# Oppgåve 1

a) i) $f(x) = \sqrt{x}\cos x = x^{1/2}\cos x$

$$f'(x) = (x^{1/2})'\cos x + x^{1/2}(\cos x)' =$$

$$\frac{1}{2}x^{1/2-1}\cos x + x^{1/2}(-\sin x) =$$

$$\frac{1}{2}x^{-1/2}\cos x - \sqrt{x}\sin x = \underline{\frac{\cos x}{2\sqrt{x}} - \sqrt{x}\sin x}$$

ii) $g(x) = \dfrac{\ln x^2}{\sin x} + e^{3x} = 2\dfrac{\ln x}{\sin x} + e^{3x}$

$$g'(x) = 2\frac{(\ln x)'\sin x - \ln x \cdot (\sin x)'}{(\sin x)^2} + e^{3x}\cdot(3x)'$$

$$= 2\frac{\frac{1}{x}\sin x - \ln x\cos x}{\sin^2 x} + 3e^{3x} =$$

$$\underline{2\frac{\sin x - x\ln x\cos x}{x\sin^2 x} + 3e^{3x}}$$

b) i) $\int(\sqrt{x} + \sin(3x))dx = \int x^{1/2}dx + \int\sin(3x)dx =$

$$\frac{1}{\frac{1}{2}+1}x^{1/2+1} + \frac{1}{3}(-\cos(3x)) + C =$$

$$\underline{\frac{2}{3}x^{3/2} - \frac{1}{3}\cos(3x) + C}$$

ii) $\int \frac{x^3}{x^2+1}\,dx$

Polynomdivision:

$$x^3 : (x^2+1) = x - \frac{x}{x^2+1}$$
$$\frac{-(x^3+x)}{-x}$$

$$\int \frac{x^3}{x^2+1}\,dx = \int \left(x - \frac{x}{x^2+1}\right)dx = \frac{1}{2}x^2 - \int \frac{x}{x^2+1}\,dx$$

Variabelbyte: $u = x^2+1 \Leftrightarrow \frac{du}{dx} = 2x, \quad dx = \frac{1}{2x}\,du$

$$\int \frac{x}{x^2+1}\,dx = \int \frac{x}{u}\,\frac{1}{2x}du = \frac{1}{2}\int \frac{du}{u} =$$
$$\frac{1}{2}\ln|u| + C' = \frac{1}{2}\ln(x^2+1) + C' = \ln\sqrt{x^2+1} + C'$$

$$\int \frac{x^3}{x^2+1}\,dx = \underline{\frac{1}{2}x^2 - \ln\sqrt{x^2+1} + C'}$$

c) i) Delvis int.: $\int_a^b u \cdot v'\,dx = [uv]_a^b - \int_a^b u'v\,dx$

$u = x, \quad v' = \sin x$
$u' = 1, \quad v = -\cos x$

$$\int_{-1}^{1} x\sin x\,dx = [x\cdot(-\cos x)]_{-1}^{1} - \int_{-1}^{1} 1\cdot(-\cos x)\,dx =$$
$$-[x\cos x]_{-1}^{1} + \int_{-1}^{1}\cos x\,dx =$$
$$-(1\cos 1 - (-1)\cos(-1)) + [\sin x]_{-1}^{1} =$$
$$-2\cos 1 + (\sin 1 - \sin(-1)) = \underline{2(\sin 1 - \cos 1)}$$

$$\approx 0.602$$

ii) $\int_{-1}^{0} x^2 \sin(x^3+1)\,dx$

Variabelbyte: $u = x^3+1$, $\frac{du}{dx} = 3x^2$, $dx = \frac{1}{3x^2}du$

$u(-1) = (-1)^3+1 = 0$, $u(0) = 1$

$\int_{-1}^{0} x^2 \sin(x^3+1)\,dx = \int_0^1 x^2 \sin u \frac{1}{3x^2}du =$

$\frac{1}{3} \int_0^1 \sin u\,du = \frac{1}{3}\left[-\cos u\right]_0^1 =$

$\frac{1}{3}(-\cos 1 - (-\cos 0)) = \underline{\frac{1}{3}(1-\cos 1)}$

## Oppgave 2

a) $$f(x) = \begin{cases} \dfrac{x^2+x-2}{x^2-1}, & x \neq 1 \\ a, & x = 1 \end{cases}$$

Krav til kontinuitet: $\lim_{x \to 1} f(x) = f(1)$

$f(1) = a$

$\lim_{x \to 1} f(x) = \lim_{x \to 1} \frac{x^2+x-2}{x^2-1}$

Faktoriserer teller: $x^2+x-2=0$,

$x = \frac{-1 \pm \sqrt{1^2-4\cdot 1\cdot(-2)}}{2\cdot 1} = \frac{-1 \pm 3}{2}$

$x = \frac{-1-3}{2} = -2$ eller $x = \frac{-1+3}{2} = 1$

$x^2+x-2 = 1\cdot(x-(-2))(x-1) = (x+2)(x-1)$

$\lim_{x \to 1} f(x) = \lim_{x \to 1} \frac{(x+2)(x-1)}{(x-1)(x+1)} = \lim_{x \to 1} \frac{x+2}{x+1} =$

$\frac{1+2}{1+1} = \frac{3}{2}$

Vi må altså velge $\underline{a = \frac{3}{2}}$

b) $g(x) = \cos x - x^2$

- Elementær funksjon som er kontinuerlig på heile $\mathbb{R}$ (inkludert intervallet $[0, \pi]$).

- $g(0) = \cos 0 - 0 = 1 > 0$

$g(\pi) = \cos \pi - 2\pi = -1 - 2\pi < 0$

Sidan $g(0)$ og $g(\pi)$ har ulike fortegn, må $g$ ha minst eitt nullpunkt på $[0, \pi]$ ved skjeeringssetninga.

$g'(x) = -\sin x + 2 > 0$

Sidan $-\sin x \geq 1$, er $g' > 0$ for alle $x$

Altså er $g$ strengt aukande og kan derfor ha maksimalt eitt nullpunkt.

Difor har $g$ eitt – og berre eitt – nullpunkt på $[0, \pi]$ $\square$

c) Vi undersøker funksjonsverdien i midtpunktet på intervallet:

$g\left(\frac{\pi}{2}\right) = \cos\frac{\pi}{2} - 2\cdot\frac{\pi}{2} = -\pi < 0$

$g(0) > 0$ og $g\left(\frac{\pi}{2}\right) < 0$, difor ligg nullpunktet mellom $0$ og $\frac{\pi}{2}$

Vi itererar igjen:

$g\left(\frac{\pi}{4}\right) = \cos\frac{\pi}{4} - 2\cdot\frac{\pi}{4} = \frac{\sqrt{2}}{2} - \frac{\pi}{2} < 0$

Nullpkt. mellom $0$ og $\frac{\pi}{4}$

$g\left(\frac{\pi}{8}\right) = \cos\frac{\pi}{8} - 2\cdot\frac{\pi}{8} \approx 0.1385 > 0$

Nullpunktet må ligge i intervallet $\underline{\left[\frac{\pi}{8}, \frac{\pi}{4}\right]}$.

## Oppgåve 3

a) $A = \begin{bmatrix} 0 & 1 & 1 \\ -2 & 3 & -6 \\ 1 & 0 & 5 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 1 & -3 \\ 4 & 5 & -2 \end{bmatrix}$, $C = \begin{bmatrix} 4 & 0 & -1 \\ 1 & 3 & 2 \end{bmatrix}$

A og B har ulike format. Difor er ikkje $A - B$ definert.

A har 3 søyler og B har 2 rekker. Difor er heller ikkje produktet $AB$ definert.

$$BA = \begin{bmatrix} 2 & 1 & -3 \\ 4 & 5 & -2 \end{bmatrix} \begin{bmatrix} 0 & 1 & 1 \\ -2 & 3 & -6 \\ 1 & 0 & 5 \end{bmatrix} =$$

$$\begin{bmatrix} 2\cdot 0 - 1\cdot 2 - 3\cdot 1 & 2\cdot 1 + 1\cdot 3 - 0 & 2 - 6 - 3\cdot 5 \\ 0 - 2\cdot 5 - 2 & 4 + 3\cdot 5 - 0 & 4 - 5\cdot 6 - 2\cdot 5 \end{bmatrix} =$$

$$\underline{\begin{bmatrix} -5 & 5 & -19 \\ -12 & 19 & -36 \end{bmatrix}}$$

$$2B - 3C = 2\cdot\begin{bmatrix} 2 & 1 & -3 \\ 4 & 5 & -2 \end{bmatrix} - 3\cdot\begin{bmatrix} 4 & 0 & -1 \\ 1 & 3 & 2 \end{bmatrix} =$$

$$\begin{bmatrix} 4 & 2 & -6 \\ 8 & 10 & -4 \end{bmatrix} - \begin{bmatrix} 12 & 0 & -3 \\ 3 & 9 & 6 \end{bmatrix} = \underline{\begin{bmatrix} -8 & 2 & -3 \\ 5 & 1 & -10 \end{bmatrix}}$$

b) $$\det A = \begin{vmatrix} 0 & 1 & 1 \\ -2 & 3 & -6 \\ 1 & 0 & 5 \end{vmatrix} \begin{matrix} \\ \leftarrow \\ 2 \end{matrix} = \begin{vmatrix} 0 & 1 & 1 \\ 0 & 3 & 4 \\ 1 & 0 & 5 \end{vmatrix} =$$

$$1 \cdot \begin{vmatrix} 1 & 1 \\ 3 & 4 \end{vmatrix} = 1 \cdot (4-3) = \underline{1} \neq 0$$

$\underline{\det A \neq 0 \Leftrightarrow A \text{ er invertibel.}}$

c)
$$[A \mid I_3] = \left[\begin{array}{ccc|ccc} 0 & 1 & 1 & 1 & 0 & 0 \\ -2 & 3 & -6 & 0 & 1 & 0 \\ 1 & 0 & 5 & 0 & 0 & 1 \end{array}\right] \begin{array}{c} \\ \\ 2 \end{array} \sim$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 5 & 0 & 0 & 1 \\ 0 & 3 & 4 & 0 & 1 & 2 \\ 0 & 1 & 1 & 1 & 0 & 0 \end{array}\right] \begin{array}{c} \\ \\ -3 \end{array} \sim \left[\begin{array}{ccc|ccc} 1 & 0 & 5 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -3 & 1 & 2 \end{array}\right] \begin{array}{c} \\ \\ -1 \quad -5 \end{array} \sim$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 15 & -5 & -9 \\ 0 & 1 & 0 & 4 & -1 & -2 \\ 0 & 0 & 1 & -3 & 1 & 2 \end{array}\right] = [I_3 \mid A^{-1}]$$

$$\underline{A^{-1} = \begin{bmatrix} 15 & -5 & -9 \\ 4 & -1 & -2 \\ -3 & 1 & 2 \end{bmatrix}}$$

Kontroll:

$$A \cdot A^{-1} = \begin{bmatrix} 0 & 1 & 1 \\ -2 & 3 & -6 \\ 1 & 0 & 5 \end{bmatrix} \begin{bmatrix} 15 & -5 & -9 \\ 4 & -1 & -2 \\ -3 & 1 & 2 \end{bmatrix} =$$

$$\begin{bmatrix} 4-3 & -1+1 & -2+2 \\ -30+12+18 & +10-3-6 & 18-6+12 \\ 15-15 & -5+5 & -9+10 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I_3$$

d) $\begin{bmatrix} 5 & a \\ 2 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 3 \\ b \end{bmatrix}$

For at vi ikkje skal ha eintydig løysing:

$$\begin{vmatrix} 5 & a \\ 2 & 4 \end{vmatrix} = 0 \Leftrightarrow 5 \cdot 4 - a \cdot 2 = 0 \Leftrightarrow a = 10$$

Totalmatrise:

$$\begin{bmatrix} 5 & 10 & 3 \\ 2 & 4 & b \end{bmatrix} \leftarrow \tfrac{1}{2} \sim \begin{bmatrix} 5 & 10 & 3 \\ 1 & 2 & \frac{b}{2} \end{bmatrix} \begin{matrix} \\ -5 \end{matrix} \sim$$

$$\begin{bmatrix} 1 & 2 & \frac{b}{2} \\ 0 & 0 & 3 - \frac{5}{2}b \end{bmatrix}$$

Inkonsistens: $3 - \frac{5}{2}b \neq 0 \Leftrightarrow b \neq \frac{6}{5}$

Altså må vi ha at $\underline{a = 10 \text{ og } b \neq \frac{6}{5}}$.

## Oppgåve 4

I linje 15 ser vi at ledda i summen har forma $\Delta x \cdot \sin x \, e^{\cos x}$ – eller $\Delta x_i \cdot f(x_i)$ med $f(x_i) = \sin x_i \, e^{\cos x_i}$.

$x_i$ blir bestemt i linje 14:

$x_i = a + (i-1) \cdot \Delta x$

$\Delta x$ er det same heile tida. Denne blir bestemt i linje 7: $\Delta x = \frac{b-a}{N}$, der $N = 100$ (linje 6).

Vi ser at $x_i$ startar på $i = 1$ (linje 13), slik at den fyrste verdien er $x_1 = a$

Den siste er $a + (N-1) \cdot \Delta x = b - \Delta x$.

Altså er summen $\sum_{i=1}^{N} \Delta x \, f(x_i)$ – ein Riemann-sum på ein regulær partisjon. Seleksjonen er her ein venstre-seleksjon.

Vi veit at når $N \to \infty$, skal Riemann-summen nærme seg integralet $\int_a^b f(x)\,dx$. $f(x)$ er gitt over, og grensene er gitt i linje 2 og 3.

Summen skal altså gå mot

$$\int_0^{\pi/2} \sin x \; e^{\cos x} \, dx$$

Variabelbyte: $u = \cos x$, $\frac{du}{dx} = -\sin x$, $dx = \frac{1}{-\sin x} du$

$u(0) = 1$, $u\left(\frac{\pi}{2}\right) = 0$

$$\int_0^{\pi/2} \sin x \; e^{\cos x} \, dx = \int_1^0 \sin x \; e^u \; \frac{1}{-\sin x} \, du =$$

$$-\int_1^0 e^u \, du = + \int_0^1 e^u \, du = \left[e^u\right]_0^1 = \underline{e-1}.$$

# Oppgåve 5

$$T'(t) = -k\,(T(t) - T_{ute}) + P$$

a) Konstant temperatur: $T'(t) = 0$

Set inn:

$$0 = -k\,(T - T_{ute}) + P$$

Gitt: $T = 22$, $T_{ute} = 10$, $k = 0.1$

$$0 = -0.1\cdot(22-10) + P, \quad \underline{P = 1.2}$$

b) $P = 0.5$:

$$T'(t) = -0.1(T - 10) + 0.5 = -0.1T + 1.5$$

$$-10\cdot\frac{dT}{dt} = T - 15$$

$$\frac{dT}{T-15} = -\frac{1}{10}\,dt, \quad \int \frac{dT}{T-15} = \int -\frac{1}{10}\,dt$$

$$\ln|T-15| = -\frac{1}{10}t + C_1$$

$$T - 15 = \pm e^{-\frac{1}{10}t + C_1} = \pm e^{C_1}\cdot e^{-t/10} = C e^{-t/10}$$

$$T(t) = 15 + C\,e^{-t/10}$$

Veit: $T(0) = 22$

$$15 + C\cdot e^{0} = 22, \quad C = 22 - 15 = 7$$

Altså:

$$\underline{T(t) = 15 + 7\,e^{-t/10}}$$

c) Seinare: ny k

$T_{ute} = 8$

$T(0) = 22$

$T(10) = 16$

$p = 0$

$$T' = -k(T-8)$$

$$\frac{dT}{T-8} = -k\,dt \quad , \quad \int \frac{dT}{T-8} = \int -k\,dt$$

$$\ln|T-8| = -kt + C_1$$

$$T-8 = \pm e^{-kt+C_1} = \pm e^{C_1} \cdot e^{-kt} = C e^{-kt}$$

$$T = 8 + C e^{-kt}$$

Initialkrav: $T(0) = 22$, $22 = 8 + C \cdot e^0$, $C = 14$

$$T(t) = 8 + 14 e^{-kt}$$

Skal ha: $T(10) = 16$

$$8 + 14 e^{-k \cdot 10} = 16$$

$$e^{-k \cdot 10} = \frac{16-8}{14} = \frac{4}{7}$$

$$-10k = \ln \frac{4}{7}$$

$$k = -\frac{\ln \frac{4}{7}}{10} = \frac{\ln \frac{7}{4}}{10} \approx 0.05596 \approx \underline{0.056}$$

# Oppgave 6

a) $$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - x_2 \\ x_2 \\ -x_1 \\ x_1 + x_2 \end{bmatrix} = x_1 \begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix} + x_2 \begin{bmatrix} -1 \\ 1 \\ 0 \\ 1 \end{bmatrix} =$$

$$\begin{bmatrix} 1 & -1 \\ 0 & 1 \\ -1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = A\vec{x}$$

der $\vec{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ og $A = \underline{\begin{bmatrix} 1 & -1 \\ 0 & 1 \\ -1 & 0 \\ 1 & 1 \end{bmatrix}}$ $\square$

Alternativt:

$$T(\vec{e}_1) = T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 1 - 0 \\ 0 \\ -1 \\ 1 + 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix}$$

$$T(\vec{e}_2) = T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 0 - 1 \\ 1 \\ -0 \\ 0 + 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ 0 \\ 1 \end{bmatrix}$$

$$A = \left[T(\vec{e}_1) \mid T(\vec{e}_2)\right] = \underline{\begin{bmatrix} 1 & -1 \\ 0 & 1 \\ -1 & 0 \\ 1 & 1 \end{bmatrix}}$$

b) $S\left(\begin{bmatrix}2\\1\end{bmatrix}\right) = \begin{bmatrix}4\\7\\10\end{bmatrix}$ og $S\left(\begin{bmatrix}6\\2\end{bmatrix}\right) = \begin{bmatrix}10\\18\\26\end{bmatrix}$

Vi leitar standardmatrisa til $S$ for $B$:

$$B\begin{bmatrix}2\\1\end{bmatrix} = \begin{bmatrix}4\\7\\10\end{bmatrix} \quad \text{og} \quad B\begin{bmatrix}6\\2\end{bmatrix} = \begin{bmatrix}10\\18\\26\end{bmatrix}$$

Kan skrivast som

$$B\begin{bmatrix}2 & 6\\1 & 2\end{bmatrix} = \begin{bmatrix}4 & 10\\7 & 18\\10 & 26\end{bmatrix}$$

$$B\begin{bmatrix}2 & 6\\1 & 2\end{bmatrix} \cdot \begin{bmatrix}2 & 6\\1 & 2\end{bmatrix}^{-1} = \begin{bmatrix}4 & 10\\7 & 18\\10 & 26\end{bmatrix} \begin{bmatrix}2 & 6\\1 & 2\end{bmatrix}^{-1}$$

Formel: $\begin{bmatrix}a & b\\c & d\end{bmatrix}^{-1} = \frac{1}{ad-bc}\begin{bmatrix}d & -b\\-c & a\end{bmatrix}$

$$\underline{B} = \begin{bmatrix}4 & 10\\7 & 18\\10 & 26\end{bmatrix} \cdot \frac{1}{2\cdot2-6\cdot1}\begin{bmatrix}2 & -6\\-1 & 2\end{bmatrix} =$$

$$-\frac{1}{2}\begin{bmatrix}4\cdot2-10 & -4\cdot6+2\cdot10\\7\cdot2-18 & -7\cdot6+2\cdot18\\10\cdot2-26 & -10\cdot6+26\cdot2\end{bmatrix} = -\frac{1}{2}\begin{bmatrix}-2 & -4\\-4 & -6\\-6 & -8\end{bmatrix} = \underline{\begin{bmatrix}1 & 2\\2 & 3\\3 & 4\end{bmatrix}}$$

Alternativt:

$S$ er lineær: $S(c\vec{x}) = cS(\vec{x})$

$S(\vec{x}+\vec{y}) = S(\vec{x}) + S(\vec{y})$

$$S\left(\begin{bmatrix}6\\2\end{bmatrix}\right) = S\left(2\begin{bmatrix}3\\1\end{bmatrix}\right) = 2\,S\left(\begin{bmatrix}3\\1\end{bmatrix}\right) = \begin{bmatrix}10\\18\\26\end{bmatrix} \Leftrightarrow$$

$$S\left(\begin{bmatrix}3\\1\end{bmatrix}\right) = \frac{1}{2}\begin{bmatrix}10\\18\\26\end{bmatrix} = \begin{bmatrix}5\\9\\13\end{bmatrix}$$

$$S\left(\begin{bmatrix}1\\0\end{bmatrix}\right) = S\left(\begin{bmatrix}3\\1\end{bmatrix} - \begin{bmatrix}2\\1\end{bmatrix}\right) = S\left(\begin{bmatrix}3\\1\end{bmatrix}\right) - S\left(\begin{bmatrix}2\\1\end{bmatrix}\right) =$$

$$\begin{bmatrix}5\\9\\13\end{bmatrix} - \begin{bmatrix}4\\7\\10\end{bmatrix} = \begin{bmatrix}1\\2\\3\end{bmatrix}$$

$$S\left(\begin{bmatrix}0\\1\end{bmatrix}\right) = S\left(\begin{bmatrix}2\\1\end{bmatrix} - 2\begin{bmatrix}1\\0\end{bmatrix}\right) = S\left(\begin{bmatrix}2\\1\end{bmatrix}\right) - 2\,S\left(\begin{bmatrix}1\\0\end{bmatrix}\right) =$$

$$\begin{bmatrix}4\\7\\10\end{bmatrix} - 2\begin{bmatrix}1\\2\\3\end{bmatrix} = \begin{bmatrix}2\\3\\4\end{bmatrix}$$

$$B = \left[S\left(\begin{bmatrix}1\\0\end{bmatrix}\right) \;\middle|\; S\left(\begin{bmatrix}0\\1\end{bmatrix}\right)\right] = \underline{\begin{bmatrix}1 & 2\\2 & 3\\3 & 4\end{bmatrix}}$$

# Oppgåve 7

a)

$$y'' - 2y' + 5y = \sin x$$

Homogen likn.: $y'' - 2y' + 3y = 0$

Går ut frå løysing på forma $y = e^{rx}$

$y' = re^{rx}$ og $y'' = r^2 e^{rx}$

Set inn: $r^2 e^{rx} - 2re^{rx} + 3e^{rx} = 0$

$r^2 - 2r + 3 = 0 \Leftrightarrow r = \dfrac{-(-2) \pm \sqrt{(-2)^2 - 4 \cdot 1 \cdot 5}}{2 \cdot 1} =$

$\dfrac{2 \pm \sqrt{-16}}{2} = 1 \pm 2i$

Løysing av homogen likning.

$y_h = Ae^{(1-2i)x} + Be^{(1+2i)x} =$

$e^x (C \cos(2x) + D \sin(2x))$

$(C = A + B$ og $D = i(B - A))$

Antar partikulær løysing av den inhomogene likninga på forma

$y = a \sin x + b \cos x$

$y' = a \cos x - b \sin x$

$y'' = -a \sin x - b \cos x$

Set inn:

$$-a\sin x - b\cos x - 2(a\cos x - b\sin x) + 5(a\sin x + b\cos x) = \sin x$$

$$(-a + 2b + 5a)\sin x + (-b - 2a + 5b)\cos x = \sin x$$

$$4a + 2b = 1 \quad \text{og} \quad -2a + 4b = 0$$

$$-2a + 4b = 0 \Leftrightarrow a = 2b$$

$$4a + 2b = 4 \cdot 2b + 2b = 10b = 1, \quad b = \frac{1}{10}$$

$$a = 2b = \frac{2}{10} = \frac{1}{5}$$

Partikulær løysing:

$$y_s = \frac{1}{5}\sin x + \frac{1}{10}\cos x$$

Total løysing:

$$\underline{y = y_h + y_s = e^x(C\cos(2x) + D\sin(2x)) + \frac{1}{5}\sin x + \frac{1}{10}\cos x}$$

b)

I linje 13 ser vi at

$$F(x,y) = \frac{\cos(3x)}{y}$$

Initialverdiproblemet er altså

$$\underline{y' = \frac{\cos(3x)}{y}, \quad y(0) = 1}$$

Vi løyser dette:

$$\frac{dy}{dx} = \frac{\cos(3x)}{y}$$

$$y\,dy = \cos(3x)\,dx$$

$$\int y\,dy = \int \cos(3x)\,dx$$

$$\frac{1}{2}y^2 = \frac{1}{3}\sin(3x) + C''$$

$$y = \pm\sqrt{\frac{2}{3}\sin(3x) + C'} \qquad (C' = 2C'')$$

Sidan $y(0) = +1$, må vi velge pluss-løysinga

$$y = +\sqrt{\frac{2}{3}\sin(3x) + C'}$$

Initialkrav: $y(0) = 1$

$$\sqrt{\frac{2}{3}\sin 0 + C'} = 1, \quad \sqrt{C'} = 1, \quad C' = 1$$

$$y(x) = \sqrt{\frac{2}{3}\sin(3x) + 1}$$