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HPC, matematikk og bevaringslover

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Hvem er jeg?

- Forsker på Meteorologisk Institutt
- Studert ved UiO, ferdig med doktorgrad i 2010
- Jobbet 10 år på SINTEF i Oslo
- Undervist på NITH (nå Westerdals), Universitetet i Oslo, OsloMet

- Jobber med matte og data til daglig

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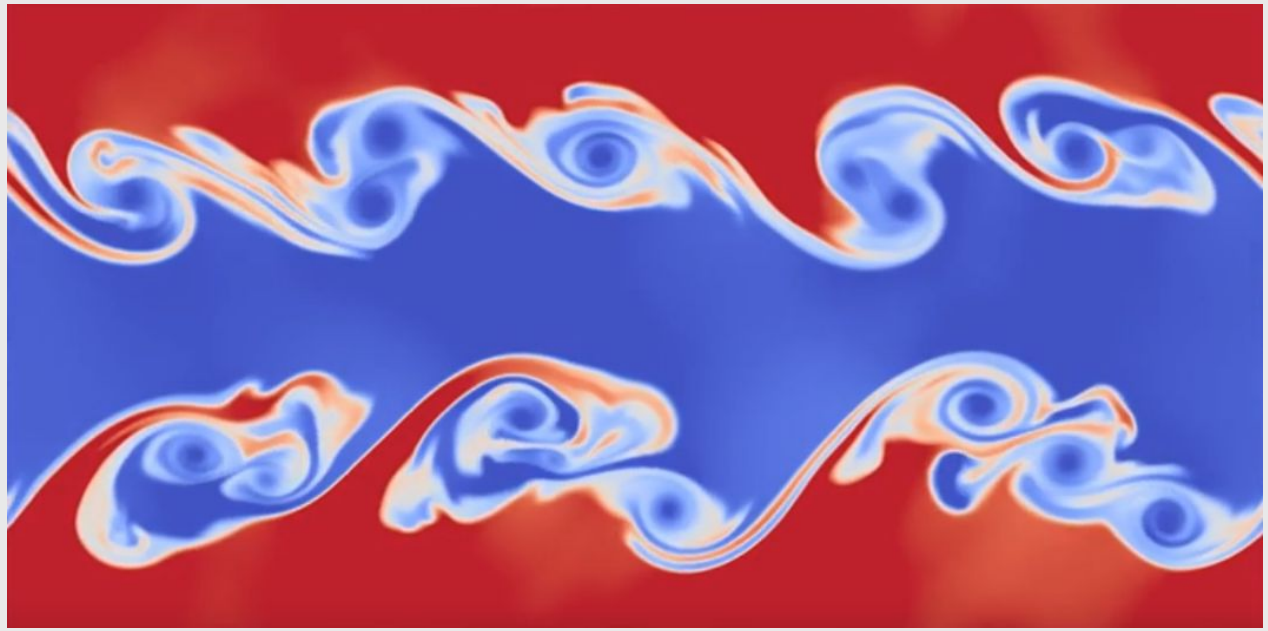
Dagens (popvit) forelesning

- Litt om matte og data: Hvorfor trenger vi matte, og hvorfor trenger vi data?
- Litt arbeid jeg har jobbet med
- Videoer!

Litt om komplisert matematikk

Fra matte til simuleringer

$$\rho\left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v}\cdot\nabla\mathbf{v}\right) = -\nabla p + \nabla\cdot\mathbb{T} + \mathbf{f}.$$



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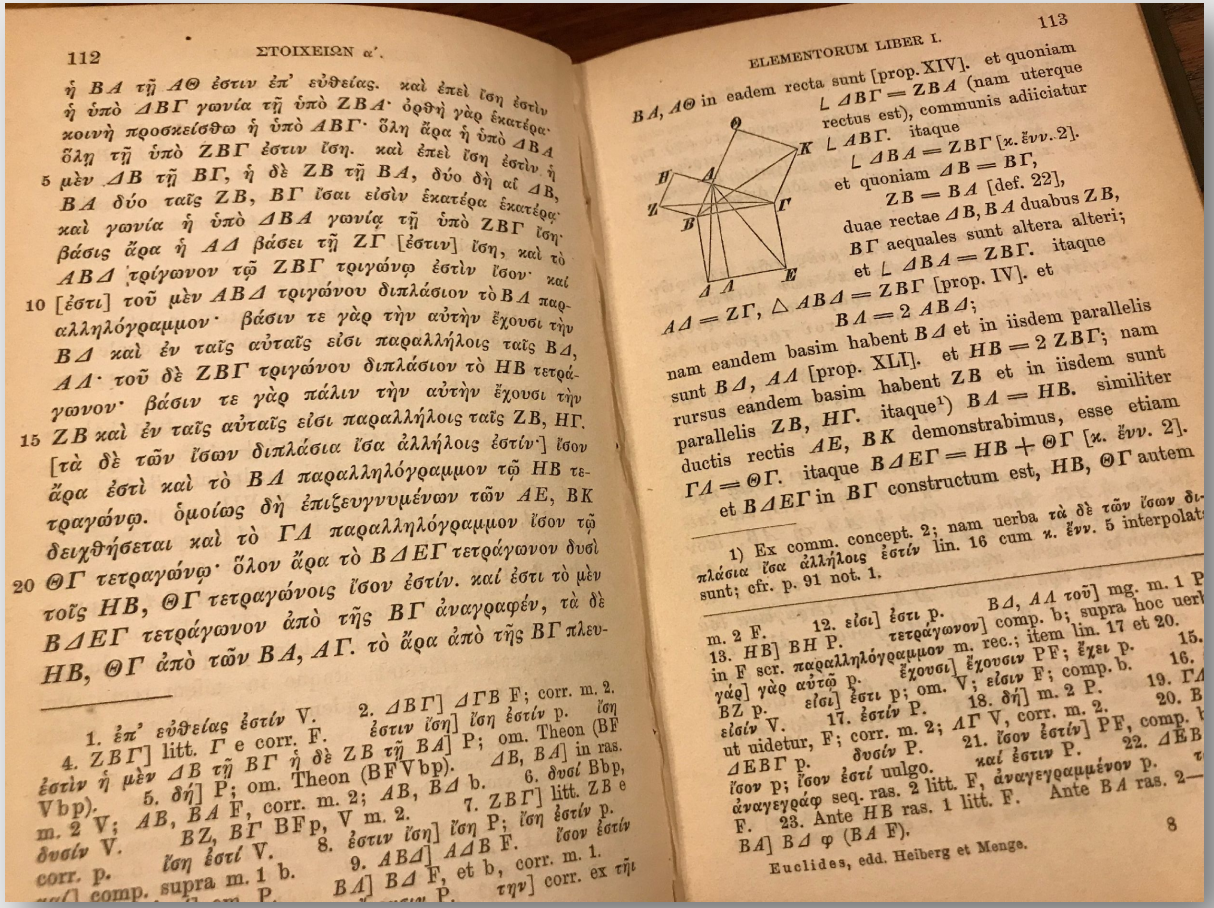
Matte er helt gresk for meg

- Euklids Elementa

  Gresk matematikkbok fra 300 fkr

  Euklidsk geometri, tallteori, …

- Wikpedia: Euclid's *Elements* has been referred to as the most successful[1][2] and influential[3] textbook ever written. It was one of the very earliest mathematical works to be printed after the invention of the printing press and has been estimated to be second only to the Bible in the number of editions published since the first printing in 1482,[3] with the number reaching well over one thousand.[4] For centuries, when the quadrivium was included in the curriculum of all university students, knowledge of at least part of Euclid's *Elements* was required of all students. Not until the 20th century, by which time its content was universally taught through other school textbooks, did it cease to be considered something all educated people had read.[5]



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Konserveringslover - bevaringslover

- Konservere – bevare

- Eksempel: Mengden vann vil ikke endres, men være konstant

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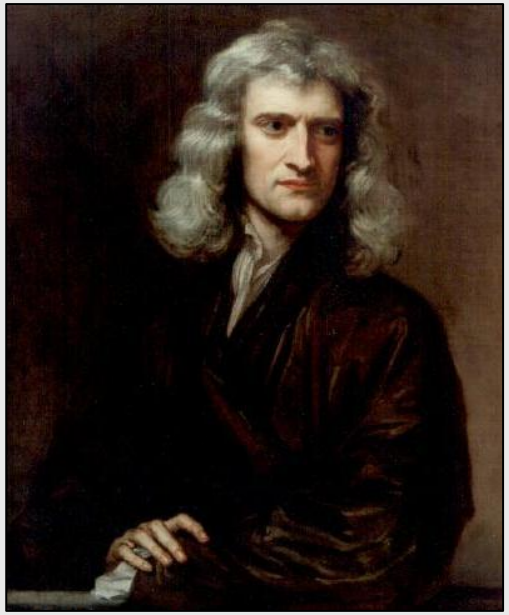
Conservation Laws



Isaac Newton, by Gottfried Kneller, public domain

- A conservation law describes that a quantity is conserved
- Comes from the physical laws of nature
- **Example: Newtons first law:** When viewed in an inertial reference frame, an object either remains at rest or continues to move at a constant velocity, unless acted upon by an external force.
- **Example: Newtons third law:** When one body exerts a force on a second body, the second body simultaneously exerts a force equal in magnitude and opposite in direction on the first body.
- More examples: conservation of mass (amount of water) in shallow water, amount of energy (heat) in the heat equation, linear momentum, angular momentum, etc.
- Conservation laws are mathematically formulated as partial differential equations: PDEs

Datamaskinarkitektur endrer seg!

The microprocessor

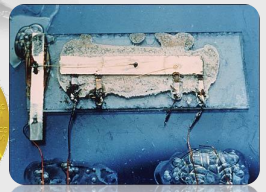
1942: Digital Electric Computer

(Atanasoff and Berry)

1956

1947: Transistor

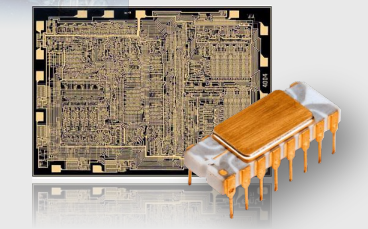
(Shockley, Bardeen, and Brattain)



2000

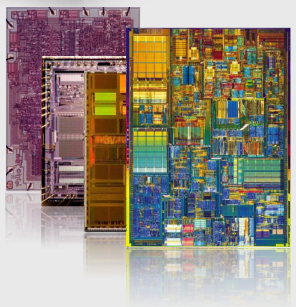
1958: Integrated Circuit

(Kilby)



1971: Microprocessor

(Hoff, Faggin, Mazor)



1971- Exponential growth

(Moore, 1965)

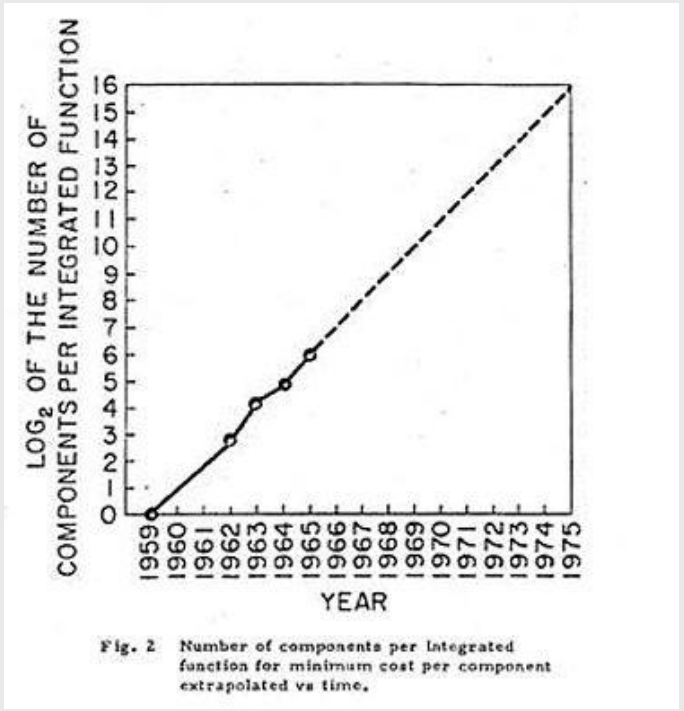


Fig. 2 Number of components per integrated function for minimum cost per component extrapolated vs time.

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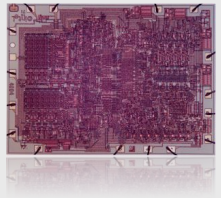
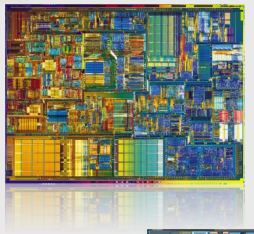
The microprocessor



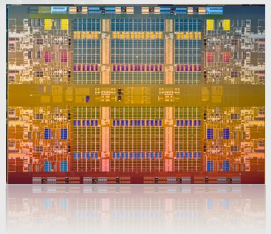
1971: 4004,
2300 trans, 740 KHz

1982: 80286,
134 thousand trans, 8 MHz

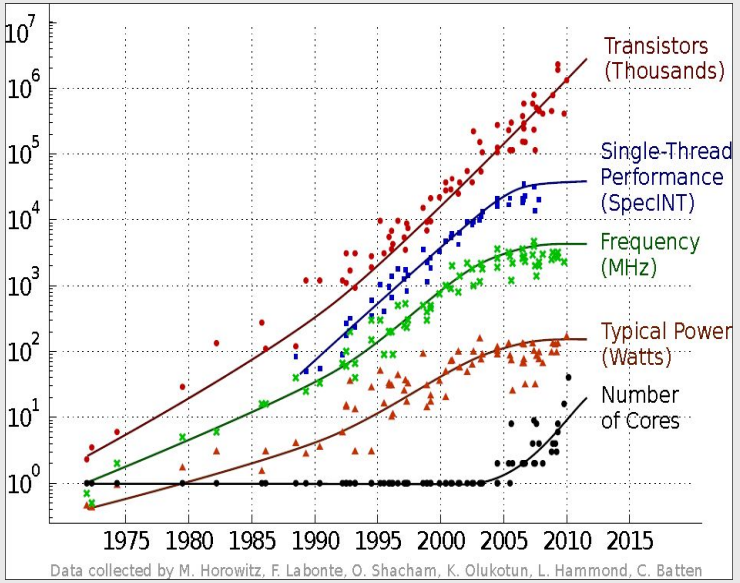
1993: Pentium P5,
1.18 mill. trans, 66 MHz



2000: Pentium 4,
42 mill. trans, 1.5 GHz



2010: Nehalem
2.3 bill. Trans, 8 cores, 2.66 GHz



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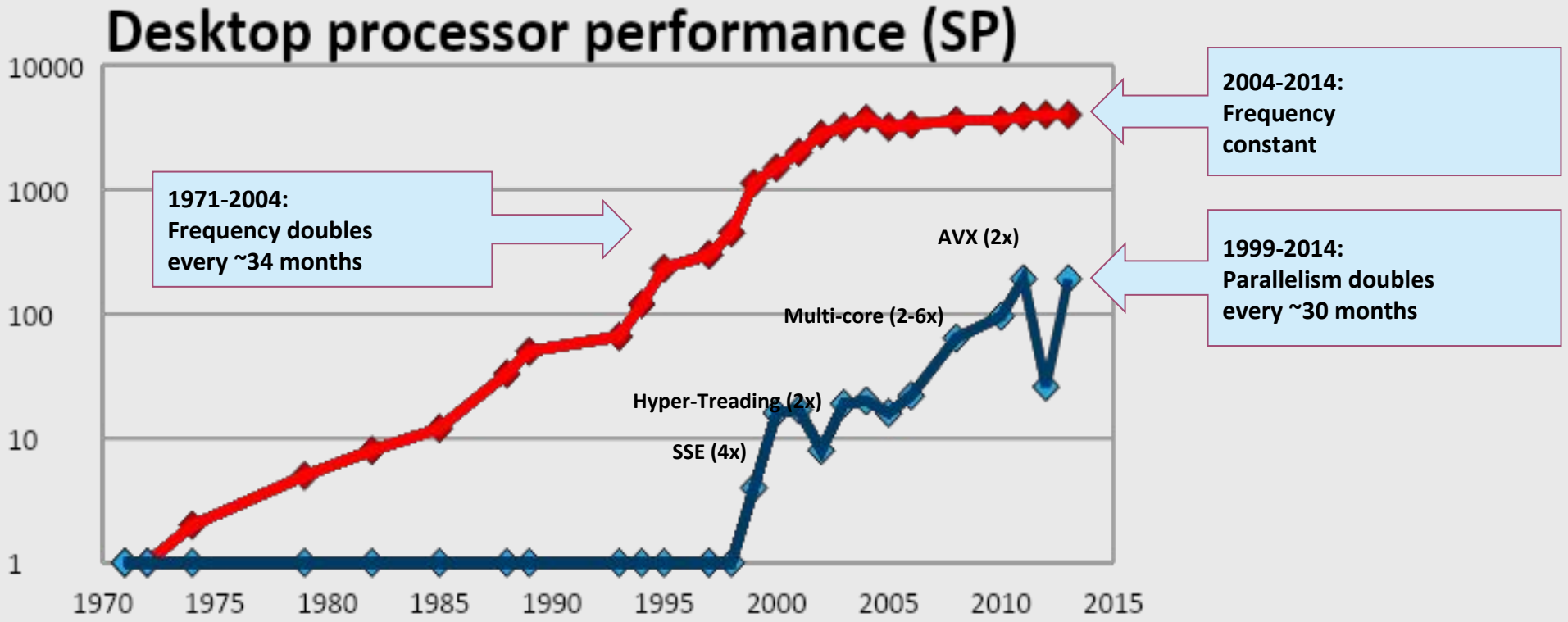
End of frequency scaling



- 1970-2004: Frequency doubles every 34 months (Moore's law for performance)
- 1999-2014: Parallelism doubles every 30 months

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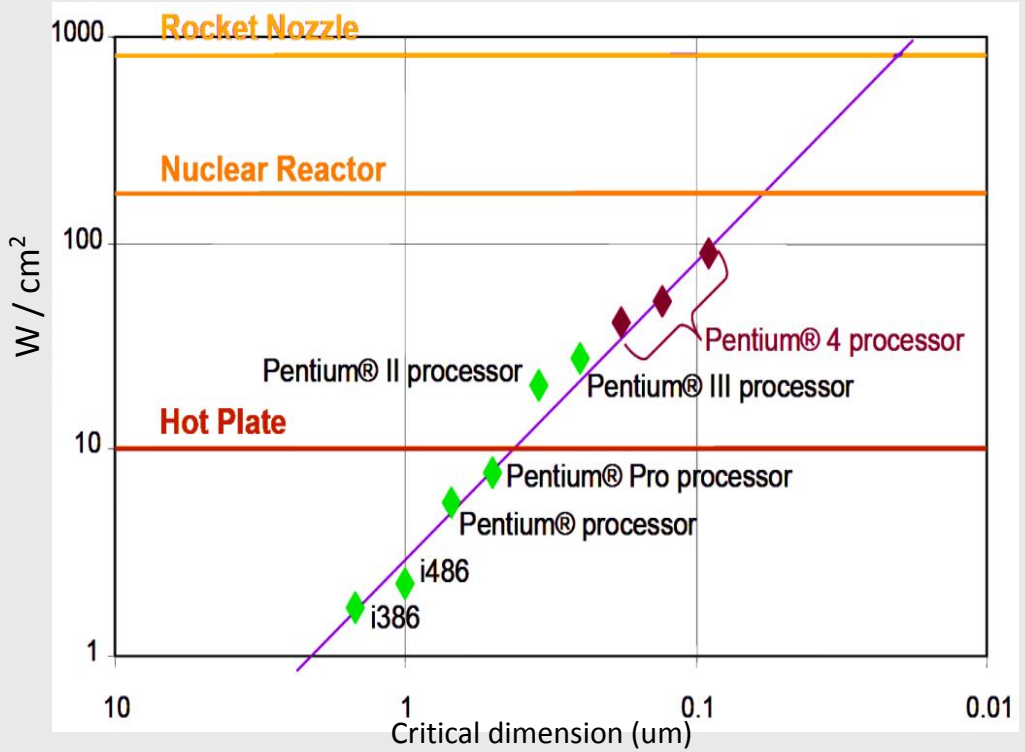
What happened in 2004?

- Heat density approaching that of nuclear reactor core: Power wall
- Traditional cooling solutions (heat sink + fan) insufficient
- Industry solution: multi-core and parallelism!



Graph taken from G. Taylor, "Energy Efficient Circuit Design and the Future of Power Delivery" EPEPS'09

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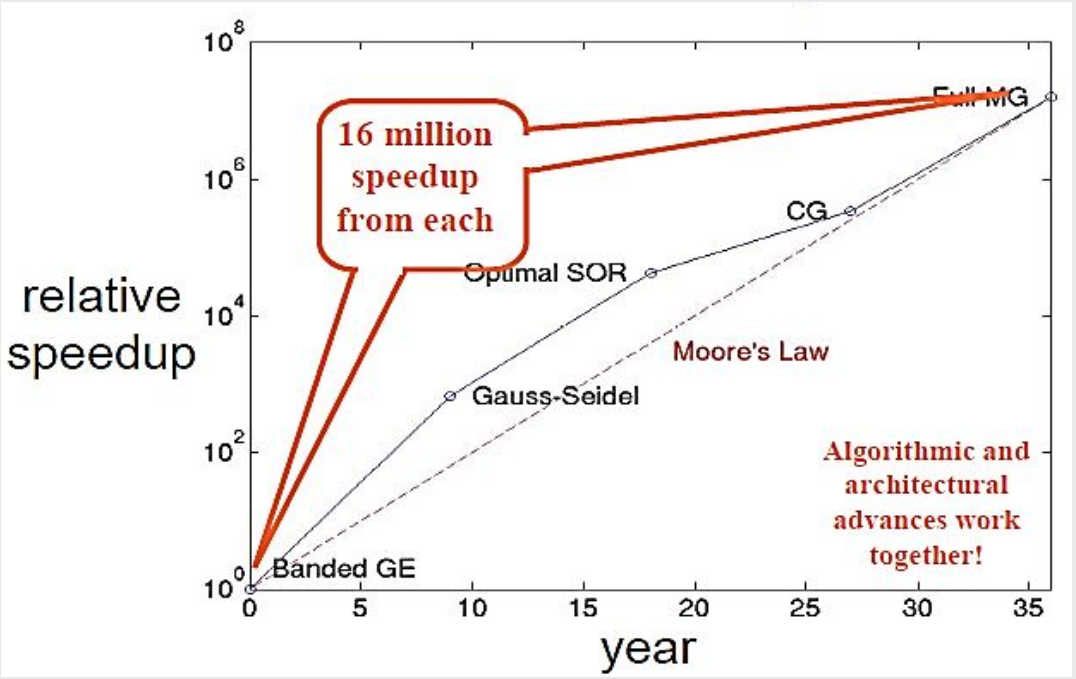
Why care about mathematics?

- The key to increasing performance, is to consider the full algorithm and architecture interaction.
- A good knowledge of both the algorithm and the computer architecture is required.



Graph from David Keyes, Scientific Discovery through Advanced Computing, Geilo Winter School, 2008

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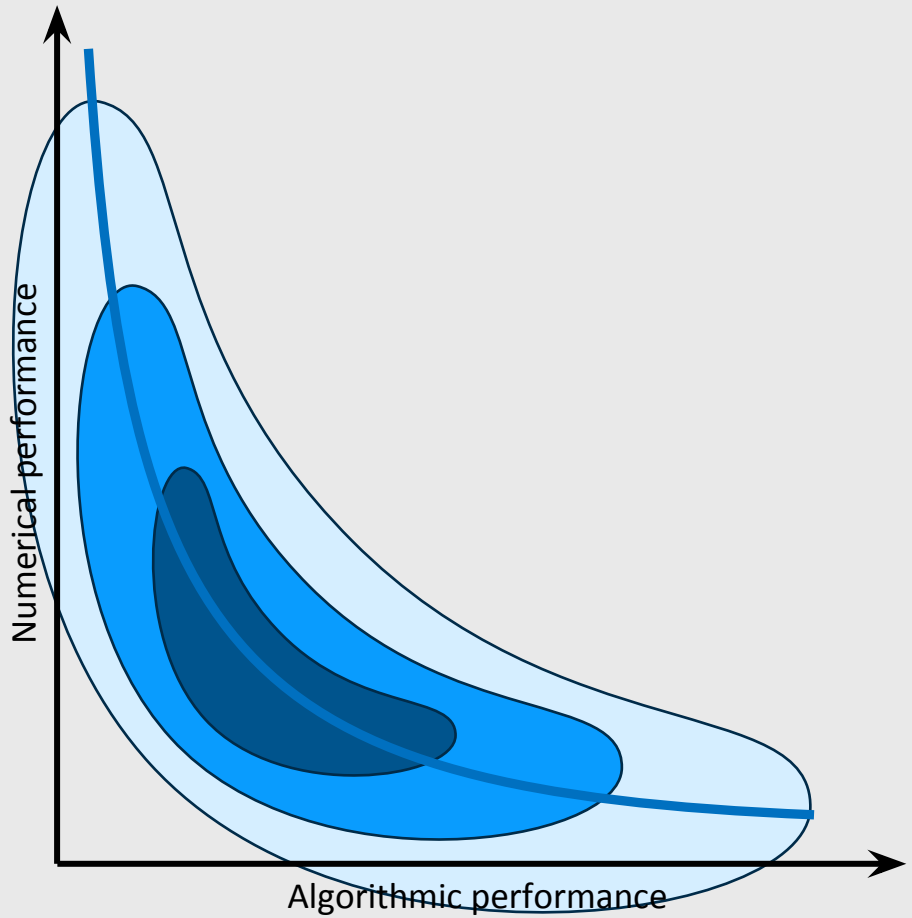
Algorithmic and numerical performance

- Total performance is the product of algorithmic **and** numerical performance
- Your mileage may vary: algorithmic performance is highly problem dependent
- Many algorithms have low numerical performance
- Need to consider both the algorithm and the architecture for maximum performance



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Når matte og data ikke spiller på lag

The patriot missile

- Designed by Raytheon (US) as an air defense system.
- Designed for time-limited use (up-to 8 hours) in mobile locations.
- Heavily used as static defenses using the Gulf war.
- Failed to intercept an incoming Iraqi Scud missile in 1991.
- 28 killed, 98 injured.

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The patriot missile

- It appears, that 0.1 seconds is not really 0.1 seconds…
  Especially if you add a large amount of them

Python:

```
> print 0.1
0.1
> print "%.10f" % 0.1
0.1000000000
> print "%.20f" % 0.1
0.10000000000000000555
> print "%.30f" % 0.1
0.100000000000000005551115123126
```

Hours	Inaccuracy (sec)	Approx. shift in Range Gate (meters)
0	0	0
1	.0034	7
8	.0025	55
20	.0687	137
48	.1648	330
72	.2472	494
100	.3433	687

http://sydney.edu.au/engineering/it/~alum/patriot_bug.html

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Matte jeg har jobbet med

Grafikkort og matte

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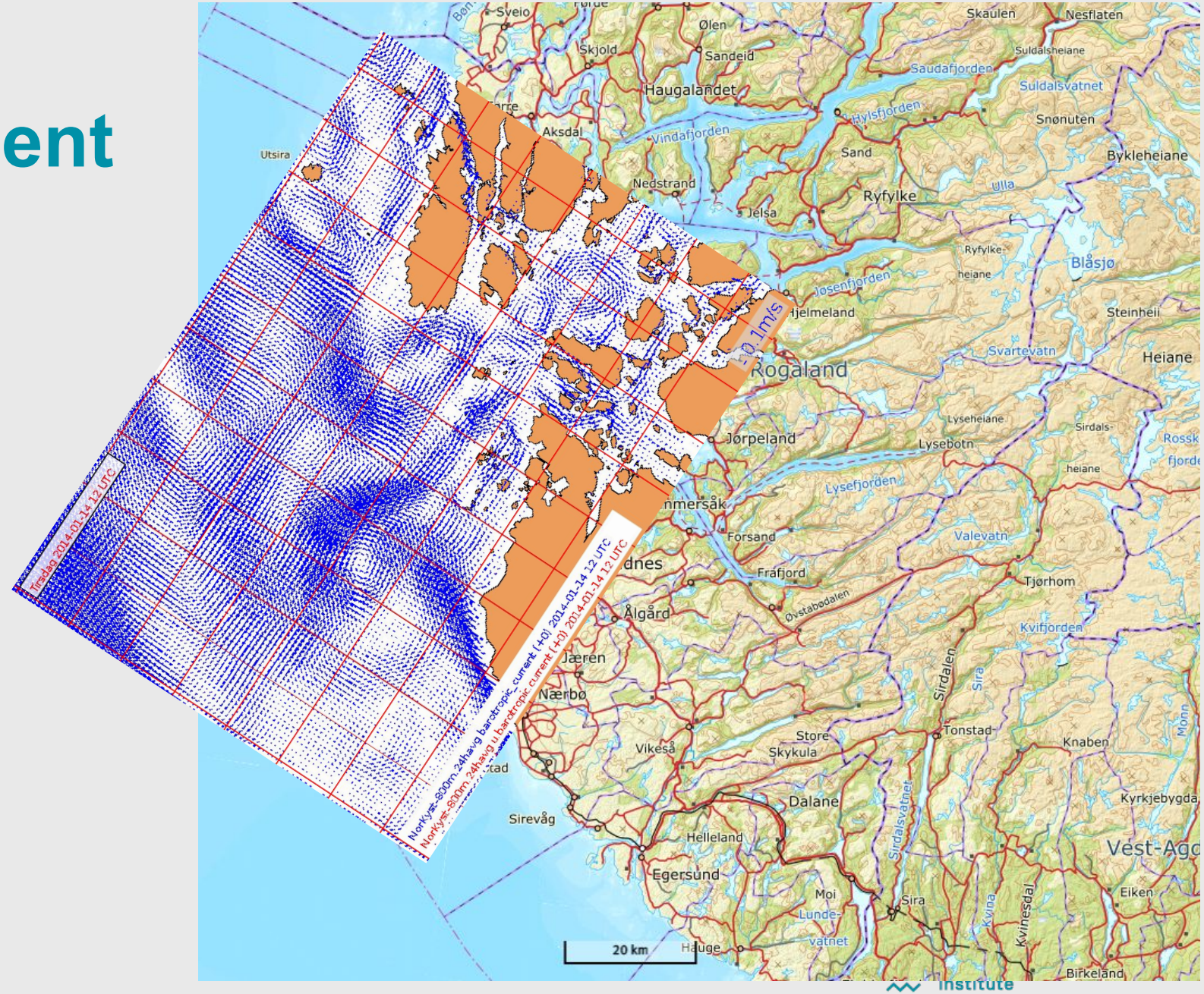
Problem statement

- Very small scale features
- Almost no measured data!
- Computationally demanding



Drift trajectory modeling - uncertainties

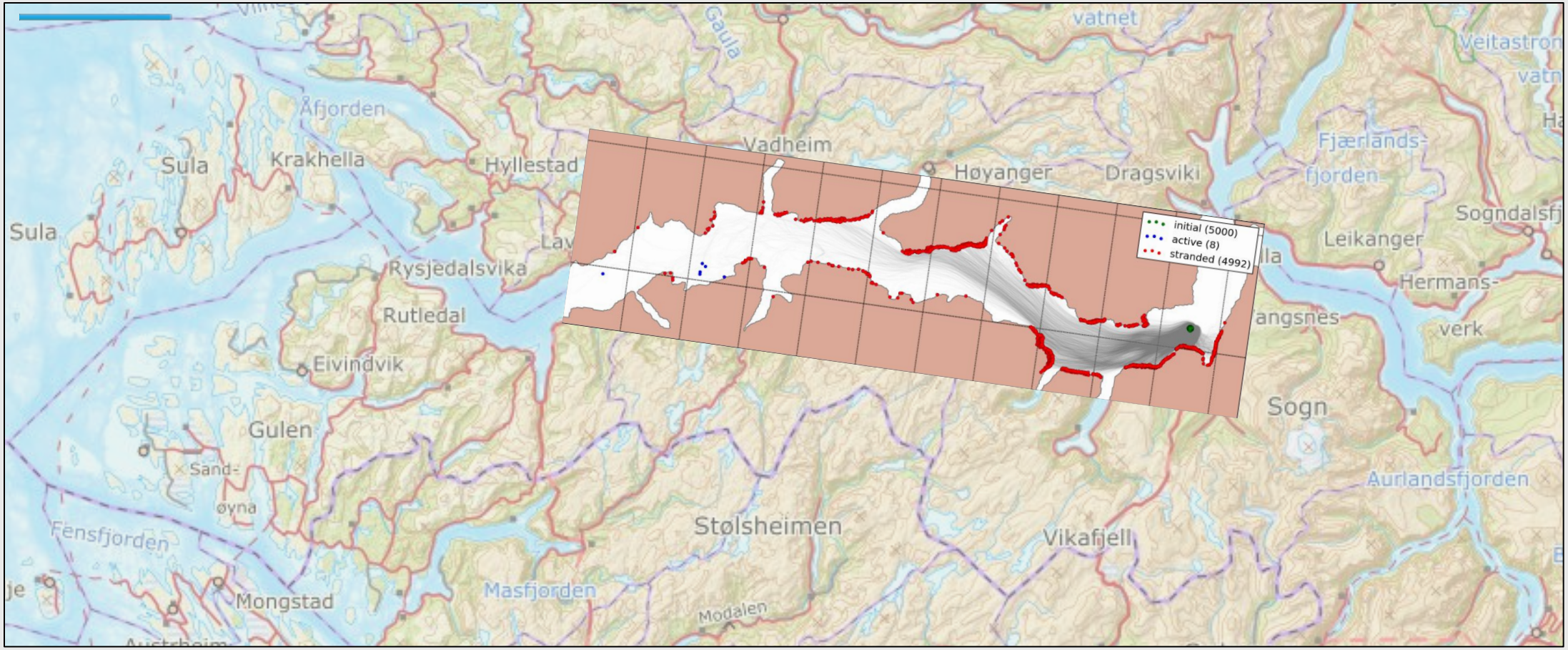


Image from Opendrift, Norwegian Meteorological Institute.
Knut-Frode Dagestad.

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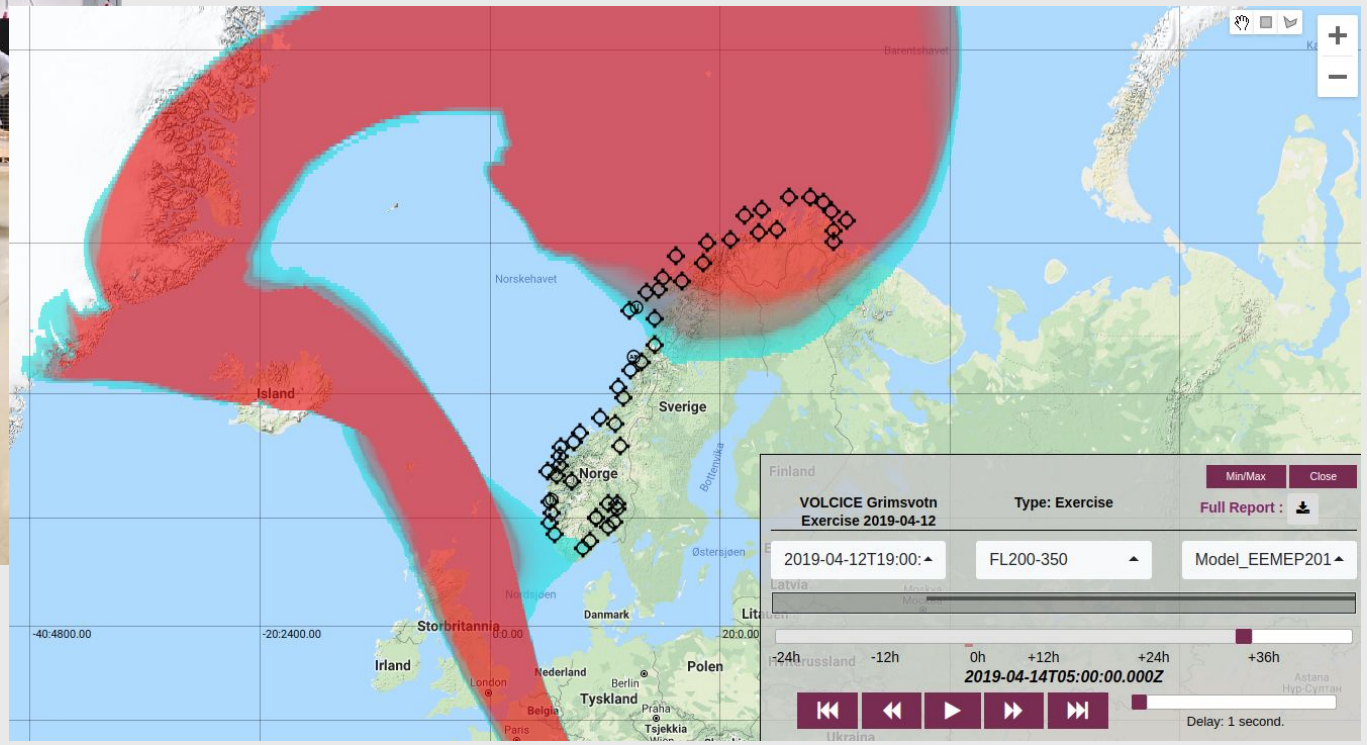
Volcanic ash

- Is it safe to fly?
- What air space should be shut down?
- http://ippc.no/



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Videoer

- Linear wave equation: https://www.youtube.com/watch?v=9kGnLusDUEU
- Shallow water: https://www.youtube.com/watch?v=RKzZgt-_EcQ
- Euler equations: https://www.youtube.com/watch?v=TKmZ5KVyJy8
- Teaching demos: https://www.youtube.com/watch?v=02iziqMqhzU
- Equelle: https://www.youtube.com/watch?v=lf9BJ2UU3Tw
- IQmulus: https://www.youtube.com/watch?v=rn_EYGt3jcU
- ADABTS: https://www.youtube.com/watch?v=JvQJHA0EI2E
- Shallow water: https://www.youtube.com/watch?v=FbZBR-FjRwY
- Kelvin-Helmholz: https://www.youtube.com/watch?v=ejCzlwIT--w
- Shock-bubble: https://www.youtube.com/watch?v=IhgFGH_kfQM
- Rayleigh-Taylor: https://www.youtube.com/watch?v=b5kzGsQlSWg
- Mandelbrot: https://www.youtube.com/watch?v=O-kxB4-Laio
- NorKyst800: https://www.youtube.com/watch?v=dRwt8Oq8imo

Oppsummering

Oppsummering

- Matte er utrolig viktig i samfunnet - alle bruker matte!
- Matte er “logisk”!
  - Ting virker ofte mye vanskeligere enn de er: konseptene er ofte enkle
  - Har man forstått konseptene så kommer detaljene på plass
- Kobling mellom matte og data er enormt viktig for effektiv problemløsing
  - Du må forstå både matematikk og data for å simulere

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