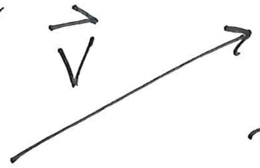
12.10
2020
1

12.1 - 3 Vektorer i planet.

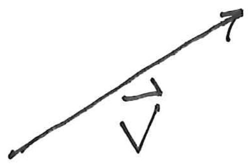
En vektor har <u>størrelse</u> og <u>retning</u>

$|\vec{v}|$

$\mathbf{v}$ eller $\vec{v}$



parallellforskyvd.

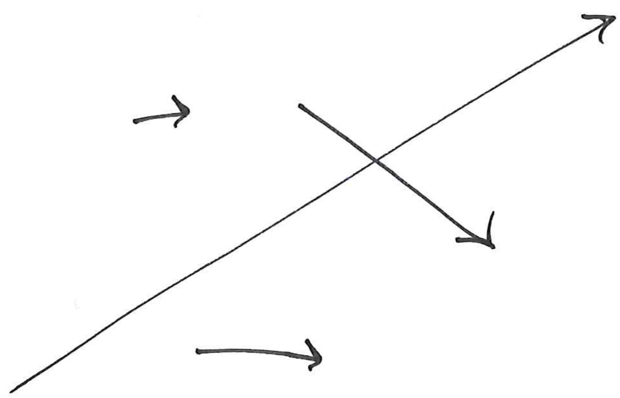


$\vec{v}$

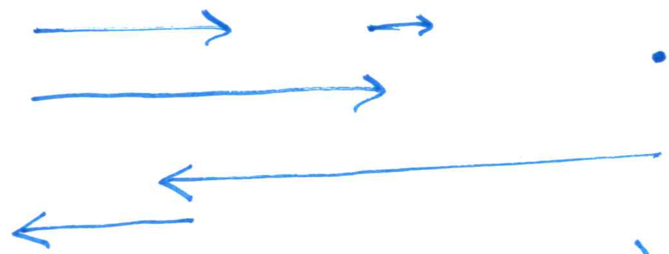
Nullvektoren $\vec{0}$

størrelsen (lengden) er lik 0

har ingen (alle?) retninger.

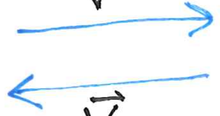


Parallelle vektorer



Alle vektorer er parallelle til $\vec{0}$.

$\vec{v}$



$-\vec{v}$

er <u>motsattvektorer</u>

samme lengde men peker i motsatte retninger.

Motsatt vektoren til $\vec{v}$ er $-\vec{v}$.

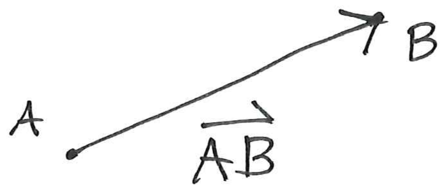
Hva er $-(-\vec{v})$, motsatt vektoren til $-\vec{v}$?

2

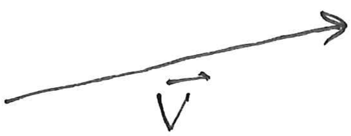
$$-(-\vec{v}) = \vec{v}$$

$$-\vec{0} = \vec{0}$$

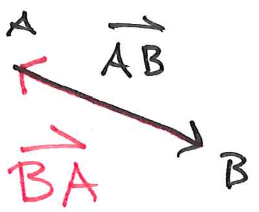
Vektorer og punkt



Er $\vec{v} = \vec{AB}$?



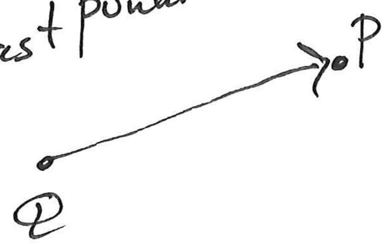
Flytt (parallellforskyv) $\vec{v}$ slik at den starter i A. Hvis endepunktet da blir B så er $\vec{v} = \vec{AB}$.



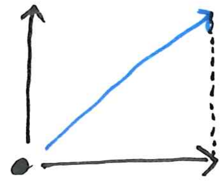
$$-\vec{AB} = \vec{BA}$$

Motsattvektoren.

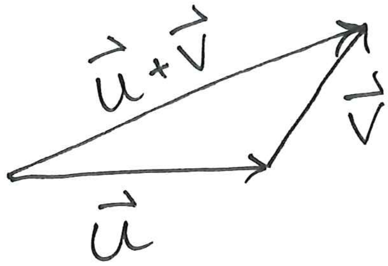
Fast punkt

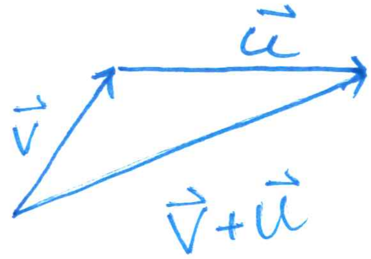


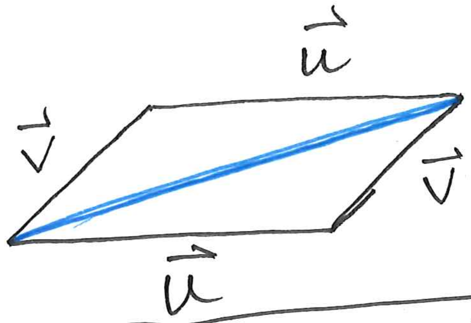
Sum av vektorer



$\vec{u}+\vec{v}$







parallellogram

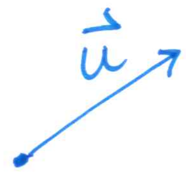
$$\boxed{\vec{u}+\vec{v}=\vec{v}+\vec{u}}$$

kommutativ

Vektoraddisjon er også assosiativ

$$(\vec{u}+\vec{v})+\vec{w}=\vec{u}+(\vec{v}+\vec{w})$$

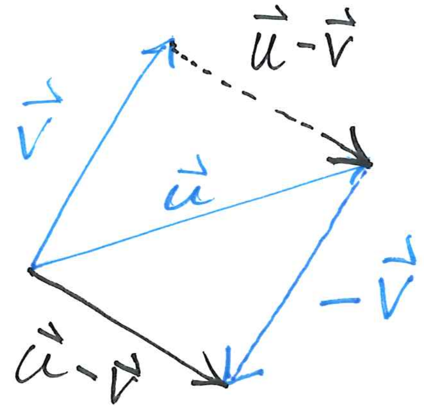
$$\vec{0}+\vec{u}=\vec{u}$$



Differanse av vektorer

$$\vec{u}-\vec{v}=\vec{u}+(-\vec{v})$$

④



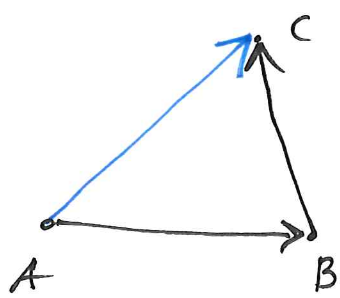
$\vec{u}-\vec{v}$: La $\vec{u}$ og $\vec{v}$ starte i samme punkt. $\vec{u}-\vec{v}$ er da vektoren fra endepunktet til $\vec{v}$ til endepunktet til $\vec{u}$.

$$\vec{v}-\vec{0}=\vec{v}$$

$$\vec{u}-\vec{u}=\vec{0}$$

$$\vec{0}-\vec{u}=-\vec{u}$$

$$\overrightarrow{AB}+\overrightarrow{BC}=\overrightarrow{AC}$$



oppg

$$\left(\overrightarrow{AC}-\overrightarrow{DC}\right)+\overrightarrow{DA}\ ?$$

$$\left(-\overrightarrow{DC}=\overrightarrow{CD}\right)$$

$$\underbrace{\overrightarrow{AC}+\overrightarrow{CD}}+\overrightarrow{DA}$$

$$\overrightarrow{AD}+\overrightarrow{DA}=\overrightarrow{AA}=\vec{0}$$

$$\overrightarrow{AC}+\overrightarrow{AC}+\overrightarrow{AC}=3\overrightarrow{AC}$$

$$-\vec{v}-\vec{v}-\vec{v}=3(-\vec{v})=-(3\vec{v})=\underline{-3\vec{v}}$$

12.3 Skalering av vektorer

5

$t \cdot \vec{v}$ har lengde $|t| \cdot |\vec{v}|$

samme retning som $\vec{v}$ hvis $t > 0$

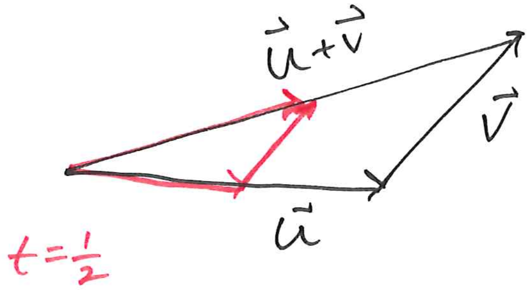
$t \in \mathbb{R}$ (reelt tall)

($\vec{0}$ hvis $t = 0$)

Motsatt retning som $\vec{v}$ hvis $t < 0$.

$$\underline{1 \cdot \vec{v} = \vec{v}}$$

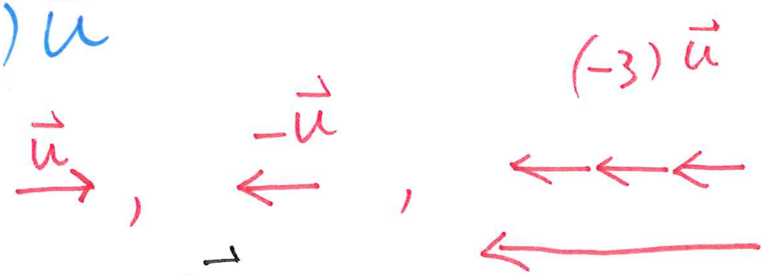
$$\underline{t(\vec{u} + \vec{v}) = t\vec{u} + t\vec{v}}$$



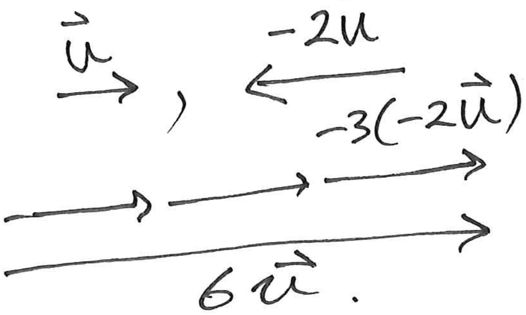
$$\underline{(s+t)\vec{u} = s\vec{u} + t \cdot \vec{u}}$$

$$t(s \cdot \vec{u}) = (t \cdot s)\vec{u}$$

$3(-1)\vec{u}$



$(-3)(-2)\vec{u}$



$$(2+3)(\vec{u}-\vec{v}) + 4(\vec{u}+2\vec{v})$$ Forenkler

(6)

$$= 5\vec{u} - 5\vec{v} + 4\vec{u} + 4\cdot(2\vec{v})$$
$$= 5\vec{u} + 4\vec{u} - 5\vec{v} + (4\cdot 2)\cdot\vec{v}$$
$$= (5+4)\vec{u} + (-5+8)\vec{v}$$
$$= \underline{9\vec{u} + 3\vec{v}} = 3(3\vec{u}+\vec{v})$$

oppg. Forenkl:

$$(2+s)(\vec{u}-3\vec{v}) + s(2\vec{v}-\vec{u}) - 2\vec{u}$$
$$= (2+s)\vec{u} + (2+s)(-3\vec{v}) + s(2\vec{v}) + s(-\vec{u}) - 2\vec{u}$$
$$= 2\vec{u} + s\vec{u} + (-6)\vec{v} + (-3s)\vec{v} + (2s)\vec{v} - s\vec{u} - 2\vec{u}$$
$$= \underbrace{2\vec{u} - 2\vec{u}}_{\vec{0}} + \underbrace{s\vec{u} - s\vec{u}}_{\vec{0}} + (-6-3s+2s)\vec{v}$$
$$= (-6-s)\vec{v} = \underline{\underline{-(6+s)\vec{v}}}$$
$$= \underline{-6\vec{v} - s\vec{v}}$$

Forenkl

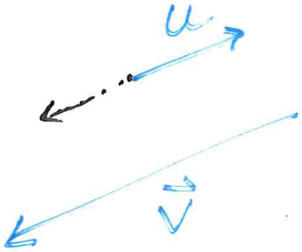
$$\vec{AD} + 2\vec{DB} + \vec{CD} - \vec{AC}$$

(brukt $-\vec{AC} = \vec{CA}$)

$$\underbrace{\underbrace{\vec{CA} + \vec{AD}}_{\vec{CD}} + \vec{CD}} + 2\vec{DB}$$
$$2\vec{CD} + 2\vec{DB}$$
$$= 2(\vec{CD} + \vec{DB}) = \underline{\underline{2\vec{CB}}}$$

7

To vektorer er parallelle hvis den ene ($\neq \vec{0}$) kan skaleres slik at den blir lik den andre vektoren.



$$3(-\vec{u}) = \vec{v}$$

$\vec{u}$ og $\vec{v}$ er parallelle.

Merk $\vec{0}$ er parallell til $\vec{v}$ ($\neq \vec{0}$)

men $s \cdot \vec{0} = \vec{0}$ kan ikke bli lik $\vec{v}$.