Mandag 4 mai 09

Selvinduksjon

el. strøm $\xrightarrow{\text{gir}}$ magnetfelt $\xrightarrow{\text{flate}}$ magnetisk fluks

endring i magnetisk fluks $\xrightarrow{\text{gir}}$ el. motorisk spenning.

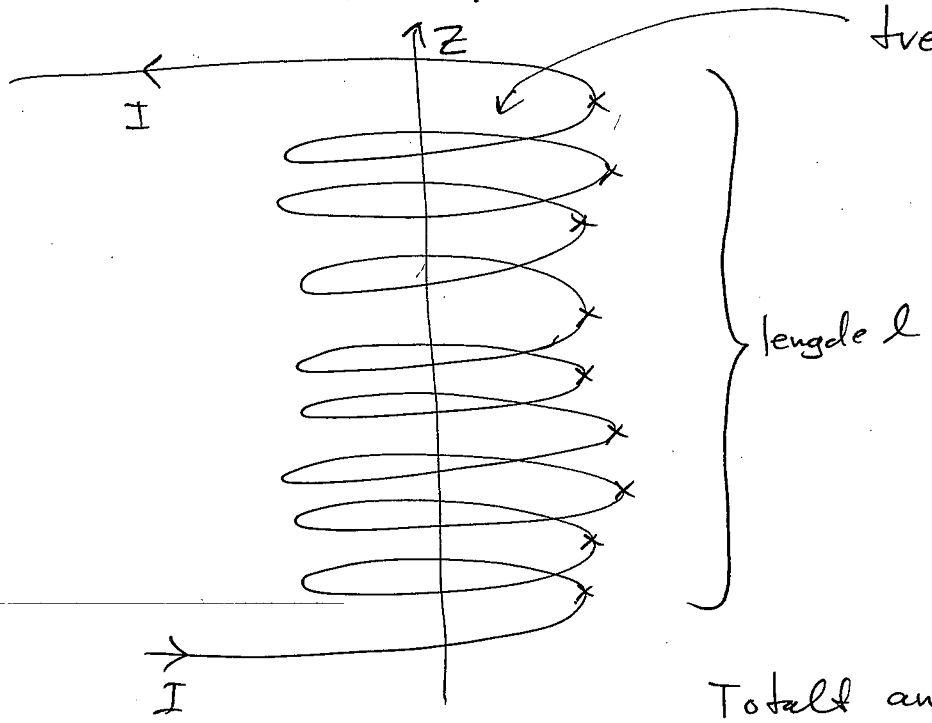
Så endring i el. strøm gir, el. motorisk spenning.

Dette gir en effekt for spoler.

Selvinduktans $\quad L = \frac{\Phi}{I} \cdot$ antall vindinger

Φ mag. fluks

I strøm



n antall vindinger per lengdeenhet

Totalt antall vindinger er $N = n \cdot l$

Magnetfeltet i spolen er $\vec{B} = \mu_0 \cdot n \cdot I \, \vec{k}$

(følger fra Ampers lov)

Fluks $\quad \Phi = \int_S \vec{B} \cdot \vec{n} \, dA \quad$ (S tverrsnittflate i spolen)

$= B \cdot a = \mu_0 \cdot n \cdot I \cdot a$

Generelt: $\Phi = L \cdot I / N$ N antall vindinger.

Faradays lov $\mathcal{E} = -\frac{d\Phi}{dt} \cdot N$

$$\mathcal{E} = -\frac{L}{N} \frac{dI}{dt} \cdot N$$

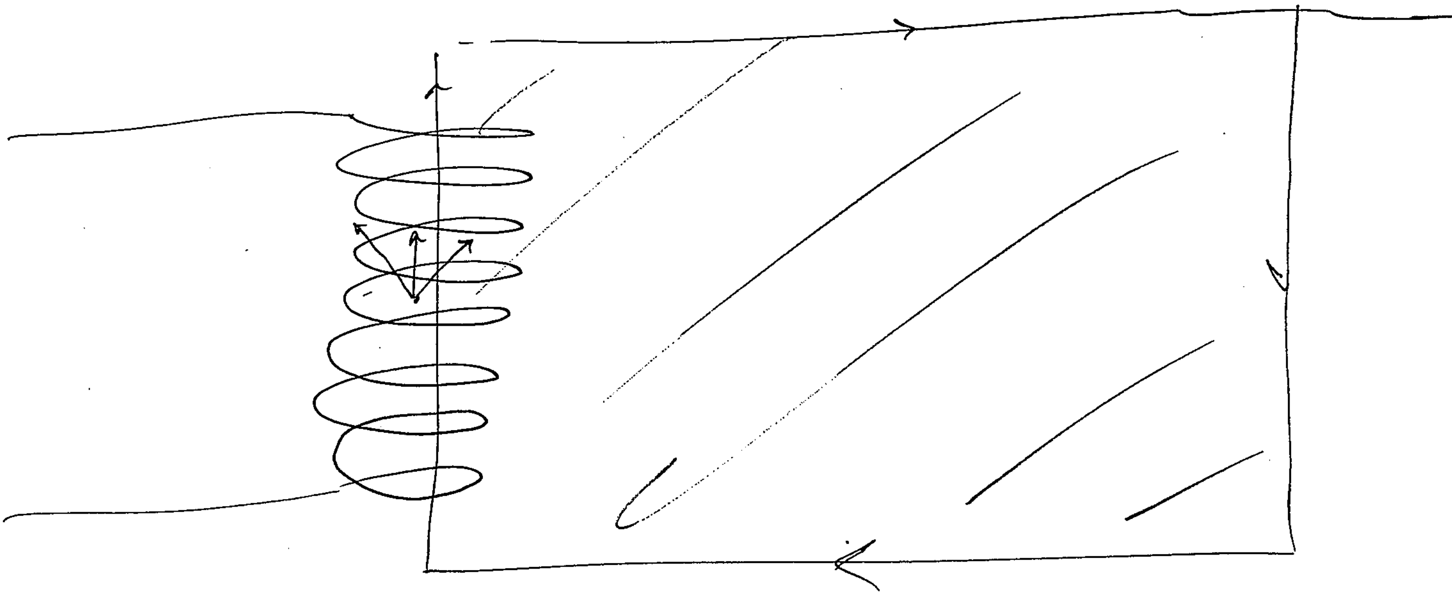
$$\mathcal{E} = -L \cdot \frac{d}{dt} I$$

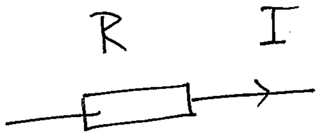
Induktansen til en (ideell) spole:

$$L = \frac{\Phi}{I} \cdot N = \frac{\mu_0 \cdot n \cdot I \cdot a}{I} \cdot n \cdot l$$

$$L = \mu_0 \cdot n^2 \cdot l \cdot a$$

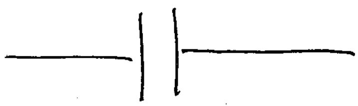
Enheten til induktans er Henry $H = T \cdot m^2 / A$





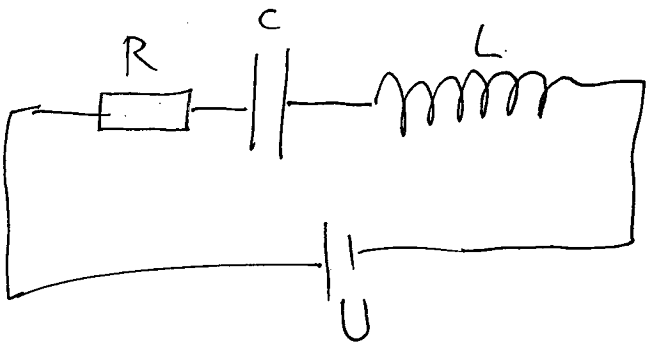
$$\mathcal{E} = -R \cdot I$$

$$\mathcal{E} = -L \cdot \frac{d}{dt} I$$



$$\mathcal{E} = -\frac{q}{C}$$

$q' = I \qquad q'' = \frac{d}{dt} I$

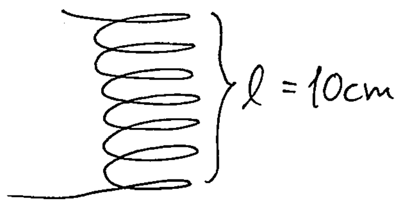


$q(t)$

$$\boxed{Lq'' + Rq' + \frac{1}{C} q = U}$$

$$-L \cdot q'' - Rq' - \frac{q}{C} + U = 0$$

Eksempel.



tverrsnittareal 1 cm^2

Antall viklinger $N = 1000$

$$n = \frac{N}{l} = \frac{1000}{0.10\,m} = 10^4\,m^{-1}$$

Hva er induktansen L?

$$L = \mu_0 \cdot n^2 \cdot l \cdot a$$

$$= 4\pi \cdot 10^{-7}\ T\cdot m/A \cdot (10^4\,m^{-1})^2 \cdot (0.10\,m) \cdot 10^{-4}\,m^2$$

$$= 4\pi \cdot 10^{-7} \cdot 10^8 \cdot 10^{-1} \cdot 10^{-4}\ Tm^2/A$$

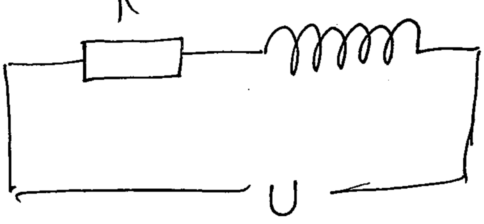
$$= 4\pi \cdot 10^{-7+8-1-4}\ Tm^2/A$$

$$= 4\pi \cdot 10^{-4}\ Tm^2/A = 12.6 \cdot 10^{-4}\ Tm^2/A$$

$$= 1.3 \cdot 10^{-3}\ Tm^2/A$$

$$= \underline{1.3\ mH}$$

($H = Tm^2/A$, Henry)



$$R \cdot I + L \cdot \frac{d}{dt} I = U$$

$$U = \begin{cases} 0 & t < 0 \\ V & t \geq 0 \end{cases}$$

kobler til i tiden $t = 0$.

stabil løsning ($t \to \infty$) $\quad I = \frac{V}{R}$.

Homogen diff. likning: $\frac{d}{dt} I = -\frac{R}{L} I$

$$I_{hom} = k\, e^{-\frac{R}{L} \cdot t}.$$

Løsningene til diff. likningen

$$R \cdot I + L \frac{d}{dt} I = U$$

er på formen $I = \frac{V}{R} + k e^{-\frac{R}{L} \cdot t}$, $t \geq 0$

$I(0) = 0$: $0 = \frac{V}{R} + k e^{0}$

så $k = -\frac{V}{R}$.

$$I(t) = \begin{cases} 0 & t < 0 \\ \frac{V}{R}\left(1 - e^{-\frac{R}{L} \cdot t}\right) & t \geq 0 \end{cases}$$

Anta $R = 1\Omega$ og $L = 1.3\,mH$
(spole fra forrige eksempel)

Hvor lang tid tar det fra vi kobler til spenningen til strømmen stabiliserer seg?

$$\frac{R}{L} = \frac{1\Omega}{1.3mH} = \frac{1000}{1.3} \simeq 770$$

$e^{-4} \simeq 0.02 = 2\%$ $\qquad e^{-5} \simeq 1\%$ (under det)

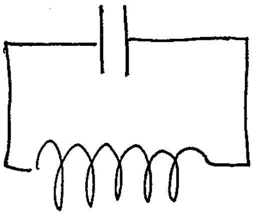
Så I er 98% av full strømstyrke når

$$\frac{R}{L} \cdot t = 4$$

$$t = \frac{4}{770} \sim \frac{1}{200} s$$

[Tiden τ er $\frac{L}{R}$.

$1 - e^{-R/L \cdot \tau} = 1 - e^{-1} = 63\%$]



q ladning

strøm: $I = \frac{d}{dt} q$

$$\frac{q}{C} + L \cdot \frac{d^2}{dt^2} q = 0$$

$$\frac{d^2}{dt^2} q + \frac{1}{L \cdot C} q = 0$$

$$q(t) = q_0 \cos\left(\frac{1}{\sqrt{L \cdot C}} (t - t_0)\right)$$

løsning slik at

$q_0 = q(t_0)$ $\quad$ $q'(t_0) = 0$