Obligatoriske innlevering nr. 2

Oppg. 1

a) $a(x) = x\cos x$

$a'(x) = x'\cos x + x\cdot(\cos x)' =$

$\cos x + x\cdot(-\sin x) = \underline{\underline{\cos x - x\sin x}}$

$b(x) = \sin(\ln(\sqrt{x}+1))$

$u = \ln(\sqrt{x}+1), \quad b = \sin u$

$$\frac{db}{dx} = \frac{db}{du}\cdot\frac{du}{dx} = \cos u\cdot\frac{du}{dx}$$

$v = \sqrt{x}+1, \quad u = \ln v$

$$\frac{du}{dx} = \frac{du}{dv}\cdot\frac{dv}{dx} = \frac{1}{v}\cdot(\sqrt{x}+1)' =$$

$$\frac{1}{v}\cdot\frac{1}{2\sqrt{x}}$$

$$\frac{db}{dx} = \cos u\cdot\frac{1}{v}\cdot\frac{1}{2\sqrt{x}} = \underline{\underline{\frac{\cos(\ln(\sqrt{x}+1))}{2\sqrt{x}(\sqrt{x}+1)}}}$$

$$c(x) = \ln\frac{x}{x^2+1} = \ln x - \ln(x^2+1)$$

$$c'(x) = \frac{1}{x} - \frac{1}{x^2+1}\cdot(x^2+1)' = \underline{\underline{\frac{1}{x} - \frac{2x}{x^2+1}}}$$

$$\left(= \frac{x^2+1-2x^2}{x(x^2+1)} = \frac{1-x^2}{x(x^2+1)}\right)$$

b) $\lim_{t \to \pi} \frac{\sin^2 t}{t - \pi}$

– Ser at både teller og nemner går mot 0. Vi bruker L'Hôpitals regel:

$$\lim_{t \to \pi} \frac{\sin^2 t}{t-\pi} = \lim_{t \to \pi} \frac{(\sin^2 t)'}{(t-\pi)'} =$$

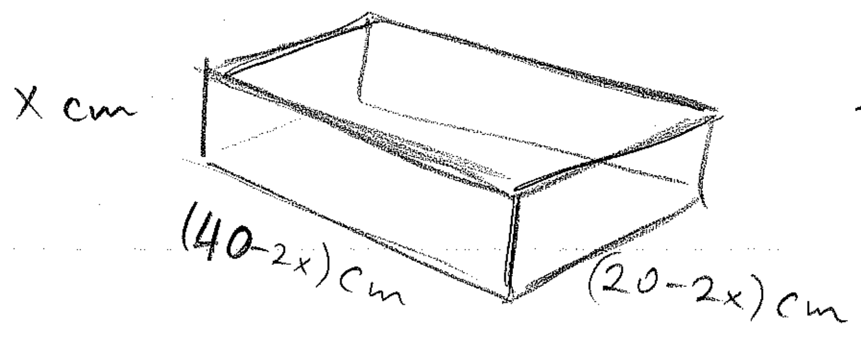
$$\lim_{t \to \pi} \frac{2 \sin t \cdot \cos t}{1 - 0} = \frac{2 \cdot \sin\pi \cdot \cos\pi}{1} = \underline{\underline{0}}$$

$$\lim_{x \to 0} \left(\frac{1}{x} - \frac{1}{x e^{ax}} \right) = \lim_{x \to 0} \left(\frac{e^{ax}}{x e^{ax}} - \frac{1}{x e^{ax}} \right) =$$

$$\lim_{x \to 0} \frac{e^{ax} - 1}{x e^{ax}} = \lim_{x \to 0} \frac{(e^{ax} - 1)'}{(x e^{ax})'} =$$

$$\lim_{x \to 0} \frac{e^{ax} \cdot a - 0}{e^{ax} + x e^{ax} \cdot a} = \frac{e^{a \cdot 0} \cdot a}{e^{a \cdot 0} + 0 \cdot e^{a \cdot 0} \cdot a} = \underline{\underline{a}}$$

c)



Krav:

$$0 < x < \frac{20}{2} = 10$$

Volumet, i cm^3, er:

$$V(x) = x\,(40 - 2x)\,(20 - 2x) = (40x - 2x^2)(20 - 2x) =$$

$$800x - 80x^2 - 40x^2 + 4x^3 =$$

$$4x^3 - 120x^2 + 800x$$

$$V'(x) = 4 \cdot 3x^2 - 120 \cdot 2x + 800 = 4(3x^2 - 60x + 200)$$

Faktoriserer:

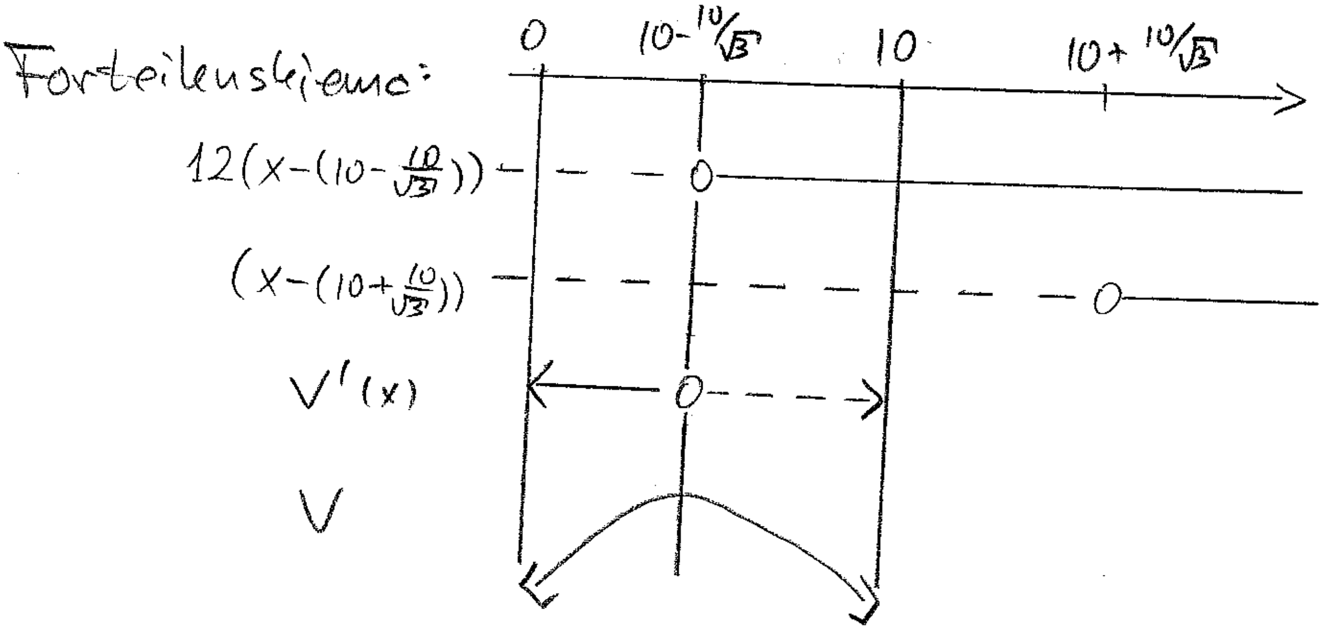
$$3x^2-60x+200=0$$

$$x=\frac{-(-60)\pm\sqrt{(-60)^2-4\cdot 3\cdot 200}}{2\cdot 3}=\frac{60\pm 20\sqrt{3}}{6}=10\pm\frac{10}{\sqrt{3}}$$

Altså:

$$V'(x)=4\cdot 3\cdot\left(x-\left(10-\frac{10}{\sqrt{3}}\right)\right)\left(x-\left(10+\frac{10}{\sqrt{3}}\right)\right)$$

Forteiknskjema:



Hugsar: $D_V=[0,10]$

Vi ser at V er maksimal for $x=\underline{\underline{10-10/\sqrt{3}}}$

Oppg. 2

a) $$V_{LJ}(r)=\varepsilon\left(\left(\frac{\sigma}{r}\right)^{12}-\left(\frac{\sigma}{r}\right)^{6}\right)=\varepsilon\left(\sigma^{12}r^{-12}-\sigma^6\cdot r^{-6}\right)$$

$$\sigma=2.0\,\text{Å}$$

$$V_{LJ}'(r)=\varepsilon\left(\sigma^{12}\cdot(-12)r^{-13}-\sigma^6\cdot(-6)r^{-7}\right)=$$

$$\frac{\varepsilon}{\sigma}\left(-12\left(\frac{\sigma}{r}\right)^{13}+6\left(\frac{\sigma}{r}\right)^{7}\right)=-6\frac{\varepsilon}{\sigma}\left(2\left(\frac{\sigma}{r}\right)^{13}-\left(\frac{\sigma}{r}\right)^{7}\right)$$

Nullpunkt for $V'_{LJ}(r)$:

$V'_{LJ}(r) = 0$

$$-6\frac{\varepsilon}{\sigma}\left(2\left(\frac{\sigma}{r}\right)^{13} - \left(\frac{\sigma}{r}\right)^{7}\right) = 0$$

$$\left(\frac{\sigma}{r}\right)^{7}\left(2\left(\frac{\sigma}{r}\right)^{6} - 1\right) = 0$$

$$2\left(\frac{\sigma}{r}\right)^{6} = 1$$

$$\left(\frac{\sigma}{r}\right)^{6} = \frac{1}{2}$$

$$\left(\frac{r}{\sigma}\right)^{6} = 2$$

$$\frac{r}{\sigma} = \sqrt[6]{2}$$

$$r = \sqrt[6]{2}\cdot\sigma$$

Av funksjonsuttrykket ser vi at V_{LJ} er kontinuerleg og deriverbar for $r > 0$. Det er gitt at V_{LJ} har eit minimum. $V'_{LJ}(r)$ må difor vere 0 her. Altså: V_{LJ} er minimal for $r = \sqrt[6]{2}\,\sigma = \underline{\sqrt[6]{2}\cdot 2.0\,\text{Å}} \approx 2.24\,\text{Å}$

b) $V_M(r) = D\left(1 - e^{-a(r-r_0)}\right)^2$

For H_2: V_M er minimal for $r = 0.74$ Å.

$\left(1 - e^{-a(r-r_0)}\right)^2$ kan bli 0 – men aldri lågare. V_M er minimal når $\left(1 - e^{-a(r-r_0)}\right)^2 = 0$

$$1 - e^{-a(r-r_0)} = 0$$

$$e^{-a(r-r_0)} = 1$$

$$-a(r-r_0) = 0$$

$$\underline{r = r_0}$$

Altså: $\underline{\underline{r_0 = 0.74 \text{ Å}}}$

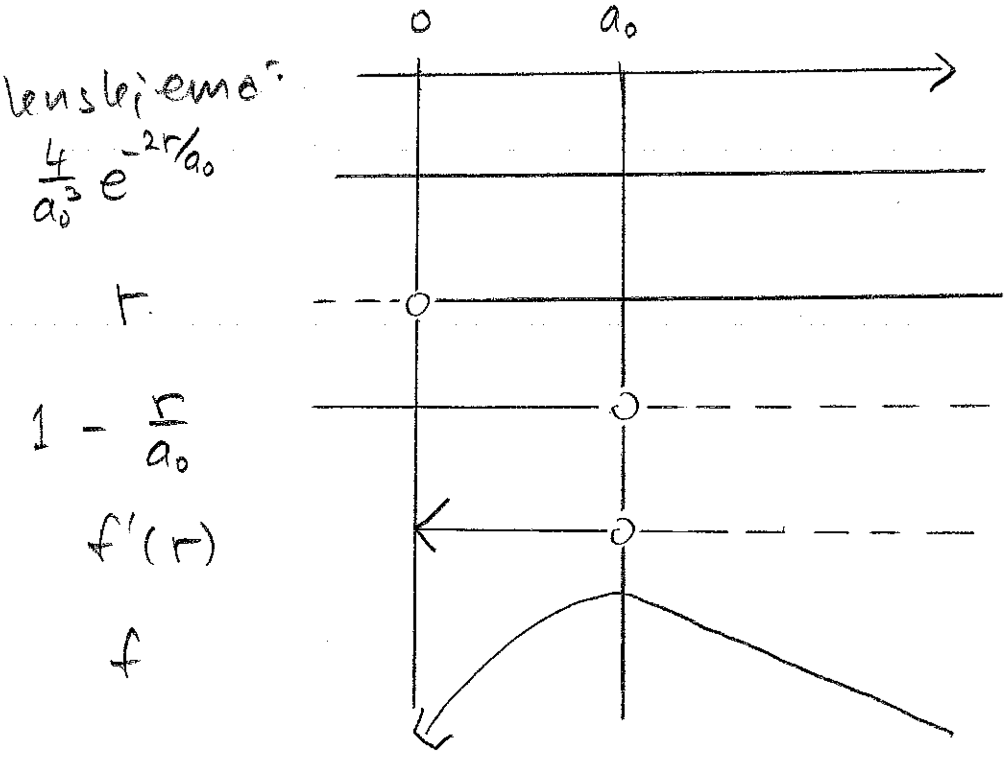
c) $f(r) = \frac{2}{a_0^3} r^2 e^{-2r/a_0}$, $a_0 = 0.529 \text{ Å}$, $r \geq 0$

$$f'(r) = \frac{2}{a_0^3}\left((r^2)' e^{-2r/a_0} + r^2 e^{-2r/a_0} \cdot \left(-\frac{2r}{a_0}\right)'\right) =$$

$$\frac{2}{a_0^3} e^{-2r/a_0} \left(2r + r^2 \cdot \left(-\frac{2}{a_0}\right)\right) =$$

$$\frac{4}{a_0^3} e^{-2r/a_0} \, r\left(1 - \frac{r}{a_0}\right)$$

Forteikenskjema:



$f(r)$ er størst for $r = a_0 = \underline{\underline{0.529\,\text{Å}}}$

Oppg. 3

$$
\begin{aligned}
x_1 + \quad\;\; 5x_3 &= 12 \\
5x_1 + 2x_2 \quad\;\; &= 10 \\
-2x_2 - x_2 + 3x_3 &= 2
\end{aligned}
$$

a) Utvida koeffisient matrise:

$$
\begin{array}{c}
\underline{\begin{pmatrix} 1 & 0 & 5 & 12 \\ 5 & 2 & 0 & 10 \\ -2 & -1 & 3 & 2 \end{pmatrix}}
\end{array}
\begin{array}{cc} -5 & 2 \\ & \\ & \end{array}
\sim
$$

b)

$$
\begin{pmatrix} 1 & 0 & 5 & 12 \\ 0 & 2 & -25 & -50 \\ 0 & -1 & 13 & 26 \end{pmatrix}
\begin{array}{c} \\ \\ 2 \leftarrow (-1) \end{array}
\sim
\begin{pmatrix} 1 & 0 & 5 & 12 \\ 0 & 1 & -13 & -26 \\ 0 & 0 & 1 & 2 \end{pmatrix}
\begin{array}{cc} & \\ & \\ 13 & -5 \end{array}
\sim
$$

$$
\begin{pmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 \end{pmatrix}
$$

Løysing: $\underline{x_1 = 2,\ x_2 = 0,\ x_3 = 2}$.