27.01.2016

Inversmatriser

$$A = \begin{bmatrix} 13 & 17 \\ 1 & 1 \end{bmatrix}$$

$$\det(A) = 13 \cdot 1 - 17 \cdot 1 = -4 \neq 0$$

1

$$\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \quad \left(\det\begin{bmatrix} a & b \\ c & d \end{bmatrix} = ad - bc \neq 0 \right) \right.$$

$$\left. \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \begin{bmatrix} ad-bc & 0 \\ 0 & ad-bc \end{bmatrix} \right)$$

$$A^{-1} = \frac{1}{-4} \begin{bmatrix} 1 & -17 \\ -1 & 13 \end{bmatrix} = \frac{1}{4} \begin{bmatrix} -1 & 17 \\ 1 & -13 \end{bmatrix}$$

$$A \cdot A^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = A^{-1} A$$

$$13x + 17y = 4$$

$$x + y = 0$$

$$\begin{bmatrix} 13 & 17 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \end{bmatrix}$$

$$A \vec{x} = \vec{b}$$

$$A^{-1} A \vec{x} = A^{-1} \vec{b}$$

$$\vec{x} = 1_2 \vec{x} = A^{-1} \vec{b}$$

Benytter dette for likningssystemet ovenfor

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{4} \begin{bmatrix} -1 & 17 \\ 1 & -13 \end{bmatrix} \begin{bmatrix} 4 \\ 0 \end{bmatrix} = \frac{1}{4} \begin{bmatrix} -4 \\ 4 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

Modifiserer likningssystemet

$$A \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \underbrace{\frac{1}{4} \begin{bmatrix} -1 & 17 \\ 1 & -13 \end{bmatrix}}_{A^{-1}} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\underline{\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 16 \\ -12 \end{bmatrix} = \begin{bmatrix} 4 \\ -3 \end{bmatrix}}$$

2

$$13x + 17y = a$$
$$x + y = b$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = A^{-1} \begin{bmatrix} a \\ b \end{bmatrix}$$
$$= \frac{1}{4} \begin{bmatrix} -1 & 17 \\ 1 & -13 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix}$$

$$\underline{\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{4} \begin{bmatrix} -a + 17b \\ a - 13b \end{bmatrix}}$$

Prosedyre for å finne inversmatrise

$$[A \mid 1_n] \xrightarrow[\text{til redusert trappeform}]{\text{rad.op}} [1_n \mid A^{-1}]$$

$[\text{ikke } 1_n \mid \quad]$ da eksisterer ikke A^{-1}.

Forklaring →

$$\begin{pmatrix} \text{Rad op. på} \\ M \cdot N \end{pmatrix} = \begin{pmatrix} \text{Rad op. på} \\ M \end{pmatrix} \cdot N$$

③ $$\begin{bmatrix} \vec{m}_1 \\ \vdots \\ \vec{m}_k \end{bmatrix} \underbrace{\begin{bmatrix} \vec{n} \end{bmatrix}}_{\text{én kolonne vektor}} = \begin{bmatrix} \vec{m}_1 \cdot \vec{n} \\ \vdots \\ \vec{m}_k \cdot \vec{n} \end{bmatrix}$$

(definisjon av mult. med kolonne-vektor fra tidligere)

$$A\vec{x} = \vec{b} = 1_n \cdot \vec{b}$$

Hvis $[A \mid 1_n] \overset{\text{rad.op}}{\sim} [1_n \mid C]$

så
$$A\vec{x} = 1_n \cdot \vec{b}$$
$$\vdots \text{ rad op}$$
$$\vec{x} = 1\vec{x} = C \cdot \vec{b}$$

Ganger med A fra venstre:

$$\vec{b} = A \cdot X = (A \cdot C) \cdot \vec{b}$$

siden $(A \cdot C) \cdot \vec{b} = \vec{b}$ for alle $\vec{b}$

så må $\underline{A \cdot C = 1_n}$

Eksempel

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \qquad \text{Finn } A^{-1}$$

4

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \end{array}\right]$$

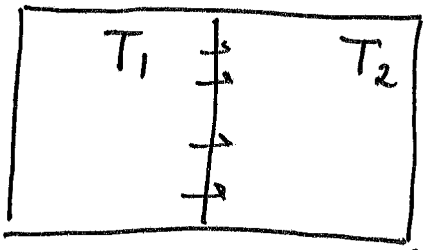
$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 1 & 1 & -1 & 0 & 1 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & 0 & -1 & 1 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

Eksempel

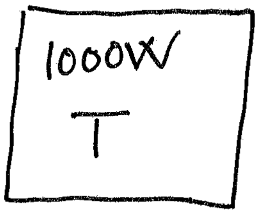
5



Varmeledningskoeffisient k

Varme overføring fra system 1 til system 2

$$\underbrace{(\text{Joule}/\text{sekund})}_{\text{Watt}} = \underline{k\,(T_1 - T_2)}$$



$k = 100\,\text{W}/°\text{C}$

omgivelsen

0°

Hva er T når den er stabil?

Da er $\frac{\text{varme ut}}{\text{tid}} = 0$

Varme ut:
(per tidsenhet)

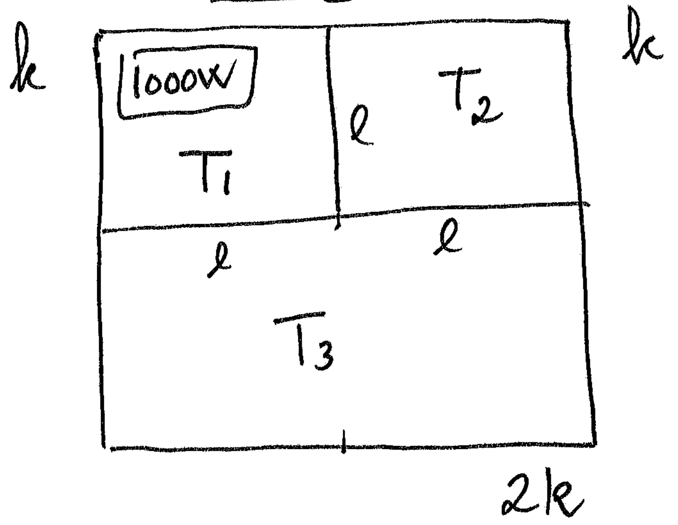
$$k \cdot (T - 0°\text{C}) - 1000\,\text{W}$$

$$k \cdot T - 1000\,\text{W} = 0$$

$$T = \frac{1000\,\text{W}}{k}$$

$$= \frac{1000\,\text{W}}{100\,\text{W}/°\text{C}} = \underline{10\,°\text{C}}$$

Leilighet



⑥

ute temp

$T_0 = 10°C$

$k = 25 W/°C$

$l = 100 W/°C = 4 \cdot k$

Hva er temperaturene T_1, T_2 og T_3 når de har stabilisert seg?

Varme strøm ut av rom

$$
\begin{array}{c|l}
1 & -1000W + l(T_1 - T_2) + l(T_1 - T_3) + k(T_1 - T_0) \\
2 & l(T_2 - T_1) + l(T_2 - T_3) + k(T_2 - T_0) \\
3 & l(T_3 - T_1) + l(T_3 - T_2) + 2k(T_3 - T_0)
\end{array}
$$

Temperaturene er stabile når varmestrømmen er lik 0.

$$k[4(T_1-T_2)+4(T_1-T_3)+(T_1-T_0)] = 1000\,W$$

$$k[4(T_2-T_1)+4(T_2-T_3)+(T_2-T_0)] = 0$$

$$k[4(T_3-T_1)+4(T_3-T_2)+2(T_3-T_0)] = 0$$

(7) $$\frac{1000\,W}{k} = \frac{1000\,W}{25\,W/°C} = 40\,°C$$

$$9T_1 \quad -4T_2 \quad -4T_3 = 40\,°C + T_0$$

$$-4T_1 \quad 9T_2 \quad -4T_3 = T_0$$

$$(-4T_1 \quad -4T_2 \quad 10T_3 = 2T_0) \cdot \tfrac{1}{2}$$

$$\begin{bmatrix} 9 & -4 & -4 \\ -4 & 9 & -4 \\ -2 & -2 & 5 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} = \begin{bmatrix} 50\,°C \\ 10\,°C \\ 10\,°C \end{bmatrix}$$

Dette gir

$$T_1 = 22.65\,°C$$

$$T_2 = 19.57\,°C$$

$$T_3 = 18.89\,°C$$

Eksempel

$$A = \begin{bmatrix} 1 & -1 & 1 \\ 1 & 1 & 1 \\ -1 & 1 & 1 \end{bmatrix} \qquad \text{Finn } A^{-1}$$

8

$$[A \mid 1_3] \sim [1_3 \mid A^{-1}]$$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 \\ -1 & 1 & 1 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} (-1) \quad 1 \\ \\ \\ \end{array}$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & -1 & 1 & 1 & 0 & 0 \\ 0 & 2 & 0 & -1 & 1 & 0 \\ 0 & 0 & 2 & 1 & 0 & 1 \end{array}\right] \begin{array}{l} \\ \cdot 1/2 \\ \cdot 1/2 \end{array}$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & -1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1/2 & 1/2 & 0 \\ 0 & 0 & 1 & 1/2 & 0 & 1/2 \end{array}\right] \begin{array}{l} \\ 1 \quad (-1) \\ \\ \end{array}$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 1/2 & -1/2 \\ 0 & 1 & 0 & -1/2 & 1/2 & 0 \\ 0 & 0 & 1 & 1/2 & 0 & 1/2 \end{array}\right]$$

$$\qquad 1_3 \qquad\qquad\qquad A^{-1}$$