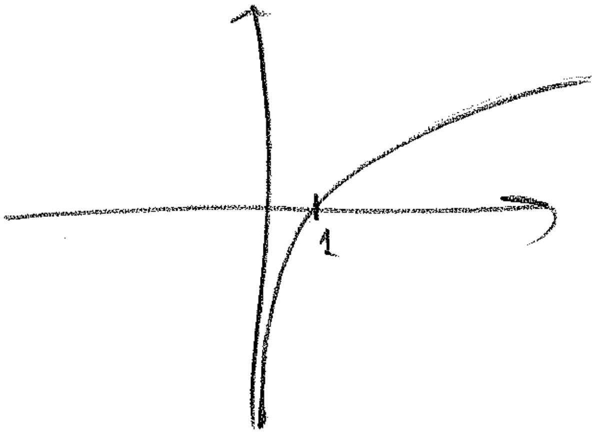
Førelesing 14/3

① Takke for midtvegsevalueringane
Om løysingsforslag

② Generelt om ln-funksjonen:



$f(x) = \ln x$

$D_f = \langle 0, \rightarrow \rangle, \; V_f = \mathbb{R}$

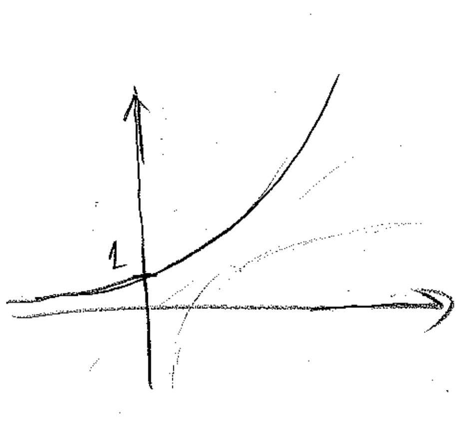
Nullpunkt: $x = 1$

$x \to 0^+ \Rightarrow f(x) \to -\infty$

$x \to \infty \Rightarrow f(x) \to \infty$

$f'(x) = \frac{1}{x}$

③ Generelt om eksp.-funksjonen:



$g(x) = e^x, \quad (g(x) = f^{-1}(x))$

$D_g = \mathbb{R} \; (= \langle \leftarrow, \rightarrow \rangle)$

$V_g = \langle 0, \rightarrow \rangle$

Ingen nullpunkt

$x \to +\infty \Rightarrow g(x) \to +\infty$

$x \to -\infty \Rightarrow g(x) \to 0$

$g'(x) = e^x$

④ Eksempel på drøfting av ein logaritme-funksjon

Gitt funksjonen $f(x) = \ln \frac{x^2}{x-1}$

Finn:

a) – største mogelege definisjonsmengd

b) – eventuelle asymptotar

c) – — " — nullpunkt

d) – — " — ekstremalpunkt

e) – — " — vendepunkt

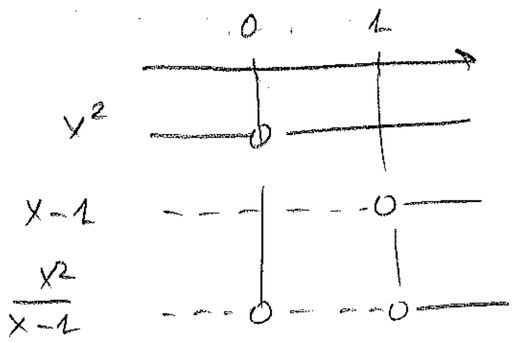
f) Skissér grafen til f.

a) Kan ikkje ha 0 i nemnaren. Vi må kreve at $x \neq 1$

Funksjonen $\ln u$ er berre definert for $u > 0$. Vi må kreve at

$$\frac{x^2}{x-1} > 0$$

Forteiknskjema:



Krav: $x > 1$

$\underline{D_f = \langle 1, \rightarrow \rangle}$

b) Når $x \to 1^+$:

$$\frac{x^2}{x-1} \to +\infty \Rightarrow f(x) = \ln \frac{x^2}{x-1} \to +\infty$$

$\underline{x = 1}$ er horisontal asymptote for $f(x)$

Veit: $\ln u \to -\infty$ når $u \to 0^+$

Men $\frac{x^2}{x-1} = 0$ når $x = 0$, som ligg utanfor definisjonsmengda D_f.

$x \to +\infty$: $\ln \frac{x^2}{x-1} \to +\infty$

Vi har ingen horisontale asymptotar.

c) $f(x) = 0$

$$\ln \frac{x^2}{x-1} = 0$$

$$\frac{x^2}{x-1} = e^0 = 1$$

$$x^2 = x - 1$$

$$x^2 - x + 1 = 0$$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4 \cdot 1 \cdot 1}}{2 \cdot 1} = \frac{1 \pm \sqrt{-3}}{2}$$

$\sqrt{-3}$ er ikkje definert; $\underline{f(x) \text{ har ingen nullpunkt}}$.

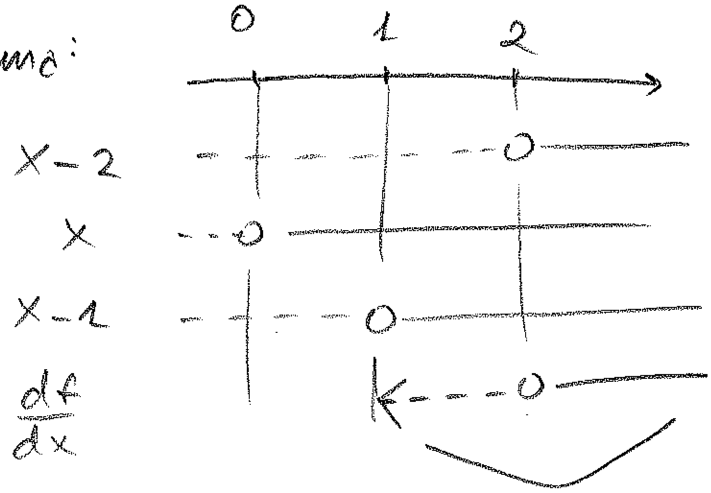
d) $f(x) = \ln u(x)$ der $u(x) = \frac{x^2}{x-1}$

$$\frac{df}{dx} = \frac{df}{du} \cdot \frac{du}{dx} = \frac{1}{u} \cdot \frac{du}{dx}$$

$$\frac{du}{dx} = \frac{2x\cdot(x-1) - x^2\cdot 1}{(x-1)^2} = \frac{2x^2-2x-x^2}{(x-1)^2} = \frac{x^2-2x}{(x-1)^2} = \frac{x(x-2)}{(x-1)^2}$$

$$\frac{df}{dx} = \frac{1}{\frac{x^2}{x-1}} \cdot \frac{x(x-2)}{(x-1)^2} = \frac{1\cdot \cancel{x}(x-2)}{\frac{x^{\cancel{2}}}{\cancel{x-1}}\cdot (x-1)^{\cancel{2}}} = \frac{x-2}{x(x-1)}$$

Forteiknskjema:



$(D_f = \langle 1, \rightarrow \rangle)$

- Botnpunkt for $x = 2$

$f(2) = \ln \frac{2^2}{2-1} = \ln 2^2 = 2\ln 2 \ (\approx 1{,}38)$

Botnpunkt: $\underline{(2,\ 2\ln 2)}$

e) $\dfrac{df}{dx} = \dfrac{x-2}{x(x-1)} = \dfrac{x-2}{x^2-x}$

$$f''(x) = \frac{d^2f}{dx^2} = \frac{1\cdot(x^2-x) - (x-2)\cdot(2x-1)}{(x^2-x)^2} =$$

$$\frac{x^2-x-(2x^2-x-4x+2)}{(x^2-x)^2} = \frac{x^2-x-2x^2+5x-2}{(x^2-x)^2} =$$

$$\frac{-x^2+4x-2}{(x^2-x)^2}$$

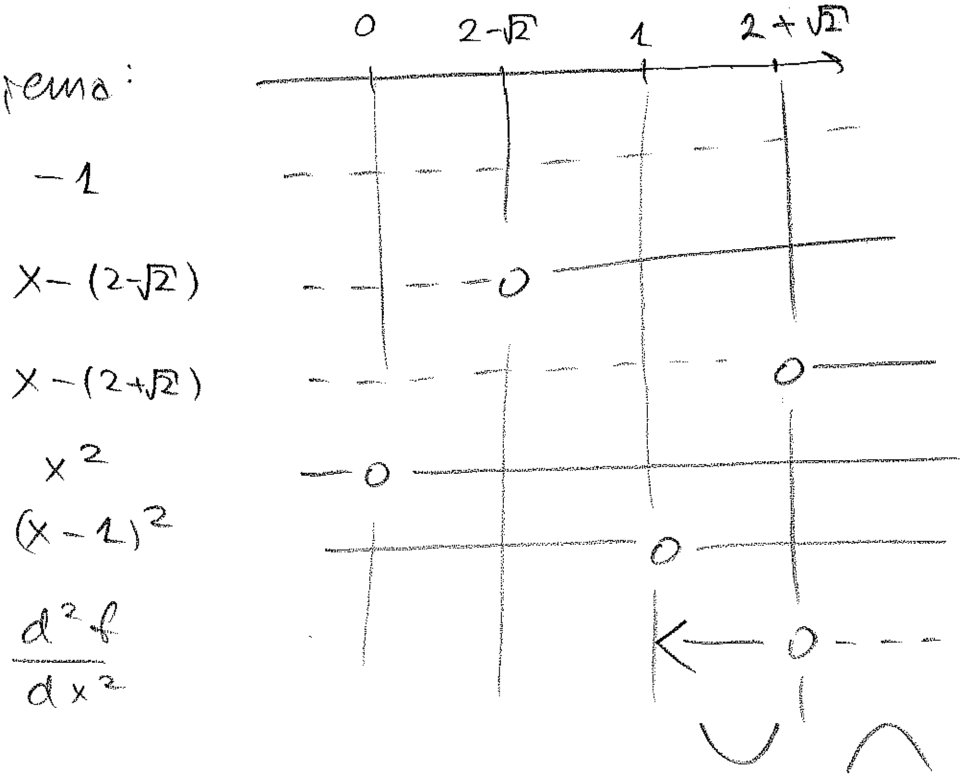
Faktoriserer:

$-x^2+4x-2=0$

$$x=\frac{-4\pm\sqrt{4^2-4\cdot(-1)\cdot(-2)}}{2\cdot(-1)}=\frac{-4\pm\sqrt{8}}{-2}=\frac{4\mp2\sqrt{2}}{2}=2\mp\sqrt{2}$$

$$\frac{d^2f}{dx^2}=\frac{-(x-(2-\sqrt{2}))\,(x-(2+\sqrt{2}))}{x^2(x-1)^2}$$

Forteiknskjema:



Vendepunkt for $x=2+\sqrt{2}$

$$f(2+\sqrt{2})=\ln\frac{(2+\sqrt{2})^2}{2+\sqrt{2}-1}=\ln\frac{4+4\sqrt{2}+2}{1+\sqrt{2}}=\ln\frac{6+4\sqrt{2}}{1+\sqrt{2}}$$

$(\approx 1{,}57)$

Vendepunkt: $\underline{\left(2+\sqrt{2},\ \ln\frac{6+4\sqrt{2}}{1+\sqrt{2}}\right)}$

$(\approx (3{,}41,\ 1{,}57))$

f) Enkel tabell:

x	1,5	2	3,4	7
f(x)	1,5	1,4	1,6	2,1