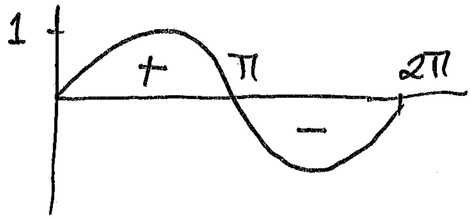
25.03.2015

$$\int_0^{2\pi} \sin x \, dx = 0$$



(1) Fundamentalteoremet

$$\int_0^{2\pi} \sin x \, dx = (-\cos x)\Big|_0^{2\pi} = -1-(-1) = 0.$$

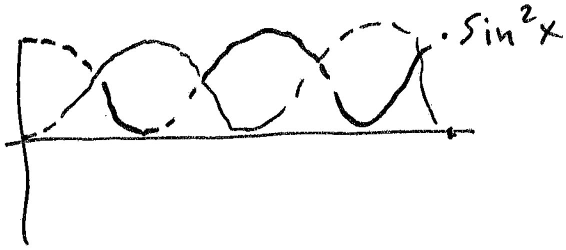
Hva er arealet mellom grafen til $f(x)$ og x-aksen fra $x=0$ til $x=2\pi$?

$$\int_0^{2\pi} |\sin x| dx = \int_0^{\pi} \sin x \, dx + \int_{\pi}^{2\pi} (-\sin x) dx$$

(fra symmetri) $= 2\int_0^{\pi} \sin x \, dx$

$$= 2\,(-\cos x)\Big|_0^{\pi} = 2(-(-1)-(-1))$$
$$= \underline{\underline{4}}$$

$$\int_0^{2\pi} \sin^2 x \, dx$$



Viser at $\int_0^{2\pi} \sin^2 x \, dx = \int_0^{2\pi} \cos^2 x \, dx$

$$= \frac{1}{2}\int_0^{2\pi} \underbrace{\sin^2 x + \cos^2 x}_{1} \, dx = \frac{1}{2}\int_0^{2\pi} 1 \, dx$$
$$= \frac{1}{2} 2\pi = \underline{\underline{\pi}}$$

Vi bestemmer integralet ved bruk av antiderivate:

$$\int \sin^2 x \, dx$$
$$\int \frac{1}{2}(1-\cos(2x))dx$$
$$= \underline{\frac{1}{2}\left[x - \frac{1}{2}\sin(2x)\right] + c}$$

$$\cos(2x) = \underbrace{\cos^2 x}_{1-\sin^2 x} - \sin^2 x$$

$$\sin^2 x = \frac{1}{2}(1-\cos 2x)$$

② $\int \sin^3 x \, dx = \int \sin x \cdot \underbrace{\sin^2 x}_{1-\cos^2 x} \, dx$

$u = \cos x$

$u' = -\sin x$

$u' dx \sim du$

$$= \int \sin x \left(1 - \cos^2 x\right) dx$$

$$= \int (-u')(1 - u^2)\, dx$$

$$= -\int (1 - u^2)\, du$$

$$= -\left(u - \frac{u^3}{3}\right) + c$$

$$= -\cos x + \frac{\cos^3 x}{3} + c$$

$$\int_0^{2\pi} \sin^3 x \, dx = 0$$

$$\int_0^{\pi} \sin^3 x \, dx = -\cos x + \frac{\cos^3 x}{3} \Big|_0^{\pi}$$

$$= 2 + \frac{1}{3}\left((-1)^3 - 1^3\right) = 2 - \frac{2}{3}$$

$$= 1 + \frac{1}{3}$$

$$\int_0^{2\pi} \sin^4 x \, dx = \frac{3\pi}{4} \quad \text{(se nedenfor...)}$$

$$\int \sin^4 x \, dx = \int \left(\sin^2 x\right)^2 dx$$

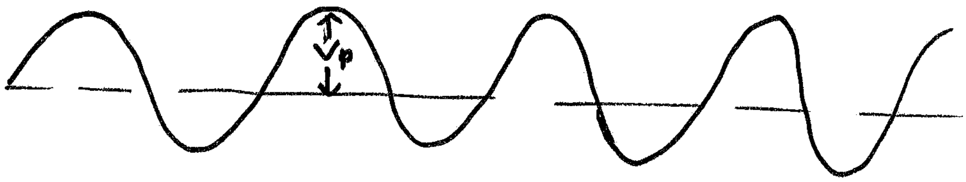
$$= \int \left(\frac{1}{2}\left(1 - \cos(2x)\right)\right)^2 dx = \frac{1}{4}\int \left(1 - 2\cos(2x) + \cos^2(2x)\right) dx$$

$$= \frac{1}{4}\int 1 - 2\cos 2x + \frac{1}{2}\left(1 + \cos(2 \cdot 2x)\right) dx$$

$$= \frac{1}{4}\left[x - \sin(2x) + \frac{x}{2} + \frac{1}{2}\left(\frac{\sin(4x)}{4}\right)\right] + c$$

$$= \frac{1}{4}\left[\frac{3x}{2} - \sin(2x) + \frac{1}{8}\sin(4x)\right] + c$$

③ VEKSELSTRØM



V_p
"peak"-spenning

$V_{rms} = 220V.$ Frekvens 50 Hz

50 svingninger i sekundet.

R resistans

ohms lov : $U = R \cdot I$

Effekten : $P = U \cdot I = \frac{U^2}{R}.$

$$V(t) = V_p \sin(\omega t)$$

Periode : T $\qquad \omega \cdot T = 2\pi$

$(\frac{1}{50})s$

$$\omega = 2\pi \cdot 50/s = 100\pi/s.$$

Effekt $\frac{\int_0^{\Upsilon} \frac{(V_p \sin(\omega t))^2}{R} dt}{\Upsilon}$

$$= \frac{V_p^2}{R} \frac{\int_0^{\Upsilon} \sin^2(\omega t) dt}{\Upsilon}$$

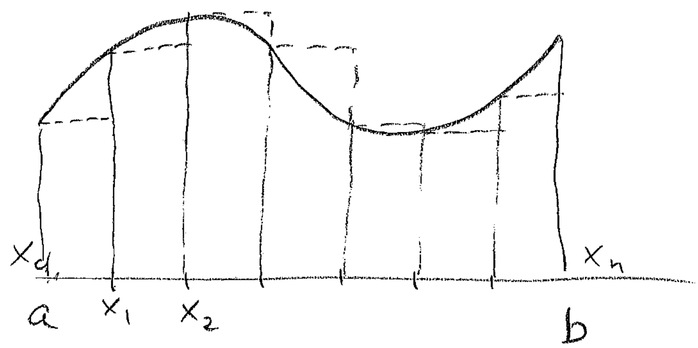
integrerer vi over mange perioder er $\int_0^{\Upsilon} \sin^2(\omega t) dt \approx \frac{\Upsilon}{2}$

Effekt gjennomsnittlig $\frac{V_p^2}{R} \cdot \frac{1}{2} = \underline{\frac{1}{R}\left(\frac{V_p}{\sqrt{2}}\right)^2}.$

$V = 220V = \frac{V_p}{\sqrt{2}}.$ $\qquad V_p = 220V \cdot \sqrt{2} \sim \underline{\underline{311V}}$

④

Numerisk integrasjon



Estimat for integralet hvis vi deler intervallet i n deler, og velger funksjonsverdien til punktet til venstre i hver delintervall:

bredden til delintervall: $d = \frac{b-a}{n}$

$$x_i = a + id$$

$$S_V = \sum_{i=0}^{n-1} f(x_i) \cdot d$$

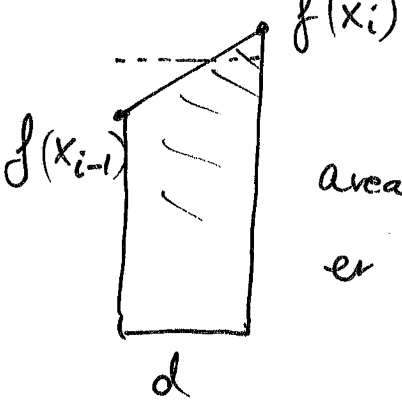
"Høyre estimat" $S_H = \sum_{i=1}^{n} f(x_i) d$

$$S_H = S_V + (f(x_n) - f(x_0)) \cdot d$$

$$= \underline{S_V + (f(b) - f(a)) \cdot d}$$

Trapesmetoden

5



arealet til trapeset er lik $\left(\frac{f(x_i)+f(x_{i-1})}{2}\right)\cdot d$

$$S_T = \frac{f(a)+f(x_1)}{2}\cdot d + \frac{f(x_1)+f(x_2)}{2}\cdot d + \cdots \frac{f(x_{n-1})+f(x_n)}{2}\cdot d$$

$$= \frac{1}{2}\cdot d\left(f(a) + 2f(x_1) + 2f(x_2) + \cdots + 2f(x_{n-1}) + f(x_n)\right)$$

$$S_T = \frac{1}{2}(S_V + S_H) = S_V + \frac{1}{2}(f(b)-f(a))d$$

Feil ved bruk av trapesmetoden er begrenset av: $\underline{\frac{(b-a)^3}{12\cdot n^2} M_2}$ hvor $M_2 \geq |f''(x)|$ for $x \in [a,b]$.