Eksamen 2008

Oppg. 1

a) $f(x) = 5x^2 + 3\sin x + e^x - e$

$f'(x) = 5 \cdot 2x + 3 \cdot \cos x + e^x - 0 =$

$\underline{10x + 3\cos x + e^x}$

b) $f(x) = \dfrac{2x+1}{(x+1)^2}$

$f'(x) = \dfrac{2 \cdot (x+1)^2 - (2x+1) \cdot 2(x+1) \cdot 1}{((x+1)^2)^2} =$

$\dfrac{(x+1)(2(x+1) - 2(2x+1))}{(x+1)^4} =$

$\dfrac{2x+2-4x-2}{(x+1)^3} = \underline{-\dfrac{2x}{(x+1)^3}}$

c) $f(x) = \cos(x^3 - 3x + 1)$

$f'(x) = -\sin(x^3 - 3x + 1) \cdot (3x^2 - 3) =$

$\underline{-3(x^2-1)\sin(x^3 - 3x + 1)}$

d) $f(x) = (x-3)^2 \cdot e^x$

$f'(x) = 2(x-3)e^x + (x-3)^2 \cdot e^x =$

$e^x(2x - 6 + x^2 - 6x + 9) =$

$\underline{e^x(x^2 - 4x + 3)}$

e) $f(x) = \sqrt[3]{x^2+4} = (x^2+4)^{1/3}$

$f'(x) = \frac{1}{3}(x^2+4)^{-2/3} \cdot 2x = \underline{\underline{\dfrac{2x}{3(x^2+4)^{2/3}}}}$

f) Stigningstal for tangenten: $y'(0)$

NB! Ikkje eksamensrelevant

$2y\frac{dy}{dx} + \frac{dy}{dx} = 4\cos x - 4x\sin x$

$\frac{dy}{dx}(2y+1) = 4(\cos x - x\sin x)$

$\frac{dy}{dx} = \frac{4(\cos x - x\sin x)}{2y+1}$

$x = y = 0$: $\frac{dy}{dx} = \frac{4(\cos 0 - 0 \cdot \sin 0)}{2 \cdot 0 + 1} = 4$

Likning: $y - y_0 = 4(x - x_0)$

$x_0 = y_0 = 0$

$\underline{\underline{y = 4x}}$

Oppg. 2

$\vec{u} = [2, 1, 1]$, $\vec{v} = [-2, 1, 3]$, $\vec{w} = [1, -1, 6]$

a) $|\vec{u}| = \sqrt{2^2 + 1^2 + 1^2} = \underline{\underline{\sqrt{6}}}$

$|\vec{v}| = \sqrt{(-2)^2 + 1^2 + 3^2} = \underline{\underline{\sqrt{14}}}$

$|\vec{w}| = \sqrt{1^2 + (-1)^2 + 6^2} = \underline{\underline{\sqrt{38}}}$

$\vec{u}\cdot\vec{v} = |\vec{u}|\cdot|\vec{v}|\cdot\cos\alpha$, der α er vinkelen mellom $\vec{u}$ og $\vec{v}$

$$\cos\alpha = \frac{\vec{u}\cdot\vec{v}}{|\vec{u}|\cdot|\vec{v}|} = \frac{[2,1,1]\cdot[-2,1,3]}{\sqrt{6}\cdot\sqrt{14}} = \frac{2\cdot(-2)+1\cdot1+1\cdot3}{\sqrt{6\cdot14}} = 0$$

$$\alpha = \underline{\underline{90^\circ}}$$

β er vinkelen mellom $\vec{u}$ og $\vec{w}$

$$\cos\beta = \frac{\vec{u}\cdot\vec{w}}{|\vec{u}|\cdot|\vec{w}|} = \frac{[2,1,1]\cdot[1,-1,6]}{\sqrt{6}\cdot\sqrt{38}} = \frac{2-1+6}{\sqrt{6\cdot38}} = \frac{7}{\sqrt{228}}$$

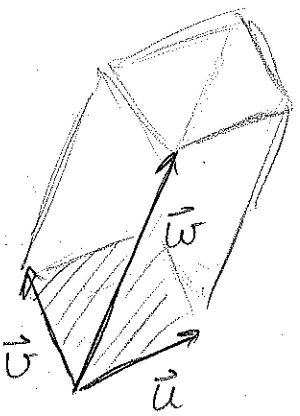
$$\beta = \cos^{-1}\frac{7}{\sqrt{228}} \approx \underline{\underline{62{,}4^\circ}}$$

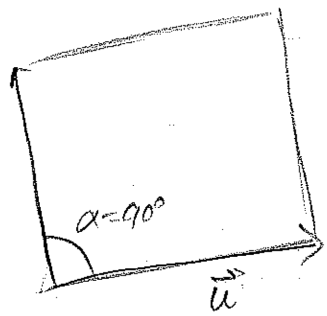
γ er vinkelen mellom $\vec{v}$ og $\vec{w}$

$$\cos\gamma = \frac{\vec{v}\cdot\vec{w}}{|\vec{v}|\cdot|\vec{w}|} = \frac{[-2,1,3]\cdot[1,-1,6]}{\sqrt{14}\cdot\sqrt{38}} = \frac{-2-1+18}{\sqrt{14\cdot38}} = \frac{15}{\sqrt{532}}$$

$$\gamma = \cos^{-1}\frac{15}{\sqrt{532}} = \underline{\underline{49{,}4^\circ}}$$

b)





Sideflata er eit rektangel

Areal $A = |\vec{u}|\cdot|\vec{v}| = \sqrt{6}\cdot\sqrt{14} = \underline{\underline{2\cdot\sqrt{21}}}$

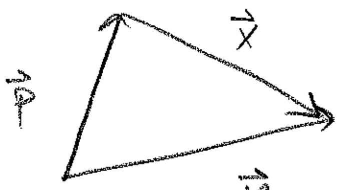
c) Volumet kan finnast ved å rekne ut determinanten

$$\begin{vmatrix} 2 & 1 & 1 \\ -2 & 1 & 3 \\ 1 & -1 & 6 \end{vmatrix} = 2\cdot1\cdot6 + 1\cdot3\cdot1 + 1\cdot(-2)\cdot(-1) - 2\cdot3\cdot(-1) - 1\cdot(-2)\cdot6 - 1 =$$

$12+3+2+6+12-1=34$

Volumet er absolutverdien av denne determinanten. Altså er volumet $\underline{34}$.

d)



$$\vec{n}=\vec{u}\times\vec{v}=\begin{vmatrix}\vec{e}_1 & \vec{e}_2 & \vec{e}_3\\ 2 & 1 & 1\\ -2 & 1 & 3\end{vmatrix}=[3-1,-2-6,2+2]=$$

$\underline{[2,-8,4]}$

$\vec{p}=\vec{u}+r\cdot\vec{v},\ r=1:$

$\vec{p}=\vec{u}+\vec{v}=[2,1,1]+[-2,1,3]=[2-2,\ 1+1,1+3]=\underline{[0,2,4]}$

Sidene i trekanten:

$|\vec{n}|=\sqrt{2^2+(-8)^2+4^2}=\sqrt{84}=\underline{2\sqrt{21}}$

$|\vec{p}|=\sqrt{2^2+4^2}=\sqrt{20}=\underline{2\sqrt{5}}$

$\vec{x}=-\vec{p}+\vec{n}=[0,-2,-4]+[2,-8,4]=[2,-10,0]$

$|\vec{x}|=\sqrt{2^2+(-10)^2}=\sqrt{104}=\underline{2\sqrt{26}}$

e) $\vec{n}\cdot\vec{p}=(\vec{u}\times\vec{v})\cdot(\vec{u}+r\cdot\vec{v})=(\vec{u}\times\vec{v})\cdot\vec{u}+r\,(\vec{u}\times\vec{v})\cdot\vec{v}$

$\vec{u}\times\vec{v}\perp\vec{u}$ og $\vec{u}\times\vec{v}\perp\vec{v}$

$\Rightarrow\ (\vec{u}\times\vec{v})\cdot\vec{u}=(\vec{u}\times\vec{v})\cdot\vec{v}=0\ \Rightarrow$

$\vec{n}\cdot\vec{p}=0+r\cdot 0=0$

Det betyr at anden er vinkelen

mellom $\vec{n}$ og $\vec{p}$ $90°$ eller så er $\vec{n}$ eller $\vec{p}$ lik $\vec{0}$.

$$\vec{n} = [0,2,4] \neq \vec{0}$$

$$\vec{p} = [2,1,1] + r[-2,1,3] = [2-2r, 1+r, 1+3r]$$

Kan $\vec{p}$ vere $\vec{0}$? $\vec{p} = \vec{0} \Leftrightarrow \begin{cases} 2-2r=0 \\ 1+r=0 \\ 1+3r=0 \end{cases} \Leftrightarrow \begin{cases} r=1 \\ r=-1 \\ r=-\frac{1}{3} \end{cases}$

Ser: $\vec{p} \neq \vec{0}$

Difor: $\vec{n} \perp \vec{p} \Leftrightarrow$ trekanten er rettvinkla for alle verdiar av r.

Oppg. 3

a) $\int (3x^2 + 5\cos x)\,dx = \underline{x^3 + 5\sin x + C}$

b) $\int \left(14e^{2x} + \frac{1}{1-x}\right)dx = 14 \cdot \frac{1}{2}e^{2x} + \frac{1}{-1}\ln|1-x| + C =$

$\underline{\underline{7e^{2x} - \ln|x-1| + C}}$

c) $\int_{1/3}^{1} \frac{3x+1}{3x^2+2x}\,dx$

Faktorisering av nemnaren: $3x^2+2x = x(3x+2)$

Delbrøksoppspalting:

$$\frac{3x+1}{3x^2+2x} = \frac{A}{x} + \frac{B}{3x+2}$$

$$3x+1 = A(3x+2) + Bx$$

$x=0$: $\quad 3\cdot 0+1 = A\cdot(3\cdot 0+2) + B\cdot 0$

$$2A = 1$$

Alternativt: Variabelskifte

$u = 3x^2+2x$, $\frac{du}{dx} = 6x+2 = 2(3x+1)$

$dx = \frac{1}{2(3x+1)}\,du$

$u(\frac{1}{3}) = 1$, $u(1) = 5$

$\int_{1/3}^{1} \frac{3x+1}{3x^2+2x}\,dx = \int_1^5 \frac{3x+1}{u} \cdot \frac{du}{2(3x+1)} =$

$\frac{1}{2}\int_1^5 \frac{1}{u}\,du = [\ln|u|]_1^5 = \frac{1}{2}(\ln 5 - \ln 1)$

$= \underline{\ln\sqrt{5}}$

$A = \frac{1}{2}$

$x = -\frac{2}{3}$: $\quad 3\cdot(-\frac{2}{3})+1 = A\cdot 0 + B\cdot(-\frac{2}{3})$

$-\frac{2}{3}B = -1$

$B = +\frac{3}{2}$

$$\Rightarrow \frac{3x+1}{3x^2+2x} = \frac{1/2}{x} + \frac{3/2}{3x+2}$$

$$\int_{1/3}^{1} \frac{3x+1}{3x^2+2x}\,dx = \frac{1}{2}\int_{1/3}^{1}\frac{1}{x}\,dx + \frac{3}{2}\int_{1/3}^{1}\frac{1}{3x+2}\,dx =$$

$$\frac{1}{2}\left[\ln|x|\right]_{1/3}^{1} + \frac{3}{2}\left[\frac{1}{3}\ln|3x+2|\right]_{1/3}^{1} =$$

$$\frac{1}{2}\left(\ln 1 - \ln\frac{1}{3}\right) + \frac{1}{2}\left(\ln 5 - \ln 3\right) =$$

$$\frac{1}{2}\left(0 + \ln 3 + \ln 5 - \ln 3\right) = \frac{1}{2}\ln 5 = \underline{\underline{\ln\sqrt{5}}}$$

d) $\int_0^{\pi/4} 2\sin^2 x\,dx$

Delvis integrasjon:

$u = 2\sin x, \quad v' = \sin x$

$u' = 2\cos x, \quad v = -\cos x$

$$\int_0^{\pi/4} 2\sin^2 x\,dx = \left[2\sin x\cdot(-\cos x)\right]_0^{\pi/4} - \int_0^{\pi/4} 2\cos x\cdot(-\cos x)\,dx =$$

$$-2\left[\sin x\cdot\cos x\right]_0^{\pi/4} + 2\int_0^{\pi/4}\cos^2 x\,dx =$$

$$-2\left(\sin\frac{\pi}{4}\cdot\cos\frac{\pi}{4} - 0\right) + 2\int_0^{\pi/4}(1-\sin^2 x)\,dx =$$

$$-2\,\frac{1}{\sqrt{2}}\cdot\frac{1}{\sqrt{2}} + 2\cdot\left[x\right]_0^{\pi/4} - \int_0^{\pi/4} 2\sin^2 x\,dx =$$

$$-1 + \frac{\pi}{2} - \int_0^{\pi/4} 2\sin^2 x\,dx$$

$$\int_0^{\pi/4} 2\sin^2 x\,dx + \int_0^{\pi/4} 2\sin^2 x\,dx = \frac{\pi}{2} - 1$$

$$\int_0^{\pi/4} 2\sin^2 x\,dx = \frac{1}{2}\left(\frac{\pi}{2}-1\right) = \underline{\underline{\frac{\pi}{4}-\frac{1}{2}}}$$

Alternativt:

$$\cos(2x) = \cos^2 x - \sin^2 x$$

$$\cos^2 x + \sin^2 x = 1$$

$$\cos(2x) = 1 - \sin^2 x - \sin^2 x = 1 - 2\sin^2 x$$

$$2\sin^2 x = 1 - \cos(2x)$$

$$\int_0^{\pi/4} 2\sin^2 x\,dx = \int_0^{\pi/4} (1-\cos(2x))dx =$$

$$\left[x - \frac{1}{2}\sin(2x)\right]_0^{\pi/4} = \frac{\pi}{4} - \frac{1}{2}\cdot\left(\sin(2\tfrac{\pi}{4}) - \sin 0\right) = \underline{\underline{\frac{\pi}{4}-\frac{1}{2}}}$$

e) $$3y' + y^2\cos x = 0$$

NB! Ikke eksamens-relevant!

$$3\frac{dy}{dx} = -y^2\cos x$$

$$3\frac{dy}{y^2} = -\cos x\,dx$$

$$3\int \frac{1}{y^2}dy = -\int \cos x\,dx$$

$$3\cdot\frac{1}{-2+1}y^{-2+1} = -\sin x + C$$

$$-\frac{3}{y} = -\sin x + C$$

$$\underline{\underline{y = \frac{3}{\sin x + C}}}$$

Kontroll:

$$y'(x) = \left(3(\sin x + C')^{-1}\right)' =$$

$$3\cdot(-1)\,(\sin x + C')^{-2}\cdot\cos x = -\frac{3\cos x}{(\sin x + C')^2}$$

$$3y' + y^2\cos x = 3\cdot\left(-\frac{3\cos x}{(\sin x + C')^2}\right) +$$

$$\left(\frac{3}{\sin x + C'}\right)^2\cdot\cos x = -\frac{9\cos x}{(\sin x + C')^2} + \frac{9\cos x}{(\sin x + C')^2} = 0$$

Vi ser at differensial-likninga er oppfyld.

f) $\int \sin^3 x\,dx = \int \sin x\cdot\sin^2 x\,dx =$

$\int \sin x\,(1-\cos^2 x)\,dx = \int \sin x\,dx - \int \sin x\cos^2 x\,dx$

$\int \sin x\,dx = -\cos x + C_1'$

$\int \sin x\cos^2 x\,dx$

Variabelskifte:

$u(x) = \cos x$

$\frac{du}{dx} = -\sin x$

$dx = -\frac{1}{\sin x}\,du$

$\int \sin x\cos^2 x\,dx = \int \sin x\cdot u^2\cdot\left(-\frac{1}{\sin x}\right)du =$

$-\int u^2\,du = -\frac{1}{3}u^3 + C_2' = -\frac{1}{3}\cos^3 x + C_2'$

$$\underline{\int \sin^3 x\,dx = -\cos x - \frac{1}{3}\cos^3 x + C'}$$

Oppg. 4

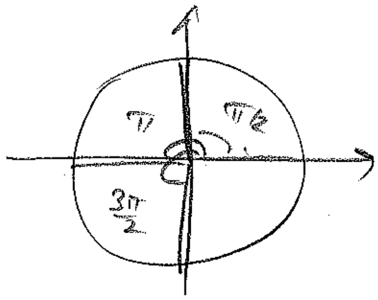
$$f(x) = \sin^2 x - \cos x - 1 , \qquad 0 < x < 2\pi$$

a) $f(x) = 0$

$$\sin^2 x - \cos x - 1 = 0$$

$$1 - \cos^2 x - \cos x - 1 = 0$$

$$-\cos x(\cos x + 1) = 0$$



$$-\cos x = 0 \quad \vee \quad \cos x + 1 = 0$$

$$\cos x = 0 \quad \vee \quad \cos x = -1$$

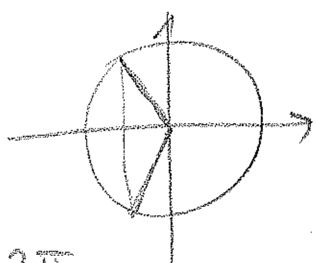
$$\underline{x = \frac{\pi}{2} \quad \vee \quad x = \frac{3\pi}{2} \quad \vee \quad x = \pi}$$

b) $f'(x) = 2\sin x \cdot \cos x - (-\sin x) - 0 = \underline{\underline{2\sin x \cdot \cos x + \sin x}}$

c) $f'(x) = \sin x(2\cos x + 1)$

Nullpunkt for $\sin x$: $x = \pi$

Nullpunkt for $2\cos x + 1$:



$$2\cos x + 1 = 0$$

$$\cos x = -\frac{1}{2}$$

$$x = \cos^{-1}\left(-\frac{1}{2}\right) = \pi - \cos^{-1}\frac{1}{2} = \pi - \frac{\pi}{3} = \frac{2\pi}{3} \quad \vee$$

$$x = 2\pi - \frac{2\pi}{3} = \frac{4\pi}{3}$$

Ser: $\sin x < 0$ når $0 < x < \pi$

$\sin x > 0$ når $\pi < x < 2\pi$

$2\cos x + 1 > 0 \iff \cos x > -\frac{1}{2}$

Oppfyld for $\quad 0 < x < \frac{2\pi}{3}$ eller $\frac{4\pi}{3} < x < 2\pi$

$\cos x < -\frac{1}{2}$ når $\quad \frac{2\pi}{3} < x < \frac{4\pi}{3}$

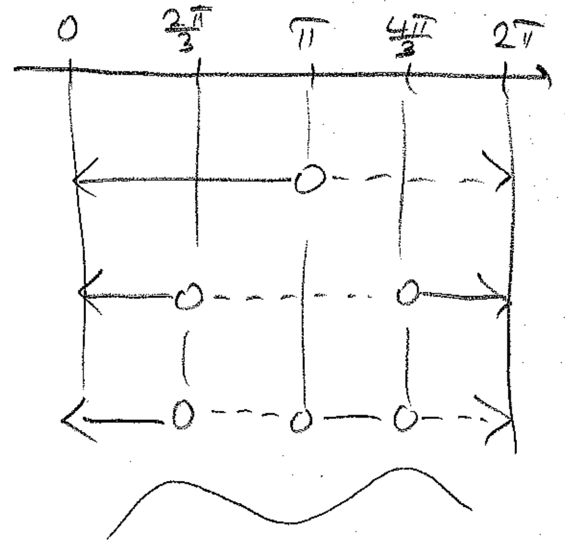
Forteiknskjema:

sin x

2cosx+1

f'(x)

f



$$f(x) = -\cos^2 x - \cos x$$

$$f\left(\frac{2\pi}{3}\right) = -\cos^2 \frac{2\pi}{3} - \cos\frac{2\pi}{3} = -\left(-\frac{1}{2}\right)^2 - \left(-\frac{1}{2}\right) = \frac{1}{4}$$

$$f(\pi) = -\cos^2\pi - \cos\pi = -(-1)^2 - (-1) = 0$$

$$f\left(\frac{4\pi}{3}\right) = -\cos^2\left(\frac{4\pi}{3}\right) - \cos\frac{4\pi}{3} = -(-1/2)^2 - \left(-\frac{1}{2}\right) = \frac{1}{4}$$

Toppunkt: $\underline{\left(\frac{2\pi}{3}, \frac{1}{4}\right) \text{ og } \left(\frac{4\pi}{3}, \frac{1}{4}\right)}$

Botnpunkt: $\underline{(\pi, 0)}$

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d) $f''(x) = (2\sin x\cos x + \sin x)' = (\sin(2x) + \sin x)' =$

$\underline{2\cos(2x) + \cos x}$

e) $f''(x) = 2\cos(2x) + \cos x =$

$2\cos^2 x - 2\sin^2 x + \cos x =$

$2\cos^2 x - 2(1 - \cos^2 x) + \cos x =$

$2\cos^2 x - 2 + 2\cos^2 x + \cos x =$

$4\cos^2 x + \cos x - 2$

$f''(x) = 0$

$4\cos^2 x + \cos x - 2 = 0$

$$\cos x = \frac{-1 \pm \sqrt{1 - 4 \cdot 4 \cdot (-2)}}{2 \cdot 4} = \frac{-1 \pm \sqrt{33}}{8}$$

$$\cos x = \frac{-1 - \sqrt{33}}{8} \quad \vee \quad \cos x = \frac{-1 + \sqrt{33}}{8}$$

$$x = \cos^{-1}\left(-\frac{1+\sqrt{33}}{8}\right) \vee \quad x = 2\pi - \cos^{-1}\left(-\frac{1+\sqrt{33}}{8}\right) \vee$$

$$x = \cos^{-1} \frac{\sqrt{33}-1}{8} \quad \vee \quad x = 2\pi - \cos^{-1} \frac{\sqrt{33}-1}{8}$$

$x \approx 2{,}57 \quad \vee \quad x = 3{,}71 \quad \vee \quad x \approx 0{,}94 \quad \vee$

$x \approx 5{,}35$

$$f''(x) = 4\left(\cos x + \frac{1+\sqrt{33}}{8}\right)\left(\cos x - \frac{\sqrt{33}-1}{8}\right)$$

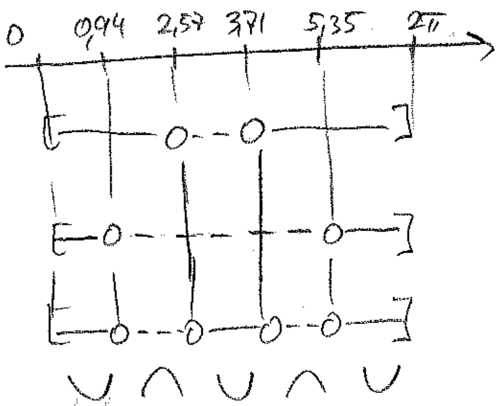
Forteilenskjema

$4\left(\cos x + \frac{1+\sqrt{33}}{8}\right)$

$\cos x - \frac{\sqrt{33}-1}{8}$

f''

f



$f(0{,}94) \approx -0{,}94$

$f(2{,}57) \approx 0{,}13$

$f(3{,}71) \approx 0{,}13$

$f(5{,}34) \approx -0{,}94$

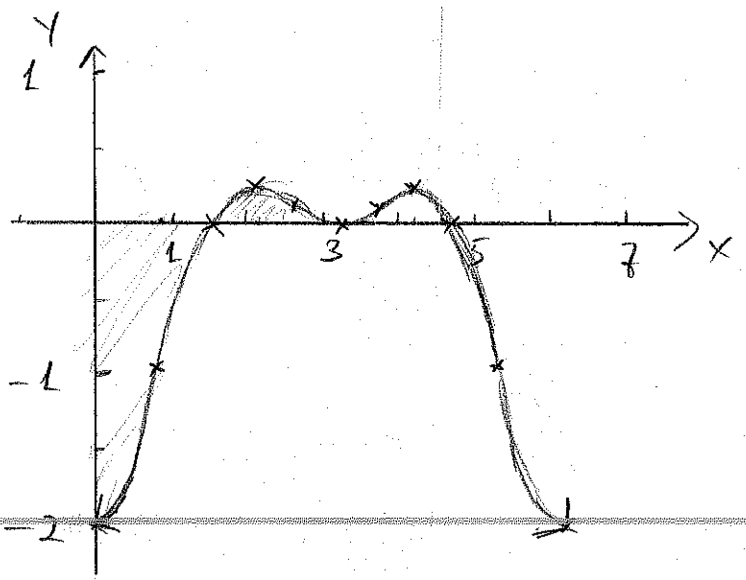
Vendepunkt: $(0{,}94,\ -0{,}94)$, $(2{,}57,\ 0{,}13)$, $(3{,}71,\ 0{,}13)$ og $(5{,}34,\ -0{,}94)$.

Tabell:

x	0,94	1,57	2,09	2,57	3,14	3,71	4,19	4,71	5,34
$f(x)$	−0,94	0	0,25	0,13	0	0,13	0,25	0	−0,94

$\lim_{x \to 0^+} f(x) = -2$

$\lim_{x \to 2\pi^-} f(x) = -2$



f) Area: $-\int_0^{\pi/2} f(x)\,dx + \int_{\pi/2}^{\pi} f(x)\,dx =$

$$f(x) = -\cos^2 x - \cos x$$

$$\int f(x)\,dx = -\int \cos^2 x\,dx - \int \cos x\,dx =$$

$$-\int \cos^2 x\,dx - \sin x + C_1$$

$\underline{\int \cos^2 x\,dx}$

1) $\cos(2x) = \cos^2 x - \sin^2 x$

$\cos^2 x + \sin^2 x = 1, \quad \sin^2 x = 1 - \cos^2 x$

$$\cos(2x) = \cos^2 x - (1 - \cos^2 x) = 2\cos^2 x - 1$$

$$\cos^2 x = \frac{1}{2}(\cos(2x) + 1)$$

$$\int \cos^2 x\,dx = \frac{1}{2}\int (\cos(2x) + 1)\,dx =$$

$$\frac{1}{2}\left(\frac{1}{2}\sin(2x) + x\right) + C_2' = \frac{1}{4}\sin(2x) + \frac{1}{2}x + C_2$$

2) Delvis:

$u = \cos x, \quad v' = \cos x$

$u' = -\sin x, \quad v = \sin x$

$$\int \cos^2 x\,dx = \cos x \cdot \sin x - \int (-\sin x) \cdot \sin x\,dx =$$

$$\cos x \sin x + \int \sin^2 x\,dx = \cos x \sin x + \int (1 - \cos^2 x)\,dx =$$

$$\cos x \sin x + x - \int \cos^2 x\,dx$$

$$2\int \cos^2 x\,dx = \cos x \sin x + x + C_3'$$

$$\int \cos^2 x\,dx = \frac{1}{2}\cos x \sin x + \frac{1}{2}x + C_3' = \frac{1}{4}\sin(2x) + \frac{1}{2}x + C_3$$

$\int f(x)\,dx = -\int \cos^2 x\,dx - \sin x + C_2' =$

$-\frac{1}{4}\sin(2x) - \frac{1}{2}x - \sin x + C'$

$-\int_0^{\pi/2} f(x)\,dx + \int_{\pi/2}^{\pi} f(x)\,dx =$

$-\left[-\frac{1}{4}\sin(2x) - \sin x - \frac{1}{2}x\right]_0^{\pi/2} + \left[-\frac{1}{4}\sin(2x) - \sin x - \frac{1}{2}x\right]_{\pi/2}^{\pi} =$

$\left[\frac{1}{4}\sin(2x) + \sin x + \frac{1}{2}x\right]_0^{\pi/2} - \left[\frac{1}{4}\sin(2x) + \sin x + \frac{1}{2}x\right]_{\pi/2}^{\pi} =$

$\frac{1}{4}\sin\pi + \sin\frac{\pi}{2} + \frac{1}{2}\cdot\frac{\pi}{2} - 0 - \Big(\frac{1}{4}\sin(2\pi) + \sin\pi + \frac{1}{2}\pi -$

$\big(\frac{1}{4}\sin\pi + \sin\frac{\pi}{2} + \frac{1}{2}\cdot\frac{\pi}{2}\big)\Big) = 1 + \frac{\pi}{4} - \Big(\frac{\pi}{2} - \big(0 + 1 + \frac{\pi}{4}\big)\Big) =$

$1 + \frac{\pi}{4} - \frac{\pi}{2} + 1 + \frac{\pi}{4} = \underline{\underline{2}}$

g) $g(x) = \sin^2 x - \sin x$

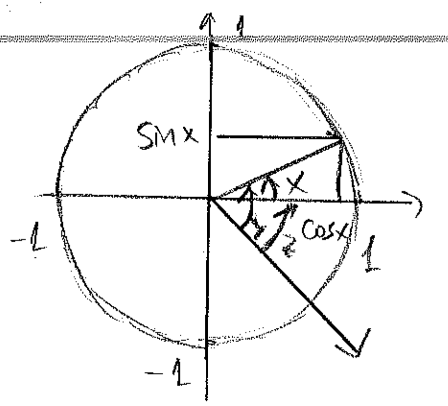
Skjæring: $g(x) = f(x)$

$\sin^2 x - \sin x = \sin^2 - \cos x - 1$

$\cos x - \sin x = -1$

NB!
-Ikke pensum

$[1, -1] \cdot [\cos x, \sin x] = -1$



$\vec{u} \cdot \vec{v} = -1,\quad \vec{u} = [1, -1],\quad \vec{v} = [\cos x, \sin x]$

$|\vec{u}| \cdot |\vec{v}| \cdot \cos y = -1$

$|\vec{u}| = \sqrt{1^2 + (-1)^2} = \sqrt{2}$

$|\vec{v}| = \sqrt{\cos^2 x + \sin^2 x} = 1$

$\sqrt{2}\cos y = -1$

$\cos y = -\frac{1}{\sqrt{2}}$

$\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right) = \pi - \cos^{-1}\frac{1}{\sqrt{2}} = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$

$y = \frac{3\pi}{4} \vee y = 2\pi - \frac{3\pi}{4} = \frac{5\pi}{4}$

$y = z + x$

$z = \frac{\pi}{4}$

$x = y - \frac{\pi}{4}$

$x = \frac{3\pi}{4} - \frac{\pi}{4} = \underline{\underline{\frac{\pi}{2}}} \vee x = \frac{5\pi}{4} - \frac{\pi}{4} = \underline{\underline{\pi}}$