Førelesing 14/2

① Minne om oblig
Kopiere opp fleire oppgåver?
Ikkje undervising torsdag

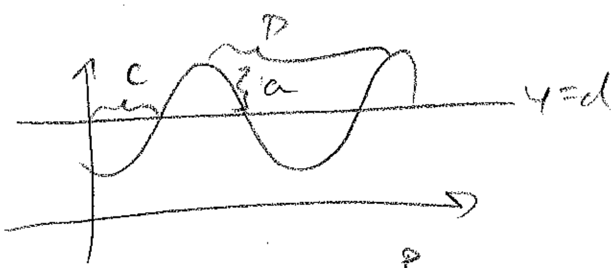
② Rette opp feil / presisere rolla til forteiknet til „a"

Skreiv: „$\cos x = \sin(x - \frac{\pi}{2})$"

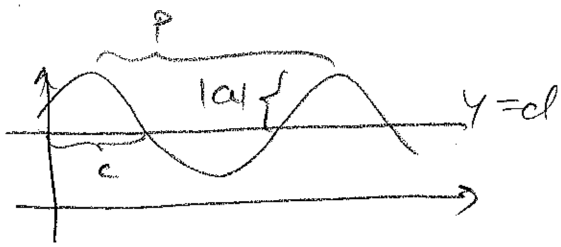
– Dette er feil
Beklagar.

$a\sin(k(x-c)) + d$

$a > 0$

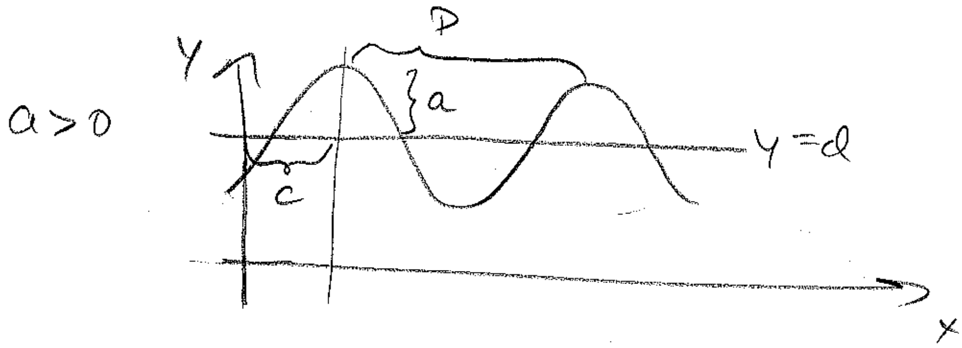


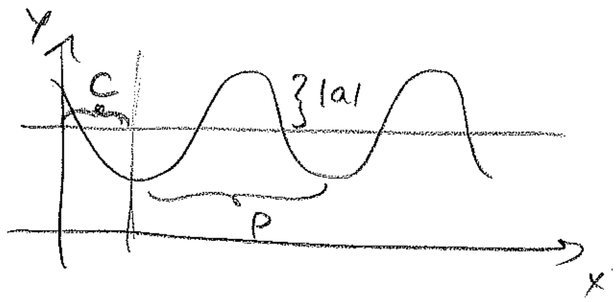
$a < 0$



$a \cos(k(x-c)) + d$

$a > 0$





For ein funksjon av typen $a \cos(k(x-c)) + d$

$|a|$ er amplituden

$k = \frac{2\pi}{p}$ der p er perioden

d: gir likevektslinja

c: $a > 0$: x-verdien til det fyrste topp-punktet til høgre for y-aksen

$a < 0$: x-verdien til det fyrste botn-punktet til høgre for y-aksen.

③ Derivasjon av trigonometriske funksjonar (10.8)

Brukar GeoGebra og teiknar $\sin x$

Også: $y = x$

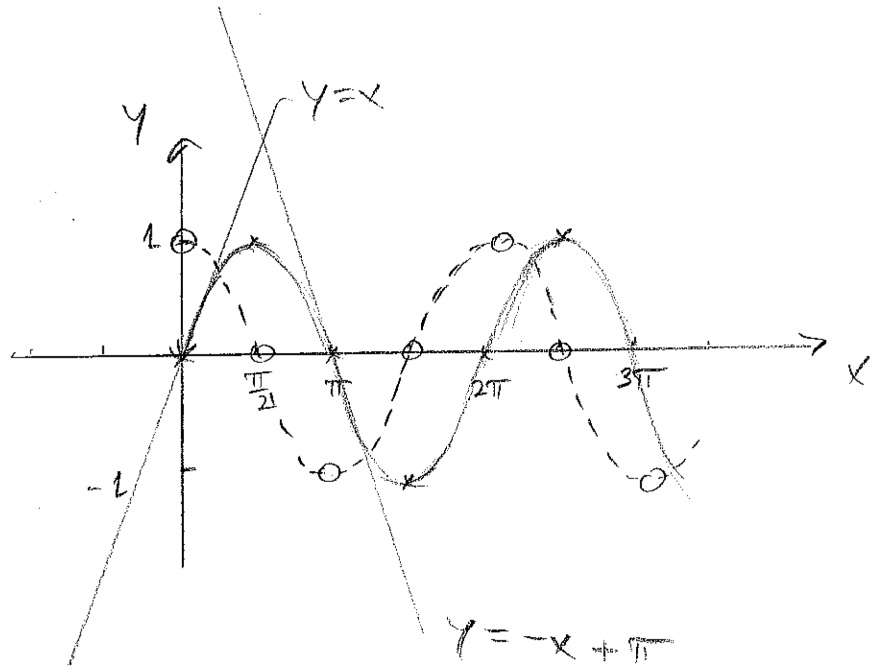
$\boxed{?}$ Hugar vi at $\sin(-0{,}381) = -0{,}391$

altså: $\sin x \approx x$

$\boxed{?}$ Når gjeld dette?

Set opp tabell:

x	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π	$\frac{5\pi}{2}$
$\sin x$	0	1	0	-1	0	1
$(\sin x)'$	1	0	-1	0	1	0
		↑ Toppunkt		↑ Botnpunkt		



Kan sjå ut til at $(\sin x)' = \cos x$

Stemmer dette? Ja.

Kva med den deriverte av $\cos x$?

Har sett: $\cos x = -\sin(x - \pi/2)$

$(\cos x)' = (-\sin(x-\frac{\pi}{2}))' = -(\sin(\underbrace{x-\frac{\pi}{2}}_{u}))' =$

$\left(-\cos u \cdot u'(x) =\right) - \cos(x-\frac{\pi}{2}) \cdot 1 =$

$-\left(\cos x \cdot \cos\frac{\pi}{2} + \sin x \cdot \sin\frac{\pi}{2}\right) =$

$-\left(\cos x \cdot 0 + \sin x \cdot 1\right) = -\sin x$

Altså:

$$\boxed{\begin{aligned}(\sin x)' &= \cos x \\ (\cos x)' &= -\sin x\end{aligned}}$$

④ Eksempel

Deriver desse funksjonane

$a(x) = \sin(x+2)$

$b(x) = \cos x^2$

$c(x) = x^2 \cdot \cos x$

$d(x) = \cos^2 x$

$e(x) = \tan x$

$a'(x) = \cos(x+2) \cdot (x+2)' = \cos(x+2) \cdot 1 = \underline{\cos(x+2)}$

$b'(x) = -\sin x^2 \cdot (x^2)' = \underline{-2x \sin x^2}$

$c'(x) = (x^2)' \cdot \cos x + x^2 \cdot (\cos x)' =$

$2x \cos x + x^2 \cdot (-\sin x) =$

$\underline{x\,(2\cos x - x \sin x)}$

$d(x) = \cos^2 x$

$\left.\begin{array}{l} u = \cos x \\ g(u) = u^2 \end{array}\right\} \Rightarrow d(x) = g(u(x))$

$d'(x) = g'(u) \cdot u'(x) = 2u \cdot (-\sin x) = \underline{-2\cos x \sin x}$

$= \underline{-\sin(2x)}$

$e(x) = \tan x = \dfrac{\sin x}{\cos x}$

$e'(x) = \dfrac{(\sin x)' \cdot \cos x - \sin x \cdot (\cos x)'}{(\cos x)^2} =$

$\dfrac{\cos x \cdot \cos x - \sin x \cdot (-\sin x)}{\cos^2 x} =$

$\dfrac{\cos^2 x + \sin^2 x}{\cos^2 x} = \underline{\dfrac{1}{\cos^2 x}}$

Alts$\overset{\circ}{a}$:

$\underline{(\tan x)' = \dfrac{1}{\cos^2 x}}$

5 Beviset for at $(\sin x)' = \cos x$

$$(\sin x)' = \lim_{h\to 0} \frac{\sin(x+h) - \sin x}{h} =$$

$$\lim_{h\to 0} \frac{\sin x \cos h + \cos x \sin h - \sin x}{h} =$$

$$\lim_{h\to 0} \frac{\cos x \sin h}{h} + \lim_{h\to 0} \frac{\sin x(\cos h - 1)}{h} =$$

$$\cos x \lim_{h\to 0} \frac{\sin h}{h} + \sin x \lim_{h\to 0} \frac{\cos h - 1}{h}$$

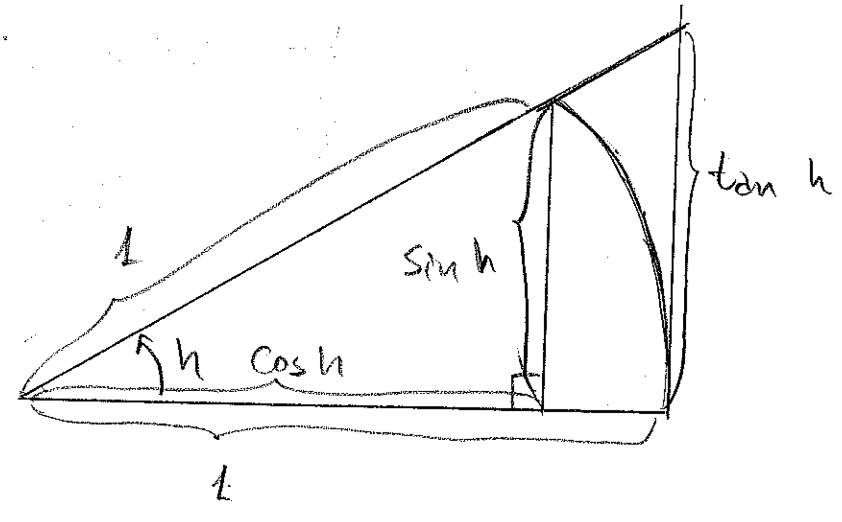
$\boxed{?}$ Hugsar vi denne?

Skal vise at a) $\lim_{h\to 0} \frac{\sin h}{h} = 1$ og at

b) $\lim_{h\to 0} \frac{\cos h - 1}{h} = 0$

Vi begynnar med den fyrste og antar at $h > 0$, $\lim_{h\to 0^+} \frac{\sin h}{h}$

a).



Areal av liten trekant: $t = \frac{1}{2} \cos h \cdot \sin h$

— " — stor " : $T = \frac{1}{2} \cdot 1 \cdot \tan h = \frac{1}{2} \tan h$

— " — sektor: $S = \frac{h}{2\pi} \cdot \pi \cdot 1^2 = \frac{1}{2} h$

Må ha:

$$t < S < T$$

$$\frac{1}{2} \cos h \sin h < \frac{1}{2} h < \frac{1}{2} \tan h$$

1) $$\frac{1}{2} h < \frac{1}{2} \tan h$$

$$h < \frac{\sin h}{\cos h}$$

$$\frac{\sin h}{h} > \cos h$$

2) $$\frac{1}{2} \cos h \sin h < \frac{1}{2} h$$

$$\frac{\cancel{\cos h} \sin h}{\cancel{\cos h}} < \frac{h}{\cos h}$$

$$\frac{\sin h}{h} < \frac{1}{\cos h}$$

Altså:

$$\cos h < \frac{\sin h}{h} < \frac{1}{\cos h}$$

$\boxed{?}$

$h \to 0 \Rightarrow \cos h \to 1^-$

$h \to 0 \Rightarrow \frac{1}{\cos h} \to 1^+$

Altså: $\lim_{h \to 0^+} \frac{\sin h}{h} = 1$

Kan visast også for $h < 0$

b) $$\lim_{h\to 0} \frac{\cos h - 1}{h} = \lim_{h\to 0} \frac{(\cos h - 1)(\cos h + 1)}{h(\cos h + 1)} =$$

$$\lim_{h\to 0} \frac{\cos^2 h - 1}{h(\cos h + 1)} = \lim_{h\to 0} \frac{-\sin^2 h}{h(\cos h + 1)} =$$

$$-\lim_{h\to 0} \frac{\sin h}{h} \cdot \lim_{h\to 0} \frac{\sin h}{\cos h + 1} =$$

$$-1 \cdot \frac{\sin 0}{\cos 0 + 1} = -1 \cdot \frac{0}{1} = 0$$

Vi får:

$$(\sin x)' = \cos x \cdot \lim_{h\to 0} \frac{\sin h}{h} + \sin x \lim_{h\to 0} \frac{\cos h - 1}{h} =$$

$$\cos x \cdot 1 + \sin x \cdot 0 = \cos x \qquad \text{(q.e.d.)}$$