac{1}{a}F'(u)\cdot u'(x) = \frac{1}{a}f(u)\cdot a = f(u) = f(ax+b)$$

$$\left(\frac{1}{a}F(ax+b)\right)' = f(ax+b) \Leftrightarrow$$

$$\int f(ax+b)\,dx = \frac{1}{a}F(ax+b) + C$$

⑤ Eksempel

Finn desse ubestemte integrala

a)* $\int e^{4x+2}\,dx$

b) $\int \frac{1}{2x+3}\,dx$

c)* $\int 5\cos(15x+4)\,dx$

d) $\int \frac{x^3-2x+3}{x-1}\,dx$

e) $\int 5^x\,dx$

f)* $\int \sqrt{2x+2}\,dx$

a) $\int e^{4x+2}\,dx = \underline{\frac{1}{4} e^{4x+2} + C}$

$$\left(\frac{1}{4} e^{4x+2} + C\right)' = \frac{1}{4} e^{4x+2} \cdot (4x+2)' + 0 =$$
$$\frac{1}{4} e^{4x+2} \cdot 4 = e^{4x+2}$$

Vi fekk integranden ved å derivere svaret.

b) $\int \frac{1}{2x+3}\,dx = \frac{1}{2} \ln|2x+3| + C = \underline{\ln\sqrt{|2x+3|} + C}$

c) $\int 5\cos(15x+4)\,dx = 5 \cdot \frac{1}{15} \sin(15x+4) + C = \underline{\frac{1}{3}\sin(15x+4) + C}$

$$\left(\frac{1}{3}\sin(15x+4) + C\right)' = \frac{1}{3} \cdot \cos(15x+4) \cdot 15 + 0 = \underline{5\cos(15x+4)}$$

d) $\int \frac{x^3 - 2x + 3}{x-1}\,dx$

Polynomdivisjon:

$$(x^3 - 2x + 3) : (x-1) = x^2 + x - 1 + \frac{2}{x-1}$$
$$\underline{-(x^3 - x^2)}$$
$$x^2 - 2x$$
$$\underline{-(x^2 - x)}$$
$$-x + 3$$
$$\underline{-(-x+1)}$$
$$2$$

$$\int \frac{x^3 - 2x + 3}{x-1}\,dx = \int \left(x^2 + x - 1 + \frac{2}{x-1}\right) dx =$$
$$\frac{1}{3}x^3 + \frac{1}{2}x^2 - x + 2\int \frac{1}{x-1}\,dx =$$
$$\frac{1}{3}x^3 + \frac{1}{2}x^2 - x + 2 \cdot \frac{1}{1} \ln|x-1| + C =$$
$$\frac{1}{3}x^3 + \frac{1}{2}x^2 - x + \ln(x-1)^2 + C$$

e) $\int 5^x dx$

$5 = e^{\ln 5}$

$\int 5^x dx = \int (e^{\ln 5})^x dx = \int e^{\ln 5 \cdot x} dx =$

$\frac{1}{\ln 5} e^{\ln 5 x} + C' = \frac{1}{\ln 5} \cdot 5^x + C'$

Generelt: $\int a^x dx = \frac{1}{\ln a} a^x + C'$

f) Veit: Dersom $f(x) = \sqrt{x} = x^{1/2}$, er

$F(x) = \frac{1}{\frac{1}{2}+1} x^{1/2+1} = \frac{2}{3} x^{3/2}$ ein anti-derivert til $f(x)$

$\int \sqrt{2x+2}\, dx = \frac{1}{2} \cdot \frac{2}{3} (2x+2)^{3/2} + C =$

$\underline{\underline{\frac{1}{3} (2x+2)^{3/2} + C'}}$

Derivasjon:

$\left(\frac{1}{3}(2x+2)^{3/2} + C'\right)' = \frac{1}{\cancel{3}} \cdot \frac{\cancel{3}}{\cancel{2}} (2x+2)^{1/2} \cdot \cancel{2} + 0 = \underline{\sqrt{2x+2}}$