Førelesing 18/1

① Eksempel

– Droppar eksempelet frå i går – enn så lenge

Nytt:

Gitt funksjonen

$$f(x) = \frac{1}{4}x^2 - 1$$

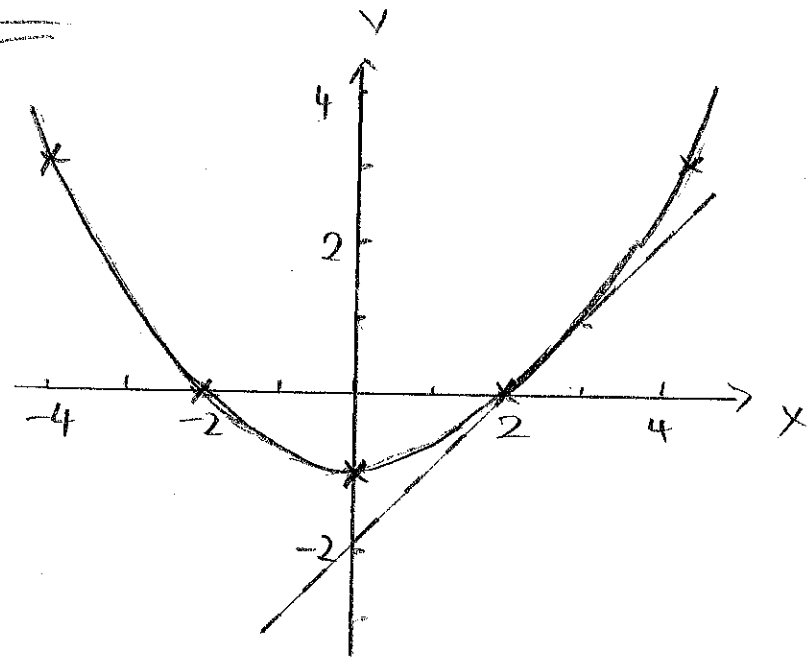
a) Finn gjennomsnittleg vekstfart i intervallet $[2, 3]$

b) ——— " ——— $[2, 2{,}5]$

c) ——— " ——— $[2, 2{,}1]$

d) Finn vekstfarten i punktet $x = 2$

e) Finn klikninga for tangenten i punktet $(2, f(2))$.



a) Gjennomsnittleg stigning: $\frac{f(3) - f(2)}{3 - 2}$

$f(2) = \frac{1}{4} 2^2 - 1 = 0$

$f(3) = \frac{1}{4} \cdot 3^2 - 1 = \frac{5}{4}$

Vi får: $\dfrac{\frac{5}{4}-0}{3-2}=\underline{\underline{\dfrac{5}{4}}}$

b) $f(2{,}5)=0{,}5625$

$$\frac{f(2{,}5)-f(2)}{2{,}5-2}=\frac{0{,}5625-0}{0{,}5}=\underline{\underline{1{,}125}}$$

c) $f(2{,}1)=0{,}1025$

$$\frac{f(2{,}1)-f(2)}{2{,}1-2}=\frac{0{,}1025-0}{0{,}1}=\underline{\underline{1{,}025}}$$

d) $f(2{,}01)=0{,}010025$

$$\frac{f(2{,}01)-f(2)}{2{,}01-2}=\dots=1{,}0025$$

$\vdots$

Grenseverdi:

$$\lim_{h\to 0}\frac{f(2+h)-f(2)}{h}$$

$$f(2+h)=\frac{1}{4}(2+h)^2-1=\frac{1}{4}(4+4h+h^2)-1=$$
$$1+h+\frac{1}{4}h^2-1=h+\frac{1}{4}h^2$$

$$\lim_{h\to 0}\frac{f(2+h)-f(2)}{h}=\lim_{h\to 0}\frac{h+\frac{1}{4}h^2-0}{h}=$$

$$\lim_{h\to 0}\frac{h\left(1+\frac{1}{4}h\right)}{h}=\lim_{h\to 0}\left(1+\frac{1}{4}h\right)=\underline{\underline{1}}$$

e) $\boxed{?}$ Stigningstallet for tangenten? $a=\underline{\underline{1}}$

Punktet på linja: $(2,0)$

Likning: $y-y_1=a(x-x_1)$

$y-0=1\cdot(x-2)$

$\underline{\underline{y=x-2}}$

Den deriverte av ein funksjon (8.7)

– Fortset med same funksjon: $f(x) = \frac{1}{4}x^2 - 1$
Kva er vekstfarten for ein vilkårleg x?

Som før:

$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\frac{1}{4}(x+h)^2 - 1 - (\frac{1}{4}x^2 - 1)}{h} =$$

$$\lim_{h\to 0} \frac{\frac{1}{4}(x^2+2hx+h^2) - 1 - \frac{1}{4}x^2 + 1}{h} =$$

$$\lim_{h\to 0} \frac{\frac{1}{4}x^2 + \frac{1}{2}hx + \frac{1}{4}h^2 - \frac{1}{4}x^2}{h} = \lim_{h\to 0} \frac{\frac{1}{2}hx + \frac{1}{4}h^2}{h} =$$

$$\lim_{h\to 0} \frac{\cancel{h}(\frac{1}{2}x + \frac{1}{4}h)}{\cancel{h}} = \lim_{h\to 0} (\frac{1}{2}x + \frac{1}{4}h) = \frac{1}{2}x + \frac{1}{4}\cdot 0 = \frac{1}{2}x$$

Dette kallar vi den deriverte til $f(x)$.
Vi skriv:

$$f'(x) = \lim_{x\to 0} \frac{f(x+h)-f(x)}{h} \quad \text{(Definisjon)}$$

I vårt eksempel: $f'(x) = \frac{1}{2}x$

$\boxed{?}$ Kva er vekstfarten for $x=2$, $x=4$, $x=1$, $x=0$?

$f'(a)$ gir vekstfarten for $x=a$
Dette er det same som stigningstalet til tangenten i punktet $(a, f(a))$.

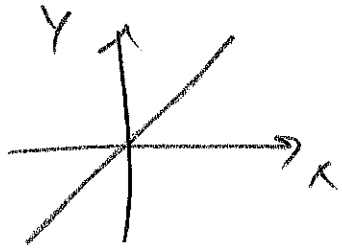
Finn den deriverte til desse funksjonane:

$a(x) = x$

$b(x) = x^2$

$c(x) = x^3$

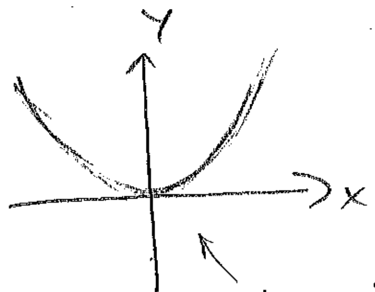
$\boxed{?}$ $a(x) = x$: Kva venter vi oss?



$$a'(x) = \lim_{h\to 0} \frac{a(x+h)-a(x)}{h} =$$

$$\lim_{h\to 0} \frac{x+h-x}{h} = \lim_{h\to 0} \frac{h}{h} = \lim_{h\to 0} 1 = \underline{\underline{1}}$$

$$x' = 1$$



$b'(0) = ?$

$\boxed{?}$

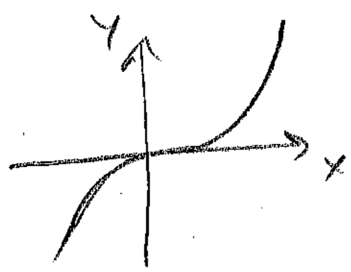
$$b'(x) = \lim_{h\to 0} \frac{b(x+h)-b(x)}{h} =$$

$$\lim_{h\to 0} \frac{(x+h)^2 - x^2}{h} = \lim_{h\to 0} \frac{x^2+2hx+h^2-x^2}{h} =$$

$$\lim_{h\to 0} \frac{2hx+h^2}{h} = \lim_{h\to 0} \frac{\cancel{h}(2x+h)}{\cancel{h}} = \lim_{h\to 0} (2x+h) =$$

$$2x + 0 = \underline{\underline{2x}}$$

$$(x^2)' = 2x$$



$$c'(x) = \lim_{h\to 0} \frac{c(x+h)-c(x)}{h} =$$

$$\lim_{h\to 0} \frac{(x+h)^3 - x^3}{h}$$

$$(x+h)^3 = (x+h)\cdot(x+h)^2 = (x+h)(x^2+2hx+h^2) =$$

$$x^3 + 2hx^2 + h^2x + hx^2 + 2h^2x + h^3 = x^3 + 3hx^2 + 3h^2x + h^3$$

$$c'(x) = \lim_{h\to 0} \frac{x^3+3hx^2+3h^2x+h^3-x^3}{h} = \lim_{h\to 0} \frac{3hx^2+3h^2x+h^3}{h} =$$

$$\lim_{h\to 0} \frac{h(3x^2+3hx+h^2)}{h} = \lim_{h\to 0}(3x^2+3hx+h^2) =$$

$$3x^2+3\cdot 0\cdot x+0^2 = \underline{\underline{3x^2}}$$

$$(x^3)' = 3x^2$$

$\boxed{?}$ Ser vi eit mønster?
Kva er $(x^4)'$?

$(x)' = 1$

$(x^2)' = 2x$

$(x^3)' = 3x^2$

$(x^4)' = 4x^3$

$(x^5)' = 5x^4$

$\vdots$

$\boxed{?}$ $(x^n)' = n x^{n-1}$

$\boxed{?}$ $(x^{54})' = 54\, x^{53}$

Vender tilbake til funksjonen frå i går:

$$f(x) = 2x^3 - 16x^2 + 40x - 28 \quad (\to \text{GeoGebra?})$$

– Skulle finne stigningstalet til tangenten i punktet $(1, f(1))$.

$\boxed{?}$ Skal finne $f'(1)$.

– Kan like godt finne $f'(x)$ og så sette inn $x = 1$.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} =$$

$$\lim_{h \to 0} \frac{2(x+h)^3 - 16(x+h)^2 + 40(x+h) - 28 - (2x^3 - 16x^2 + 40x - 28)}{h} =$$

$$\lim_{h \to 0} \left(\frac{2(x+h)^3 - 2x^3}{h} + \frac{-16(x+h)^2 - (-16x^2)}{h} + \frac{40(x+h) - 40x}{h} + \frac{-28 - (-28)}{h} \right) \overset{\boxed{?}}{\underset{S.266}{=}}$$

$$2 \lim_{h \to 0} \frac{(x+h)^3 - x^3}{h} - 16 \lim_{x \to 0} \frac{(x+h)^2 - x^2}{h} + 40 \lim_{x \to 0} \frac{(x+h) - x}{h} - 0) \overset{\boxed{?}}{=}$$

$$2 \cdot (x^3)' - 16 \cdot (x^2)' + 40 \cdot x' - 0 =$$

$$2 \cdot 3x^2 - 16 \cdot 2x + 40 \cdot 1 = 6x^2 - 32x + 40$$

$$f'(1) = 6 - 32 + 40 = \underline{\underline{14}}$$

Poeng:

$$f(x) = 2x^3 - 16x^2 + 40x - 28$$

$$f'(x) = 2 \cdot (x^3)' - 16(x^2)' + 40x' - 28'$$