Løysingsforslag

Opprg. 1

a) i) $f(x) = \sin x + x^2$

$$f'(x) = \underline{\underline{\cos x + 2x}}$$

ii) $\int x e^x dx$

Delvis integrasjon: $\int u v' dx = uv - \int u' v dx$

$u = x \Rightarrow u' = 1$

$v' = e^x \Leftarrow v = e^x$

$$\int x e^x dx = x e^x - \int 1 \cdot e^x dx = \underline{\underline{x e^x - e^x + C}}$$

$$= e^x (x-1) + C$$

b) i) $z = 8 e^{-\frac{3\pi}{2} i} = 8 \left(\cos\left(-\frac{3\pi}{2}\right) + i \sin\left(-\frac{3\pi}{2}\right)\right) =$

$$8 \left(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2} \right) = 8(0 + i) = \underline{\underline{8i}}$$

ii) $w = -1 + i = r e^{i\theta}$

$r^2 = (-1)^2 + 1^2$, $r = \sqrt{2}$

$\tan \theta = \frac{1}{-1} = -1$, $\theta = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$

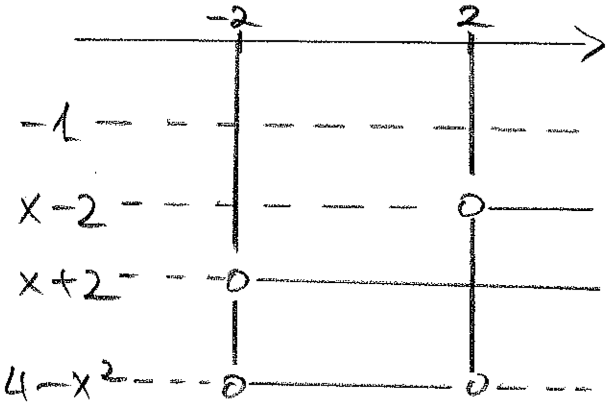
$$w = \sqrt{2}\, e^{\frac{3\pi}{4} i}$$

c) $f(x) = \ln(4-x^2) + \dfrac{1}{\sin x}$

Krav: $4-x^2 > 0$ og $\sin x \neq 0$

$4-x^2 > 0 \Leftrightarrow \quad -(x-2)(x+2) > 0 \Leftrightarrow$

Forteiknsskjema:



Krav: $-2 < x < 2$

$\sin x \neq 0 \Leftrightarrow \quad x \neq n\pi, \ n \in \mathbb{Z}$

For $x \in (-2, 2)$ er det berre ein x-verdi som gir $\sin x = 0$, nemleg $x = 0$.

Den naturlege definisjonsmengda blir

$D_f = \underline{(-2,2) \setminus \{0\}} = (-2,0) \cup (0,2)$

d) i) $\lim\limits_{x\to 1} \dfrac{x^2-1}{x^2+1} = \dfrac{1^2-1}{1^2+1} = \underline{\underline{0}}$

$\lim\limits_{x\to 1} \dfrac{x-1}{x^2-1} = \lim\limits_{x\to 1} \dfrac{\cancel{x-1}}{(\cancel{x-1})(x+1)} = \lim\limits_{x\to 1} \dfrac{1}{x+1} = \dfrac{1}{1+1} = \underline{\underline{\dfrac{1}{2}}}$

$$\lim_{x \to 1} \frac{\ln x}{\sin(\pi x)}$$

Både teljar og nemnar går mot 0. Vi bruker L'Hôpitals regel:

$$\lim_{x \to 1} \frac{\ln x}{\sin(\pi x)} = \lim_{x \to 1} \frac{(\ln x)'}{(\sin(\pi x))'} = \lim_{x \to 1} \frac{\frac{1}{x}}{\pi \cos(\pi x)} =$$

$$\frac{1}{\pi} \lim_{x \to 1} \frac{1}{x \cos(\pi x)} = \frac{1}{\pi} \cdot \frac{1}{1 \cdot \cos \pi} = \underline{\underline{-\frac{1}{\pi}}}$$

Oppg. 2

$$f(x) = e^{\sqrt{3}x} \sin x$$

- Skal finne alle globale ekstremalpunkt på $[0, \pi]$. Sidan f og $f'(x)$ er kontinuerlege, vill desse punkta vere randpunkt eller punkt der $f'(x) = 0$

$$f'(x) = \sqrt{3}\, e^{\sqrt{3}x} \sin x + e^{\sqrt{3}x} \cos x =$$

$$e^{\sqrt{3}x} (\sqrt{3} \sin x + \cos x)$$

Nullpunkt for $f'(x)$:

$$f'(x) = 0$$

$$e^{\sqrt{3}x} (\sqrt{3} \sin x + \cos x) = 0$$

$$\sqrt{3} \sin x + \cos x = 0$$

$$\sqrt{3} \frac{\sin x}{\cos x} + 1 = 0$$

$\tan x = -\frac{1}{\sqrt{3}}$

$x = \pi - \frac{\pi}{6} + n\pi = \frac{5\pi}{6} + n\pi, \quad n \in \mathbb{Z}$

For $x \in [0, \pi]$: $x = \frac{5\pi}{6}$

Funksjonsverdiar:

$f(0) = e^{\sqrt{3}\cdot 0} \sin 0 = 0$

$f(\frac{5\pi}{6}) = e^{\sqrt{3}\cdot\frac{5\pi}{6}} \sin\frac{5\pi}{6} = \frac{1}{2}\, e^{\sqrt{3}\cdot\frac{5\pi}{6}}$

$f(\pi) = e^{\sqrt{3}\cdot\pi} \sin\pi = 0$

Altså er randpunkta, $(0,0)$ og $(\pi, 0)$, globale minima, medan $(\frac{5\pi}{6}, \frac{1}{2} e^{\sqrt{3}\cdot\frac{5\pi}{6}})$ er globalt maksimum.

Oppg. 3

a) $y' = y + 5e^x \iff y' - y = 5e^x$

Integrerande faktor: $e^{F(x)}$ der $F'(x) = -1$

Vel $F = -x$.

$e^{-x}(y' - y) = e^{-x} \cdot 5e^x = 5$

$(e^{-x}y)' = 5$

$e^{-x}y = \int 5\,dx = 5x + C$

$\underline{y = e^x(5x + C)}$

b)

$y'' - y = x, \quad y(0) = y'(0) = 1$

- Løyser fyrst homogen likning:

$y'' - y = 0$

- Går ut frå at $y = e^{rx}$ slik at

$y'' = r^2 e^{rx}$:

$r^2 e^{rx} - e^{rx} = 0$

$r^2 - 1 = 0, \quad r = \pm 1$

Generell løysing:

$y_h = Ae^{x} + Be^{-x}$

- Går ut frå at vi kan finne ei partikulær løysing på forma

$y_p = ax + b$

$y_p' = a, \quad y_p'' = 0$

Set inn:

$0 - (ax+b) = x, \quad -ax - b = 1x + 0$

$-a = 1$ og $-b = 0,$

$a = -1$ og $b = 0$

$y_p = -x$

Generell løysing: $y = y_h + y_p = Ae^{x} + Be^{-x} - x$

Initialkrav: $y(0)=1$

$$A\cdot 1 + B\cdot 1 - 0 = 1, \quad A+B=1$$

$$y' = Ae^{x} - Be^{-x} - 1$$

Initialkrav: $y'(0)=1$

$$A\cdot 1 - B\cdot 1 - 1 = 1, \quad A-B=2$$

$$\left.\begin{cases} A+B=1 \\ A-B=2 \end{cases}\right\} \Leftrightarrow \left.\begin{cases} 2A=3 \\ 2B=-1 \end{cases}\right\} \Leftrightarrow \begin{cases} A=\frac{3}{2} \\ B=-\frac{1}{2} \end{cases}$$

Løysing:

$$\underline{\underline{y(x) = \frac{3}{2}e^{x} - \frac{1}{2}e^{-x} - x}}$$

c) $(x^2+1)y' = \sqrt{y}, \quad y(0)=4$

NB! Denne utgår!

$$\frac{1}{\sqrt{y}}\frac{dy}{dx} = \frac{1}{x^2+1}$$

$$\frac{1}{\sqrt{y}}dy = \frac{dx}{x^2+1}$$

$$\int y^{-1/2}\,dy = \int \frac{1}{x^2+1}\,dx$$

$$\frac{1}{-\frac{1}{2}+1}y^{-1/2+1} = \arctan x + C'$$

$$2\sqrt{y} = \arctan x + C'$$

Initialkrav: $y(0)=4$

$$2\cdot\sqrt{4} = \arctan 0 + C', \quad C' = 2\sqrt{4} = 4$$

Altså:

$$2\sqrt{y} = \arctan x + 4$$

$$\sqrt{y} = \tfrac{1}{2}\arctan x + 2$$

$$\underline{\underline{y = \left(\tfrac{1}{2}\arctan x + 2\right)^2}}$$

Oppg. 4

$$A = \begin{bmatrix} 0 & 4 & 2 \\ 0 & 2 & 2 \\ 1 & 4 & 6 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & 0 \\ 3 & 0 \\ 1 & 1 \end{bmatrix}$$

a) Dersom A er invertibel, er matrisa $[A \mid I_3]$ rekke-ekvivalent med $[I_3 \mid A^{-1}]$.

$$[A \mid I_3] = \begin{array}{c} \begin{bmatrix} 0 & 4 & 2 & 1 & 0 & 0 \\ 0 & 2 & 2 & 0 & 1 & 0 \\ 1 & 4 & 6 & 0 & 0 & 1 \end{bmatrix} \end{array} \begin{array}{l} \\ -2 \;\; -2 \;\; \leftarrow \tfrac{1}{2} \\ \\ \end{array} \sim$$

$$\begin{bmatrix} 0 & 0 & -2 & 1 & -2 & 0 \\ 0 & 1 & 1 & 0 & \tfrac{1}{2} & 0 \\ 1 & 0 & 2 & 0 & -2 & 1 \end{bmatrix} \begin{array}{l} 1 \\ \\ \\ \end{array} \sim$$

$$\begin{bmatrix} 1 & 0 & 0 & 1 & -4 & 1 \\ 0 & 1 & 1 & 0 & \tfrac{1}{2} & 0 \\ 0 & 0 & -2 & 1 & -2 & 0 \end{bmatrix} \begin{array}{l} \\ \\ \leftarrow -\tfrac{1}{2} \end{array} \sim \begin{bmatrix} 1 & 0 & 0 & 1 & -4 & 1 \\ 0 & 1 & 1 & 0 & \tfrac{1}{2} & 0 \\ 0 & 0 & 1 & -\tfrac{1}{2} & 1 & 0 \end{bmatrix} \begin{array}{l} \\ \\ -1 \end{array} \sim$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -4 & 1 \\ 0 & 1 & 0 & \frac{1}{2} & -\frac{1}{2} & 0 \\ 0 & 0 & 1 & -\frac{1}{2} & 1 & 0 \end{array}\right] = [I_3 | A^{-1}]$$

Altså: $A^{-1} = \underline{\underline{\begin{bmatrix} 1 & -4 & 1 \\ \frac{1}{2} & -\frac{1}{2} & 0 \\ -\frac{1}{2} & 1 & 0 \end{bmatrix}}}$

Kontroll: $A^{-1}A = \begin{bmatrix} 1 & -4 & 1 \\ \frac{1}{2} & -\frac{1}{2} & 0 \\ -\frac{1}{2} & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 4 & 2 \\ 0 & 2 & 2 \\ 1 & 4 & 6 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

OK

$$AB = \begin{bmatrix} 0 & 4 & 2 \\ 0 & 2 & 2 \\ 1 & 4 & 6 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 3 & 0 \\ 1 & 1 \end{bmatrix} =$$

$$\begin{bmatrix} 0\cdot 2 + 4\cdot 3 + 2\cdot 1 & 0\cdot 0 + 4\cdot 0 + 2\cdot 1 \\ 0\cdot 2 + 2\cdot 3 + 2\cdot 1 & 0\cdot 0 + 2\cdot 0 + 2\cdot 1 \\ 1\cdot 2 + 4\cdot 3 + 6\cdot 1 & 1\cdot 0 + 4\cdot 0 + 6\cdot 1 \end{bmatrix} = \underline{\underline{\begin{bmatrix} 14 & 2 \\ 8 & 2 \\ 20 & 6 \end{bmatrix}}}$$

B har 2 søyler, og A har 3 rekker.
<u>Difor er ikkje BA definert.</u>

b) Vi kjenner att vektorane som søylene i A. Vi har, ved å finne A^{-1}, vist at A er invertibel.
Difor er søylene i A, og også dei gitte vektorane, lineært uavhengige.

c) Likningssystemet $A\vec{x}=\vec{b}$, med A kvadratisk, har eintydig løysing hvis og berre hvis $\det A \neq 0$.

Vi reknar ut determinanten til koeffisientmatrisa:

$$\begin{vmatrix} 4 & 0 & 0 \\ 2 & k & 6 \\ 4 & 1 & k-1 \end{vmatrix} = 4 \begin{vmatrix} k & 6 \\ 1 & k-1 \end{vmatrix} = 4(k(k-1)-6\cdot 1) =$$

$$4(k^2-k-6)$$

Krever at denne skal vere ulik 0:

$$4(k^2-k-6) \neq 0 \Leftrightarrow k \neq \frac{-(-1) \pm \sqrt{(-1)^2-4\cdot 1\cdot(-6)}}{2}$$

$$k \neq \frac{1 \pm 5}{2}, \quad k \neq \frac{1-5}{2} = -2 \wedge k \neq \frac{1+5}{2} = 3$$

Vi har eintydig løysing for $\underline{k \notin \{-2, 3\}}$.

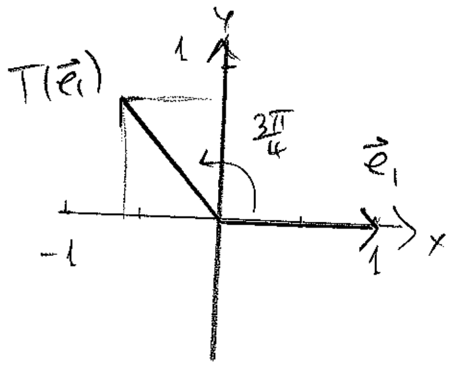
d) $T: \mathbb{R}^2 \to \mathbb{R}^2$

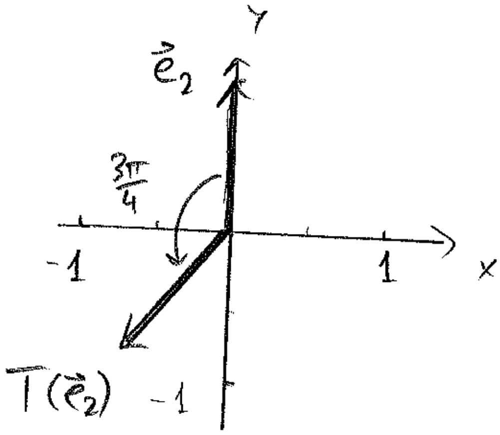
- Rotasjon $\frac{3\pi}{4}$ mot klokka

$T(\vec{x}) = A\vec{x}$

der standardmatrisa $A = [T(\vec{e}_1) \mid T(\vec{e}_2)]$

$\vec{e}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ og $\vec{e}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$





Av figurene ser vi at

$$T(\vec{e}_1) = \begin{bmatrix} \cos\frac{3\pi}{4} \\ \sin\frac{3\pi}{4} \end{bmatrix} = \begin{bmatrix} -\cos\frac{\pi}{4} \\ \sin\frac{\pi}{4} \end{bmatrix} = \begin{bmatrix} -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}$$

og at

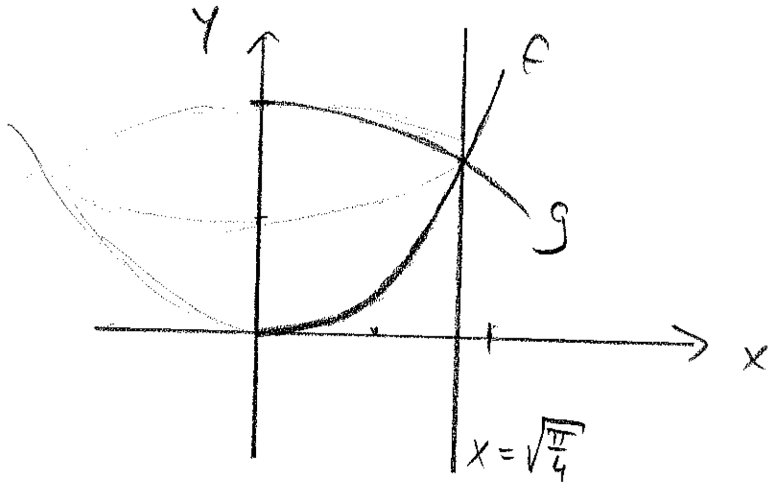
$$T(\vec{e}_2) = \begin{bmatrix} -\sin\frac{3\pi}{4} \\ \cos\frac{3\pi}{4} \end{bmatrix} = \begin{bmatrix} -\sin\frac{\pi}{4} \\ -\cos\frac{\pi}{4} \end{bmatrix} = \begin{bmatrix} -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{bmatrix}$$

Vi får

$$A = \begin{bmatrix} -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}$$

Oppg. 5

$f(x) = \sin x^2, \quad g(x) = \cos x^2$



Volum av omdreiingslekam:

$$V = 2\pi \int_0^{\sqrt{\pi/4}} x\,(g(x) - f(x))\,dx =$$

$$2\pi \int_0^{\sqrt{\pi/4}} x\,(\cos x^2 - \sin x^2)\,dx$$

Variabelbyte:

$$u = x^2, \quad \frac{du}{dx} = 2x, \quad dx = \frac{1}{2x}\,du$$

$$u(0) = 0 \quad u\left(\sqrt{\tfrac{\pi}{4}}\right) = \frac{\pi}{4}$$

$$V = 2\pi \int_0^{\pi/4} x\,(\cos u - \sin u)\,\frac{1}{2x}\,du =$$

$$\pi \int_0^{\pi/4} (\cos u - \sin u)\,du = \pi\,[\sin u + \cos u]_0^{\pi/4} =$$

$$\pi\left(\sin\frac{\pi}{4} + \cos\frac{\pi}{4} - (\sin 0 + \cos 0)\right) =$$

$$\pi\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} - 1\right) = \underline{\underline{\pi\,(\sqrt{2} - 1)}}$$

Oppg. 6

a) $M = \begin{bmatrix} -4 & 0 & 0 \\ 3 & -2 & 0 \\ 1 & 2 & 0 \end{bmatrix}$

$M = PDP^{-1}$ der P har eigenvektorane for M som søyler og dei tilsvarande eigenverdiane står langs diagonalen i diagonalmatrisa D.

Eigenv.

$$M\vec{x} = \lambda\vec{x} \Leftrightarrow (M - \lambda I)\vec{x} = \vec{0}$$

Ikkje-trivielle løysingar:

$\det(M - \lambda I) = 0 \Leftrightarrow$

$$\begin{vmatrix} -4-\lambda & 0 & 0 \\ 3 & \ldots & \ldots \end{vmatrix} = (-4-\lambda)(\ldots$$

$$\begin{bmatrix} 1 & 2 & 4 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix} \begin{matrix} \\ -2 \\ \\ \end{matrix} \sim \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix}$$

$x_1 - 2x_3 = 0 \Leftrightarrow x_1 = 2x_3$

$x_2 + 3x_3 = 0 \Leftrightarrow x_2 = -3x_3$

$$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2x_3 \\ -3x_3 \\ x_3 \end{bmatrix} = x_3 \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix}$$

Vel $x_3 = 1$ og får eigenvektoren $\vec{x}_1 = \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix}$

Eigenvektor for $\lambda_2 = -2$

$$A - (-2)I = \begin{bmatrix} -2 & 0 & 0 \\ 3 & 0 & 0 \\ 1 & 2 & 2 \end{bmatrix} \begin{matrix} \frac{3}{2} \quad \frac{1}{2} \leftarrow -\frac{1}{2} \\ \\ \\ \end{matrix} \sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 2 & 2 \end{bmatrix} \begin{matrix} \\ \\ \leftarrow \frac{1}{2} \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$x_1 = 0$

$x_2 + x_3 = 0, \quad x_2 = -x_3$

$$\vec{x} = \begin{bmatrix} 0 \\ x_3 \\ -x_3 \end{bmatrix} = x_3 \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$$

Vel $x_3 = 1$ og får eigenvektoren $\vec{x}_2 = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$

b) $N_A' = -4N_A$

$$\frac{dN_A}{dt} = -4N_A$$

$$\frac{dN_A}{N_A} = -4\,dt$$

$$\int \frac{1}{N_A}\,dN_A = \int -4\,dt$$

$$\ln|N_A| = -4t + C_1$$

$$|N_A| = e^{-4t + C_1} = e^{C_1} \cdot e^{-4t}$$

$$N_A = \pm e^{C_1} e^{-4t}$$

Med $C = \pm e^{C_1}$:

$$\underline{\underline{N_A(t) = C\,e^{-4t}}}$$

Skal ha: $N_A(t + T_{1/2}) = \frac{1}{2} N(t)$

$$C\,e^{-4\cdot(t+T_{1/2})} = \frac{1}{2} \cdot C\,e^{-4t}$$

$$e^{-4\cdot t} \cdot e^{-4\cdot T_{1/2}} = e^{-4t} \cdot \frac{1}{2}$$

$$e^{-4T_{1/2}} = \frac{1}{2}$$

$$-4\,T_{1/2} = \ln\frac{1}{2} = \ln 2^{-1} = -\ln 2$$

$$T_{1/2} = \frac{-\ln 2}{-4} = \underline{\frac{\ln 2}{4}} \quad \text{(q.e.d.)}$$

c) $\vec{N}'(t) = M \cdot \vec{N}(t)$

Generell løysing:

$$\vec{N}(t) = C_1 e^{\lambda_1 t} \vec{x}_1 + C_2 e^{\lambda_2 t} \vec{x}_2 + C_3 e^{\lambda_3 t} \vec{x}_3$$

der λ_i og $\vec{x}_i$ er eigenverdiane og -vektorane frå a).

Altså:

$$\vec{N}(t) = C_1 e^{-4t} \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix} + C_2 e^{-2t} \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} + C_3 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Initialkrav: $\vec{N}(0) = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

$$C_1 \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix} + C_2 \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} + C_3 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$P \begin{bmatrix} C_1 \\ C_2 \\ C_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

Totalmatrise:

$$\begin{bmatrix} 2 & 0 & 0 & 1 \\ -3 & 1 & 0 & 0 \\ 1 & -1 & 1 & 0 \end{bmatrix} \begin{matrix} \leftarrow \frac{1}{2} \\ \\ \\ \end{matrix} \sim \begin{bmatrix} 1 & 0 & 0 & \frac{1}{2} \\ -3 & 1 & 0 & 0 \\ 1 & -1 & 1 & 0 \end{bmatrix} \begin{matrix} 3 \quad -1 \\ \\ \\ \end{matrix} \sim$$

$$\begin{bmatrix} 1 & 0 & 0 & \frac{1}{2} \\ 0 & 1 & 0 & \frac{3}{2} \\ 0 & -1 & 1 & -\frac{1}{2} \end{bmatrix} \begin{matrix} \\ 1 \\ \\ \end{matrix} \sim \begin{bmatrix} 1 & 0 & 0 & \frac{1}{2} \\ 0 & 1 & 0 & \frac{3}{2} \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

Vi får

$$C_1 = \frac{1}{2}, \quad C_2 = \frac{3}{2} \quad \text{og} \quad C_3 = 1$$

Slik at

$$\vec{N}(t) = \begin{bmatrix} N_A(t) \\ N_B(t) \\ N_C(t) \end{bmatrix} = \underline{\underline{\frac{1}{2} e^{-4t} \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix} + \frac{3}{2} e^{-2t} \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}}}$$

d) Vi kallar summen S: $S = N_A(t) + N_B(t) + N_C(t)$

Løysinga frå c gir at

$$N_A(t) = \frac{1}{2} e^{-4t} \cdot 2 = e^{-4t}$$

$$N_B(t) = \frac{1}{2} e^{-4t} \cdot (-3) + \frac{3}{2} e^{-2t} \cdot 1 = \frac{3}{2}(e^{-2t} - e^{-4t})$$

$$N_C(t) = \frac{1}{2} e^{-4t} \cdot 1 + \frac{3}{2} e^{-2t} \cdot (-1) + 1 =$$

$$\frac{1}{2} e^{-4t} - \frac{3}{2} e^{-2t} + 1$$

Sum:

$$S = N_A + N_B + N_C = e^{-4t} + \frac{3}{2}(e^{-2t} - e^{-4t}) +$$

$$\frac{1}{2} e^{-4t} - \frac{3}{2} e^{-2t} + 1 =$$

$$e^{-4t}\left(1 - \frac{3}{2} + \frac{1}{2}\right) + e^{-2t}\left(\frac{3}{2} - \frac{3}{2}\right) + 1 = 1$$

Altså: $\underline{\underline{S = 1 \text{ heile tida}}}$.

Alternativt kan vi bruke differensial-likninga til å bestemme $S'(t)$:

$$\vec{N}' = M\vec{N}$$

$$\begin{bmatrix} N_A' \\ N_B' \\ N_C' \end{bmatrix} = \begin{bmatrix} -4 & 0 & 0 \\ 3 & -2 & 0 \\ 1 & 2 & 0 \end{bmatrix} \begin{bmatrix} N_A \\ N_B \\ N_C \end{bmatrix} = \begin{bmatrix} -4N_A \\ 3N_A - 2N_B \\ N_A + 2N_B \end{bmatrix}$$

$$S'(t) = N_A'(t) + N_B'(t) + N_C'(t) =$$

$$-4N_A + 3N_A - 2N_B + N_A + 2N_B = 0$$

$S'(t) = 0 \Leftrightarrow S$ endrar seg ikkje med tida t.

Vi kan også løyse likningssystemet i c) utan å diagonalisere; det er ikkje eigentleg kopla. Vi skriv likningane ut komponentvis:

$$N_A' = -4N_A \quad (i)$$

$$N_B' = 3N_A - 2N_B \quad (ii)$$

$$N_C' = N_A + 2N_B \quad (iii)$$

(i) har vi alt løyst og fått

$N_A = Ce^{-4t}$. Sidan $N_A(0) = 1$, må vi ha $C = 1$

Altså: $N_A = e^{-4t}$

$$N_B' = 3 \cdot e^{-4t} - 2N_B$$

$$N_B' + 2N_B = 3e^{-4t}$$

Integrerande faktor: e^{2t}

$$e^{2t}(N_B' + 2N_B) = e^{2t} \cdot 3e^{-4t} = 3e^{-2t}$$

$$(e^{2t}N_B)' = 3e^{-2t}$$

$$e^{2t}N_B = \int 3e^{-2t}\,dt = -\tfrac{3}{2}e^{-2t} + D$$

$$N_B = e^{-2t}\left(-\tfrac{3}{2}e^{-2t} + D\right) = -\tfrac{3}{2}e^{-4t} + De^{-2t}$$

Initialkrav: $N_B(0) = 0$

$$-\tfrac{3}{2}e^{0} + De^{0} = 0, \quad D = \tfrac{3}{2}$$

Altså:

$$N_B = -\frac{3}{2}e^{-4t} + \frac{3}{2}e^{-20} = \frac{3}{2}\left(e^{-2t} - e^{-4t}\right)$$

Likning (iii) løyser vi ved rein integrasjon:

$$N_C' = N_A + 2N_B = e^{-4t} + 2\cdot\frac{3}{2}\left(e^{-2t} - e^{-4t}\right) =$$

$$e^{-4t} + 3e^{-2t} - 3e^{-4t} = 3e^{-2t} - 2e^{-4t}$$

$$N_C = \int N_C'(t)\,dt = \int \left(3e^{-2t} - 2e^{-4t}\right)dt =$$

$$-\frac{3}{2}e^{-2t} - \frac{2}{-4}e^{-4t} + E =$$

$$\frac{1}{2}e^{-4t} - \frac{3}{2}e^{-2t} + E$$

Initialkrav: $N_C(0) = 0$

$$\frac{1}{2}\cdot 1 - \frac{3}{2}\cdot 1 + E = 0, \quad E = 1$$

Vi får:

$$N_A(t) = e^{-4t}$$

$$N_B(t) = \frac{3}{2}\left(e^{-2t} - e^{-4t}\right)$$

$$N_C(t) = \frac{1}{2}e^{-4t} - \frac{3}{2}e^{-2t} + 1$$

Eller:

$$\vec{N}(t) = \begin{bmatrix} e^{-4t} \\ \frac{3}{2}\left(e^{-2t} - e^{-4t}\right) \\ \frac{1}{2}e^{-4t} - \frac{3}{2}e^{-2t} + 1 \end{bmatrix} = \frac{1}{2}e^{-4t}\begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix} + \frac{3}{2}e^{-2t}\begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$