Førelesing BYFE 1000

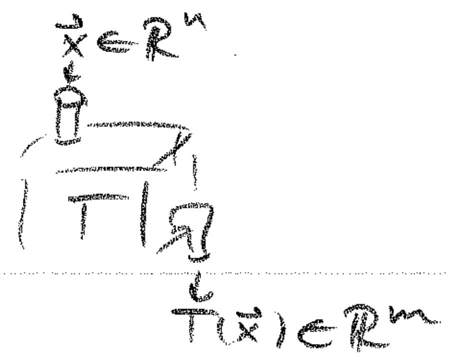
23/9 -14

(1) Beskjedar

- Oblig 1 lagt ut
- Viktig info. om obligane (Fronter)
- Obligframside

(2) Repetisjon

Transformasjon



Lineær trans.

a) $T(c\vec{x}) = cT(\vec{x})$

NB: Vektorar <u>skal ha</u> pil over

b) $T(\vec{x}+\vec{y}) = T(\vec{x})+T(\vec{y})$

Konsekvensar: $T(\vec{0}) = \vec{0}$

$T(c_1\vec{x}_1 + c_2\vec{x}_2 + \dots + c_p\vec{x}_p) = c_1T(\vec{x}_1) + \dots + c_pT(\vec{x}_p)$

Eksempel : Lorenz-transformasjonen

(3) Eksempel frå notat 16/9, s.9 og utover

(4) <u>Eksempel</u>

a) Transformasjonen $T: \mathbb{R}^2 \to \mathbb{R}^2$ går ut på å flytte $\vec{x}$ med vektoren $\vec{p} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$ (translasjon): $\vec{x} \mapsto \vec{x} + \vec{p}$. Er T lineær?

b) Trans $U: \mathbb{R}^3 \to \mathbb{R}^3$ er projeksjon ned i x_2x_3-planet (yz-planet). Vis at U er lineær.

c) Bestem matrisa A slik at $U(\vec{x}) = A\vec{x}$.

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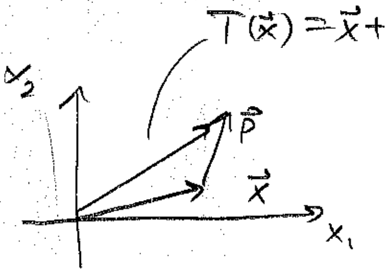
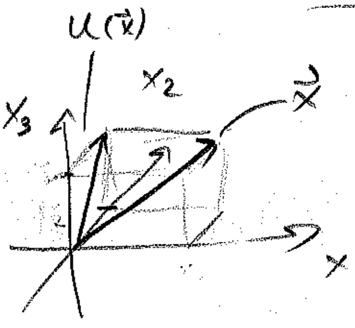
a) $T(\vec{0}) = \vec{0} + \vec{p} = \vec{p} \neq \vec{0}$

T er <u>ikkje lineær</u>



b)



Ser: $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \longmapsto \begin{bmatrix} 0 \\ x_2 \\ x_3 \end{bmatrix}$

a) $U(c\vec{x}) = U\left(\begin{bmatrix} cx_1 \\ cx_2 \\ cx_3 \end{bmatrix}\right) = \begin{bmatrix} 0 \\ cx_2 \\ cx_3 \end{bmatrix} = c \begin{bmatrix} 0 \\ x_2 \\ x_3 \end{bmatrix} = c\,U(\vec{x})$ Ok

b) $U(\vec{x}+\vec{y}) = U\left(\begin{bmatrix} x_1+y_1 \\ x_2+y_2 \\ x_3+y_3 \end{bmatrix}\right) = \begin{bmatrix} 0 \\ x_2+y_2 \\ x_3+y_3 \end{bmatrix} = \begin{bmatrix} 0 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ y_2 \\ y_3 \end{bmatrix} = U(\vec{x}) + U(\vec{y})$

Ok

U er lineær ▷

c) $U\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} 0 \\ x_2 \\ x_3 \end{bmatrix} = x_1 \cdot \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + x_3 \cdot \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} =$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Altså: $U(\vec{x}) = A\vec{x}$ der $A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

A: <u>Standardmatrisa</u> til U.

(5) Oppsummering

Alle lineære transformasjonar $T(\vec{x})$ frå $\mathbb{R}^n$ til $\mathbb{R}^m$ kan skrivast som matrisemultiplikasjon:

$T(\vec{x}) = A\vec{x}$.

[?] A må ha formatet $m \times n$. Denne matrisa, som vi kallar <u>standardmatrisa til T</u>, er unik.

Korleis bestemmer vi A?

Éin måte:

Med $\vec{x} \in \mathbb{R}^3$: $\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = x_1 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$= x_1 \vec{e}_1 + x_2 \vec{e}_2 + x_3 \vec{e}_3$

$\vec{e}_1, \vec{e}_2, \vec{e}_3$: Einingsvektorar [enhetsvektorer] langs kvar av aksane

I $\mathbb{R}^2$: $\vec{e}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $\vec{e}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ (ingen $\vec{e}_3$)

i $\mathbb{R}^5$: $\vec{e}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$, $\vec{e}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$, ..., $\vec{e}_5 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$

T lineær

$T(\vec{x}) = T(x_1 \vec{e}_1 + x_2 \vec{e}_2 + x_3 \vec{e}_3) =$

$x_1 T(\vec{e}_1) + x_2 T(\vec{e}_2) + x_3 T(\vec{e}_3) =$

$\underbrace{[T(\vec{e}_1) \mid T(\vec{e}_2) \mid T(\vec{e}_3)]}_{A} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}}_{\vec{x}} = A\vec{x}$

Generelt, med $\mathbb{R}^n$ som frårom:

Standardmatrisa til T er

$[T(\vec{e}_1) \mid T(\vec{e}_2) \mid \dots \mid T(\vec{e}_n)]$

Eksempel

Finn standardmatrisene til desse transformasjonane

a) $T : \mathbb{R}^2 \to \mathbb{R}^3$ der $T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 + x_2 \\ x_1 - x_2 \\ 2x_1 - 3x_2 \end{bmatrix}$

b) $U : \mathbb{R}^2 \to \mathbb{R}^2$ der U er spegling om x_2-aksen

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a) Som i tidlegare eksempel:

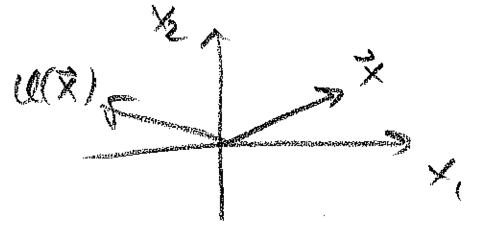
$$T(\vec{x}) = x_1 \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} + x_2 \begin{bmatrix} 1 \\ -1 \\ -3 \end{bmatrix} = \underline{\begin{bmatrix} 1 & 1 \\ 1 & -2 \\ 2 & -3 \end{bmatrix} \vec{x}}$$

Eller:

$$T(\vec{e}_1) = T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 1+0 \\ 1-0 \\ 2\cdot1 - 3\cdot0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

$$T(\vec{e}_2) = T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 0+1 \\ 0-1 \\ 2\cdot0 - 3\cdot1 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ -3 \end{bmatrix}$$

Standardmatrise: $[T(\vec{e}_1) | T(\vec{e}_2)] = \underline{\begin{bmatrix} 1 & 1 \\ 1 & -1 \\ 2 & -3 \end{bmatrix}}$

b) 

$$U(\vec{e}_1) = -\vec{e}_1 = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$$

$$U(\vec{e}_2) = \vec{e}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Standardmatrise: $[U(\vec{e}_1) | U(\vec{e}_2)] = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$

Generelt:

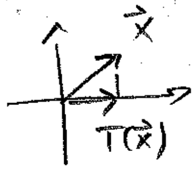
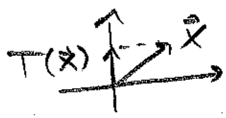
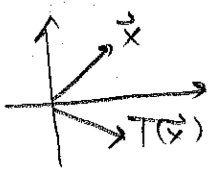
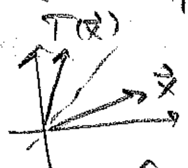
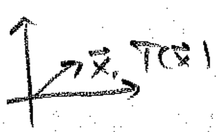
$$T\left(\begin{bmatrix}x_1\\x_2\end{bmatrix}\right)=\begin{bmatrix}-1&0\\0&1\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix}=\begin{bmatrix}-x_1\\x_2\end{bmatrix}$$

Poeng:

Ved å bestemme $T(\vec{e}_1)$, $T(\vec{e}_2)$ etc. får vi bestemt $T(\vec{x})$ for <u>hva $\vec{x}$ som helst</u>.

⑥ Geometriske trans. i $\mathbb{R}^2$

$\boxed{?}$

	Ill.	$T(\vec{x})=?$	Standardm.
Proj. på x_1-aksen		$\begin{bmatrix}x_1\\0\end{bmatrix}$	$\begin{bmatrix}1&0\\0&0\end{bmatrix}$
Proj. på x_2-aksen		$\begin{bmatrix}0\\x_2\end{bmatrix}$	$\begin{bmatrix}0&0\\0&1\end{bmatrix}$
Speiling om x_1-aksen		$\begin{bmatrix}x_1\\-x_2\end{bmatrix}$	$\begin{bmatrix}1&0\\0&-1\end{bmatrix}$
— " — x_2-aksen	(eksempel)	$\begin{bmatrix}-x_1\\x_2\end{bmatrix}$	$\begin{bmatrix}-1&0\\0&1\end{bmatrix}$
— " — $x_2=x_1$-linja		$\begin{bmatrix}x_2\\x_1\end{bmatrix}$	$\begin{bmatrix}0&1\\1&0\end{bmatrix}$
Gjøre lengre/kortere		$k\begin{bmatrix}x_1\\x_2\end{bmatrix}$	$\begin{bmatrix}k&0\\0&k\end{bmatrix}$
Uendret		$\begin{bmatrix}x_1\\x_2\end{bmatrix}$	$\underbrace{\begin{bmatrix}1&0\\0&1\end{bmatrix}}_{I_2}$

(7) Identitetsmatrisa

I_n — har $\vec{e}_1, \vec{e}_2, \dots, \vec{e}_n$ som søyler

I $\mathbb{R}^2$: $\quad I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\mathbb{R}^3$: $\quad I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

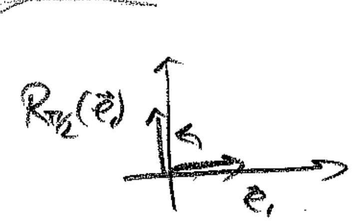
Er slik at $I\vec{x} = \vec{x}$ — uansett hva $\vec{x}$ er.

(8) Eksempel

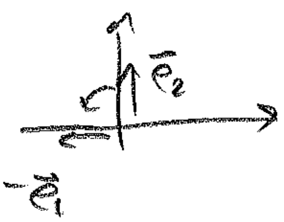
Transformasjonen $R_\theta : \mathbb{R}^2 \to \mathbb{R}^2$ roterer $\vec{x}$ θ rad mot klokka.

a) Finn standardmatrisa til $R_{\pi/2}$.

b) —— " —— R_θ.



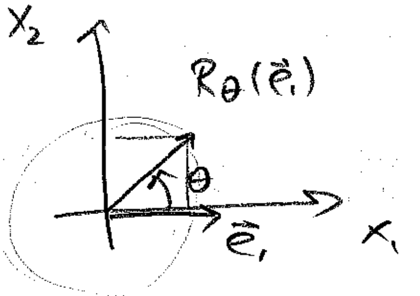
$$R_{\pi/2}(\vec{e}_1) = \vec{e}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$



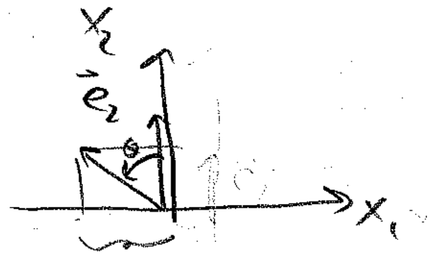
$$R_{\pi/2}(\vec{e}_2) = -\vec{e}_1 = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$$

Standardmatr.: $[R_{\pi/2}(\vec{e}_1) \mid R_{\pi/2}(\vec{e}_2)] = \underline{\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}}$

b)



$$R_\theta(\vec{e}_1) = \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix}$$



$$R_\theta(\vec{e}_2) = \begin{bmatrix} -\sin\theta \\ \cos\theta \end{bmatrix}$$

Standardmatr.: $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

Spesielt, for $\theta = \pi/2$:

$$\begin{bmatrix} \cos\frac{\pi}{2} & -\sin\frac{\pi}{2} \\ \sin\frac{\pi}{2} & \cos\frac{\pi}{2} \end{bmatrix} = \underline{\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}}$$

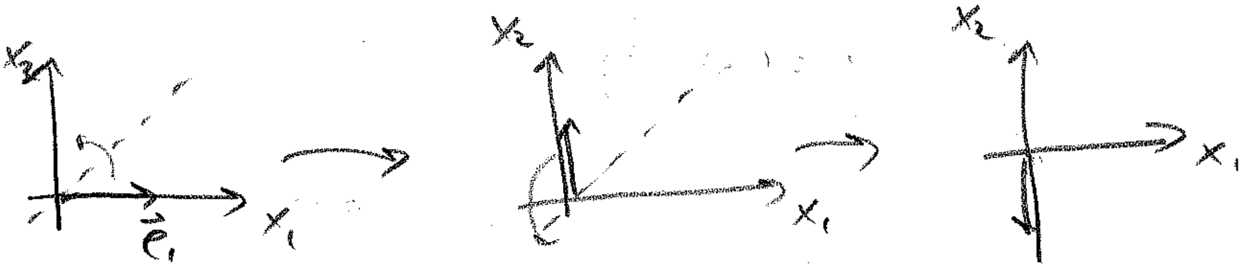
Eksempel

Trans. T består i å fyrst „flippe" $\vec{x}$ om linje $x_2 = x_1$, så spegle om x_1-aksen.

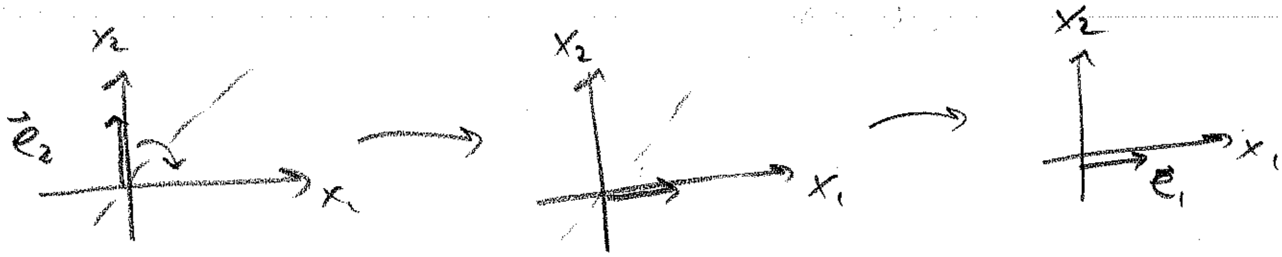
a) Finn standardmatrisa til T.

b) — " ——— " — transformasjonen U, som går ut på å gjere det same i motsett rekkefølge.

a)



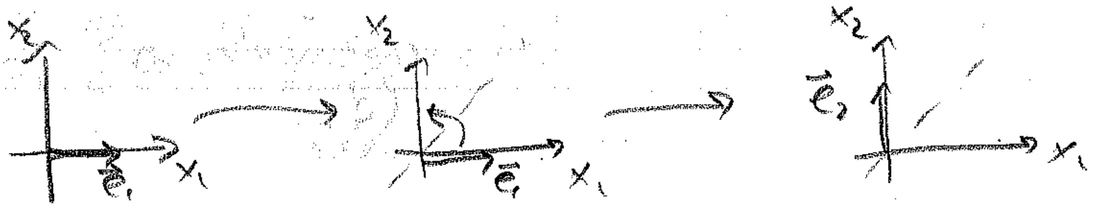
$T(\vec{e}_1) = -\vec{e}_2 = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$



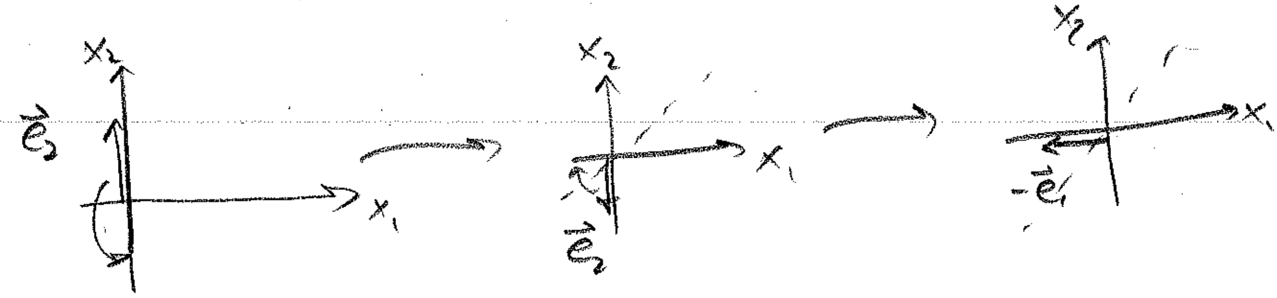
$T(\vec{e}_2) = \vec{e}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

Standardmatr.: $[T(\vec{e}_1) | T(\vec{e}_2)] = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

b)



$U(\vec{e}_1) = \vec{e}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$



$U(\vec{e}_2) = -\vec{e}_1 = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$

Standardmatr.: $[U(\vec{e}_1) | U(\vec{e}_2)] = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

[?] Spelar rekkefølga noko rolle?

→ Ja