1 Repetisjon M 1000 18 mars 2016

Faush

Integral av polynomer, rasjonale funksjoner.

potensfunksjoner (x^n).

* $\int \overbrace{2 \cdot x^3}^{\text{integrand}} dx = 2 \int x^3 dx = 2 \frac{x^{3+1}}{3+1} + c$

$= \underline{\frac{1}{2} x^4 + c}$

* $\int_1^4 3 \cdot (x-2)^2 dx$ substitusjon $u = x-2$

$du = dx$

$= 3 \int_{u(1)}^{u(4)} u^2 du$

$= 3 \int_{-1}^{2} u^2 du = 3 \left(\frac{u^3}{3}\right)\Big|_{-1}^{2} = u^3 \Big|_{-1}^{2}$

$= 2^3 - (-1)^3$

$= 8 - (-1) \underline{\underline{= 9}}$

alternativt: $\int 3(x-2)^2 dx = (x-2)^3 + c$

$\int_1^4 3(x-2)^2 dx = (x-2)^3 + c \Big|_1^4 = \dots = \underline{\underline{9}}$

$\int 5.39 (2.36x - 1.78)^8 dx$ $u = 2.36x - 1.78$

$du = 2.36 dx$

$= \int \frac{5.39}{2.36} (u)^8 du$

$= \frac{5.39}{2.36} \frac{u^9}{9} + c = \underline{\frac{5.39}{2.36 \cdot 9} (2.36x - 1.78)^9 + c}$

②

$$* \int (5x-2)^{7/3}\, dx = \int (5x-2)^2 \cdot \sqrt[3]{5x-2}\, dx$$

Linear substitusjon: $u = 5x-2$

$du = 5dx$

$$\int u^{7/3} \frac{1}{5} du = \frac{1}{5} \frac{u^{\frac{7}{3}+1}}{\frac{7}{3}+1} + c$$

$$= \frac{1}{5} \frac{u^{10/3}}{10/3} \frac{\cdot 3}{\cdot 3} + c$$

$$= \underline{\frac{3}{50} (5x-2)^{10/3} + c}$$

$$\left(= \frac{3}{50} (5x-2)^3 \cdot \sqrt[3]{5x-2} + c \right)$$

$$* \int \sqrt[3]{x} \cdot \sqrt{x} \cdot x\, dx$$

$$= \int x^{1/3} \cdot x^{1/2} \cdot x^1\, dx$$

$$= \int x^{\frac{1}{3}+\frac{1}{2}+1}\, dx = \int x^{\frac{5}{6}+1}\, dx = \int x^{\frac{11}{6}}\, dx$$

$$= \frac{x^{\frac{11}{6}+1}}{\frac{11}{6}+1} + c = \frac{x^{17/6}}{17/6} + c$$

$$= \underline{\frac{6}{17} x^{17/6} + c} \left(= \frac{6}{17} x^2 \cdot \sqrt[6]{x^5} + c \right.$$

$$\left. = \frac{6}{17} x^2 \sqrt{x}\, \sqrt[3]{x} + c \right)$$

③ $\int_1^5 \frac{1}{\sqrt{2x-1}}\,dx$

Vi forsøker med substitusjonen $u = 2x-1$

$du = 2\,dx$

($u=\sqrt{2x-1}$ fungerer også ...)

$$\int_{u(1)}^{u(5)} \frac{1}{\sqrt{u}} \cdot \frac{1}{2}du = \frac{1}{2}\int_1^9 \frac{1}{u^{1/2}}\,du$$

$$= \frac{1}{2}\int_1^9 u^{-1/2}\,du = \frac{1}{2}\cdot\left(\frac{u^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}\right)\Bigg|_1^9$$

$$\frac{1}{2}\cdot\frac{u^{1/2}}{1/2}\Bigg|_1^9 = \sqrt{u}\,\Big|_1^9 = \sqrt{9}-\sqrt{1} = \underline{\underline{2}}$$

Eksamensoppg. 12. 2015

$$\int_1^2 \frac{3}{\sqrt[4]{x^3}} + \sqrt{4-2x}\,dx$$

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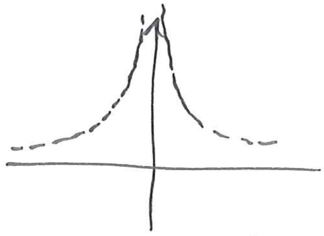
$$\int_{-1}^{1} \frac{1}{x^2}\,dx$$



$$= \cancel{-\frac{1}{x}\Big|_{-1}^{1} = \frac{-1}{1} - \left(\frac{-1}{-1}\right) = -1-1=-2}$$

GALT!

$\int_{-1}^{1} \frac{1}{x^2}\,dx$ eksisterer ikkje.

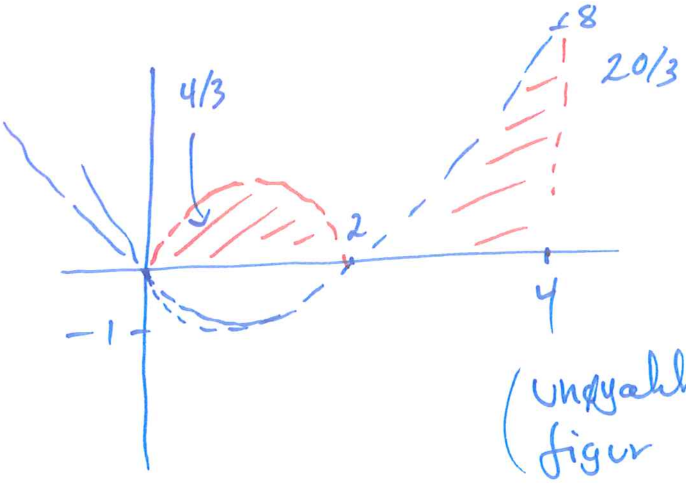
$$\lim_{a\to 0^-}\int_{-1}^{a}\frac{1}{x^2}\,dx \qquad \lim_{b\to 0^+}\int_{b}^{1}\frac{1}{x^2}\,dx$$

$$\lim_{a\to 0^-}\left(\frac{-1}{a} - \left(\frac{-1}{-1}\right)\right) + \lim_{b\to 0^+}\left(\frac{-1}{1}+\frac{1}{b}\right)$$

Eksisterer ikkje!

$$\int_0^4 |x^2-2x|\,dx$$

5



(ungefärlig figur)

$$-\int_0^2 (x^2-2x)\,dx$$

$$+\int_2^4 (x^2-2x)\,dx$$

$$= -\left(\frac{x^3}{3}-x^2\right)\Big|_0^2 + \frac{x^3}{3}-x^2\Big|_2^4$$

$$= -\left(\frac{8}{3}-4\right)\cdot 2 + \left(\frac{4^3}{3}-16\right)$$

$$= \frac{4}{3}\cdot 2 + 16\left(\frac{4}{3}-1\right)$$

$$= \frac{8}{3} + \frac{16}{3} = \underline{\underline{\frac{24}{3}}}$$

Ikke-lineære substitusjoner

Eks. oppg august 2015

* $\int x(1+x^2)^6\,dx$

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Substitusjon

$u = 1+x^2$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$$\int u^6 \cdot \frac{1}{2}du$$

$$= \frac{1}{2}\frac{u^7}{7} + C$$

$$= \underline{\frac{1}{14}(1+x^2)^7 + C}$$

ganger ut $(1+x^2)^3$

* $\int (1+x^2)^3 dx = \int 1 + 3x^2 + 3x^4 + x^6\,dx$

$$= x + x^3 + \frac{3}{5}x^5 + \frac{x^7}{7} + C$$

* $\int \frac{x^5}{1+2x^6+x^{12}}dx = \int \frac{x^5}{(1+x^6)^2}dx$

$u = 1+x^6 \qquad du = 6x^5 dx$

$$\int \frac{1}{u^2}\frac{1}{6}du = \frac{-1}{6u} + C = \underline{\frac{-1}{6(1+x^6)} + C}$$

7 $\int \frac{1}{1+x^2}\,dx = \arctan(x)$

$y = \tan(x)$

$x = \arctan(y)$

$$\frac{dy}{dx} = (\tan x)' = 1+\tan^2 x$$
$$= 1+y^2$$

Så $\frac{dx}{dy} = \frac{1}{1+y^2}$

$$\int \frac{x^3}{1+x^8}\,dx$$

forsøker med substitusjonen $u = x^4$
$du = 4x^3 dx$

$$\int \frac{1}{1+u^2}\,\frac{1}{4}du$$

$$= \frac{1}{4}\arctan(x^4)+C$$

8

$$\int \frac{1}{x^2+4x+13}\,dx$$

Fullfører kvadratet: $x^2+4x+13$

$$= (x+2)^2 + 9$$

$$\int \frac{1}{(x+2)^2+9}\,dx = \int \frac{1}{9}\cdot\frac{1}{\left(\frac{x+2}{3}\right)^2+1}\,dx$$

La $u = \frac{x+2}{3}$, $du = \frac{1}{3}dx$

$$\int \frac{1}{9}\cdot\frac{1}{u^2+1}\,3\,du$$

$$= \underline{\frac{1}{3}\arctan\left(\frac{x+2}{3}\right) + C}$$

Hva er $\int \frac{1}{x^2+4x+3}\,dx$?

$\int \frac{1}{x^2+4x+4}\,dx$?

svar →

Eksamensoppgave august 2015

$$\int_0^2 \frac{x^2}{x^2+4}\,dx$$

* $\int \frac{1}{x^2+4x+3}\,dx = \int \frac{1}{(x+2)^2-1}\,dx$

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delbrøkes oppspalting :

$= \int \frac{1}{2}\left(\frac{1}{x+2-1} - \frac{1}{x+2+1}\right)dx$

$= \frac{1}{2}\left(\ln|x+1| - \ln|x+3|\right) + c$

$= \frac{1}{2}\ln\left|\frac{x+1}{x+3}\right| + c$

* $\int \frac{1}{x^2+4x+4}\,dx$

$= \int \frac{1}{(x+2)^2}\,dx = \int (x+2)^{-2}\,dx$

$= \frac{-1}{(x+2)} + c$