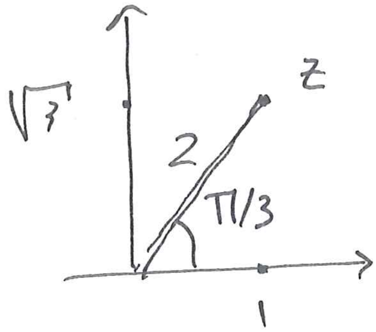
Felles øvingstime mandag 5sep 2016

$\pm\sqrt{1+\sqrt{3}i}$ (løser: $w^2 = 1+\sqrt{3}\cdot i$)

$z = 1+\sqrt{3}i$

$|z| = \sqrt{1^2+(\sqrt{3})^2} = \underline{2}$



$z = 2e^{i\pi/3}$

$w = re^{iv}$ $\qquad w^2 = r^2 e^{i\overbrace{(v+v)}^{2v}} = 2e^{i\pi/3}$

$r^2 = 2 \qquad 2v = \frac{\pi}{3} + (2\pi\cdot n)$

$r = \sqrt{2} \qquad v = \frac{\pi}{6}, \ \frac{\pi}{6}+\pi \qquad (e^{\pi i} = -1)$

Røttene er $\underline{\sqrt{2}e^{\pi i/6}}$, $\sqrt{2}e^{7\pi i/6} = \underline{-\sqrt{2}e^{\pi i/6}}$

$\pm\sqrt{1+\sqrt{3}i} = \pm\sqrt{2}\,e^{\pi i/6}$

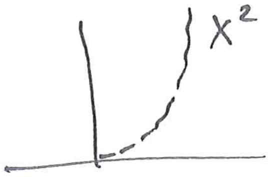
$= \pm\sqrt{2}\left(\sqrt{3}/2 + i/2\right)$

Vis følgende:

$$\sqrt{a} + \sqrt{b} \geq \sqrt{a+b} \qquad a, b \geq 0$$

Hint:

Sammenlign: $(\sqrt{a} + \sqrt{b})^2$ og $(\sqrt{a+b})^2$.



Benytt at $y = x^2$ er en voksende funksjon for $x \geq 0$

$$0 \leq x \leq y \iff x^2 \leq y^2$$

Påstanden vår er derfor ekvivalent til:

$$(\sqrt{a} + \sqrt{b})^2 \geq (\sqrt{a+b})^2$$

$$a + b + \underbrace{2\sqrt{a}\sqrt{b}}_{\geq 0} \geq a + b$$ som opplagt er sant.

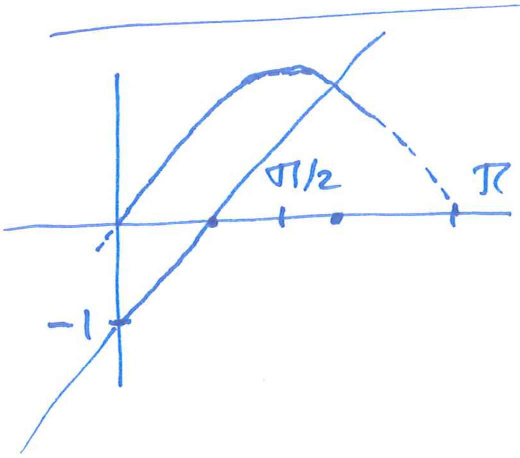
Derfor er $\sqrt{a} + \sqrt{b} \geq \sqrt{a+b}$ sann for $a, b \geq 0$.

a) Vis at $\sin x = x - 1$ har presis én løsning i intervallet $[0, \pi]$.

b) Estimer løsning ved å benytte Newtons metode ($\pi/2$ startverdi, 2 ganger)

$\sin x = x - 1$ $\overset{\text{ekvivalent}}{\Longleftrightarrow}$ ~~sinx~~ $\sin x - (x-1) = 0$

løsning $\longleftrightarrow$ nullpunkt



a) $f(x) = \sin x - x + 1$ kontinuerlig.

$f(0) = 1, \quad f(\pi) = 1 - \pi < 0$

Så skjæringssetningen gir at $f(x)$ har minst ett nullpunkt.

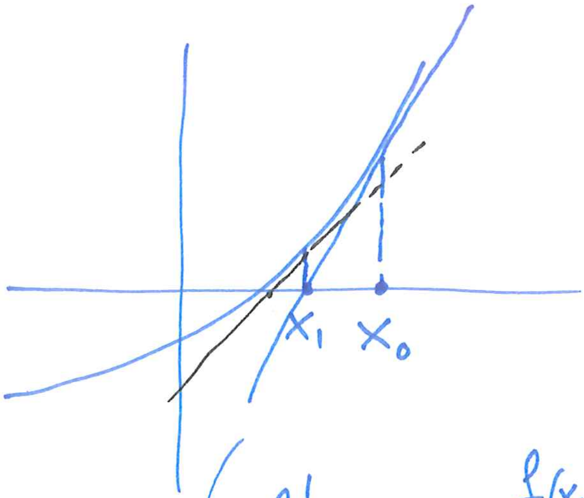
$$f'(x) = \cos x - 1 \leq 0$$
$$< 0 \quad (x > 0)$$

$f(x)$ er ekte avtagende i $[0, \pi]$.

Vi konkluderer med at likningen har akkurat en løsning.

$$f(x) = \sin x - x + 1$$
$$f'(x) = \cos x - 1$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$



$$\left(f'(x_n) = \frac{f(x_n) - 0}{x_n - x_{n+1}} \right)$$

$$\underline{x_0 = \frac{\pi}{2}}$$

$$f\left(\frac{\pi}{2}\right) = 1 - \frac{\pi}{2} + 1 = 2 - \frac{\pi}{2}$$

$$f'\left(\frac{\pi}{2}\right) = -1$$

$$x_1 = \frac{\pi}{2} - \frac{\left(2 - \frac{\pi}{2}\right)}{-1} = \frac{\pi}{2} + \left(2 - \frac{\pi}{2}\right) = \underline{2}$$

$$x_2 = 2 - \frac{f(2)}{f'(2)} = \underline{1.9345}$$

$$(x_3 =)$$