oblig 1 kan hentes i øvingen 8:30 – 12:15

Rom PI 460

P35

5.10.2020

10.1 Absolutte vinkelmål

b buelengde

(1)

$V = \frac{b}{r}$ vinkel i radianer

omkretsen $2\pi \cdot r$

$$360° = 2\pi \text{ rad}$$

$$\boxed{180° = \pi \text{ rad}}$$

$$30° = 30° \cdot 1 = 30° \frac{\pi \text{ rad}}{180°} = \frac{\pi}{6} \text{ rad}$$

$$1 \text{ rad} = 1 \text{ rad} \frac{180°}{\pi \text{ rad}} = \frac{180}{\pi}°$$

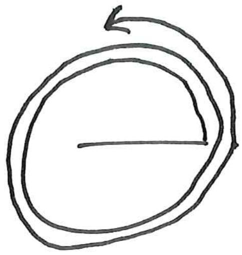
$$\underline{1 \text{ rad} \sim 57.29°}$$

$$\frac{\pi}{4} \text{ rad} = \frac{\pi}{4} \cdot \frac{180°}{\pi} = 45°$$

$$\pi° = \pi° \cdot \frac{\pi \text{ rad}}{180°} = \frac{\pi^2}{180} \text{ rad.}$$

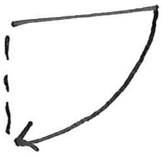
$$30 \text{ rad} = 30 \cdot \frac{180°}{\pi} = \frac{30 \cdot 180}{\pi}°$$

$$= \underline{\frac{5400}{\pi}°}$$



$$2\pi + 2\pi + \frac{\pi}{2}$$
$$= 4\pi + \frac{\pi}{2}$$
$$= \frac{9\pi}{2} \text{ rad}$$

(2)

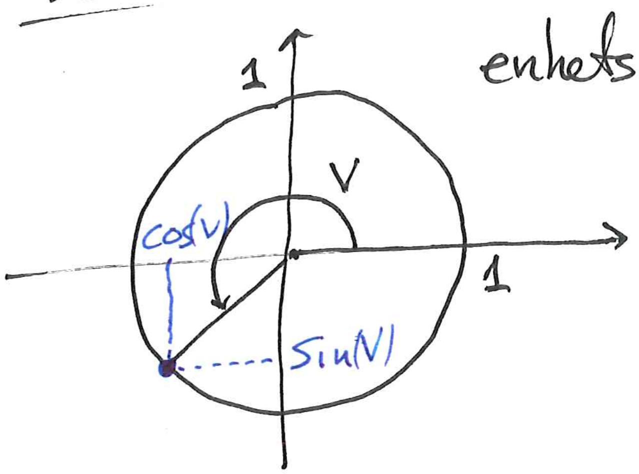


negativ retning

$-\frac{\pi}{2}$ rad

enhetssirkelen

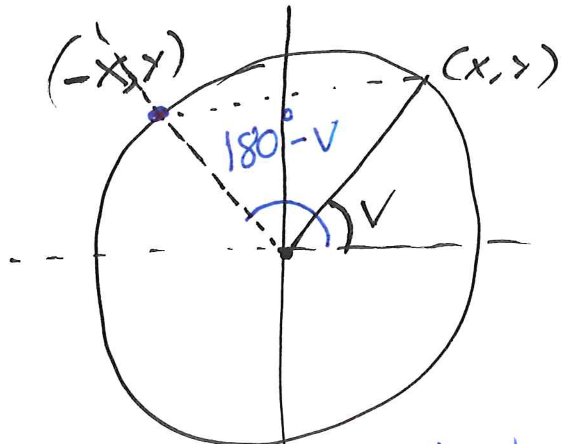
10.2



10.9 Pythagoras sin sats

$$\cos^2 v + \sin^2 v = 1$$
for alle v

Refleksjon om y-aksen



$$\sin V = \sin(180° - V)$$
$$\cos(V) = -\cos(180° - V)$$

Tilsvarende: refleksjon om x-aksen

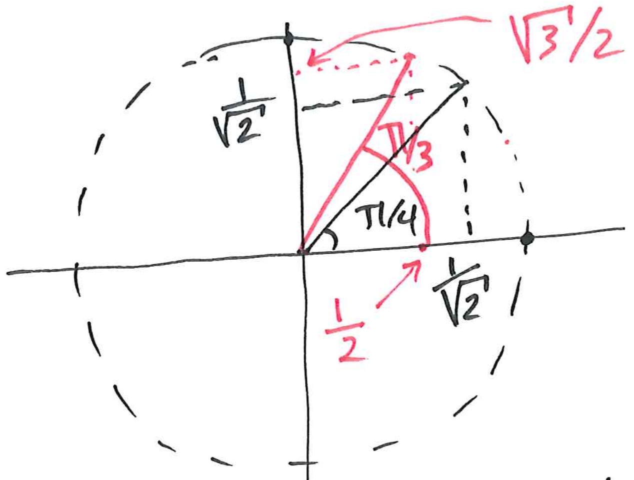
$$\sin(-v) = -\sin(v)$$
$$\cos(-v) = \cos(v)$$

Refleksjon om origo

$$\sin(v + 180°) = -\sin(v)$$
$$\cos(v + 180°) = -\cos(v)$$

Refleksjon om linjen $x = y$: $\cos\left(\frac{\pi}{2} - v\right) = \sin(v)$

③

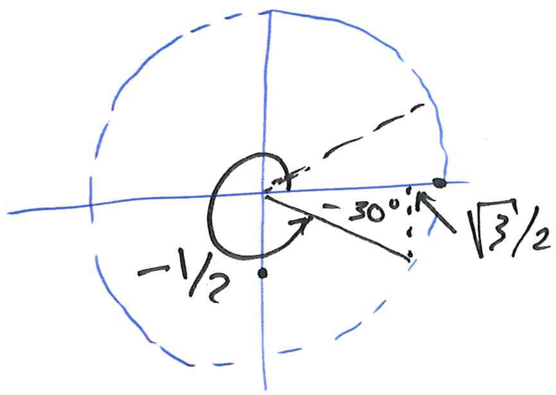


10.6

	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
$\sin v$	0	$1/2$	$1/\sqrt{2}$	$\sqrt{3}/2$	1
$\cos v$	1	$\sqrt{3}/2$	$1/\sqrt{2}$	$1/2$	0

Hva er $\sin(330°)$?

$330° = 360° - 30°$

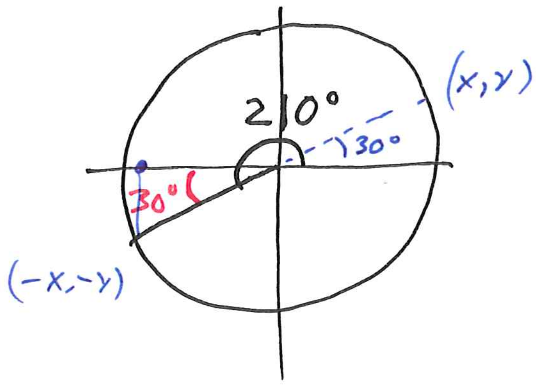


$\sin(330°) = \sin(-30°)$

$= -\sin(30°) = -\frac{1}{2}$

oppg. Finn eksakt verdi til $\cos(210°)$.

$210° = 180° + 30°$



$\cos(210°) = \cos(180° + 30°)$

$= -\cos(30°) = -\frac{\sqrt{3}}{2}$

10.3 Trigonometriske likninger

Eksempler

1) $\sin(v) = 1$

2) $\sin(v) = 1/2$ $\quad v \in [0, 2\pi]$

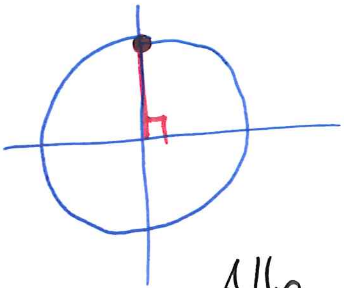
3) $\sin(v) = 2$

4) $5\sin(v) + 3 = 0$

5) $\sin^2(v) + \sin(v) = 0$

6) $\sin^2(v) + 3\sin(v) + 2 = 0$

1)



$\sin(v) = 1$

$v = \arcsin 1 = \frac{\pi}{2}$

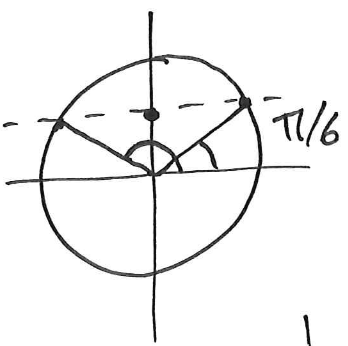
Alle løsningene er

$$\underline{v = \frac{\pi}{2} + 2\pi \cdot n}$$

$n \in \mathbb{Z}$

$\sin(v) = \frac{1}{2}$ $\quad v \in [0, 2\pi]$

2)



$\frac{\pi}{6}$ og $\pi - \frac{\pi}{6} = \frac{5\pi}{6}$

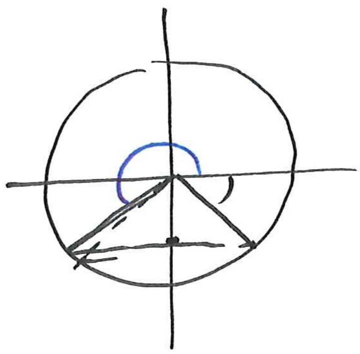
opp til hele omløp.

Løsningene er $\underline{v = \frac{\pi}{6} \text{ og } \frac{5\pi}{6}}$

3) $\sin(v) = 2$ har ingen løsning

Løsningsmengden er tom.

4) $5\sin(v) + 3 = 0$



$$\sin(v) = \frac{-3}{5}$$
$$= -0.6$$

5

$\arcsin(-0.6) = -36.9°$

og $180° - (-36.9°) = 216.9°$

Løsningene er $v = -36.9° + 360° \cdot n$

og $v = 216.9° + 360° \cdot n$

$n \in \mathbb{Z}$

5) $\sin^2 v + \sin v = 0$

$\sin v (\sin v + 1) = 0$

$(a \cdot b = 0 \Leftrightarrow a = 0$ eller $b = 0)$

$\sin v = 0$ eller $\sin(v) + 1 = 0$

$\sin(v) = -1.$

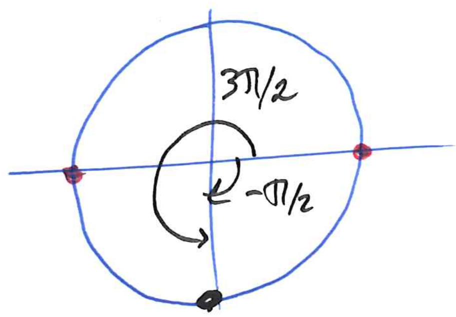
$v = 0 + 2\pi \cdot n$

og $v = \pi + 2\pi \cdot n$

$v = \pi \cdot n$, n heltall

$v = \frac{3\pi}{2} + 2\pi \cdot n$

$n \in \mathbb{Z}$



Løsningsmengden til $\sin^2 v + \sin v = 0$

er $v = \pi \cdot n$ og

$v = \frac{3\pi}{2} + 2\pi \cdot n$ for heltall n

6) $\sin^2(v) + 3\sin(v) + 2 = 0$

La $\sin v = x$

$$x^2 + 3x + 2 = 0$$

6 $$(x+2)(x+1) = 0$$

Løsningene er $x = -2$ og $x = -1$

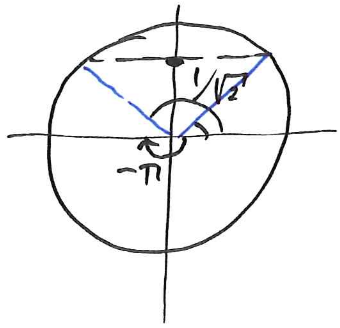
$\sin v = -2$ og $\sin v = -1$

ingen løsning $\quad v = \frac{3\pi}{2} + 2\pi \cdot n$

Løsningsmengden er $\underline{\underline{v = \frac{3\pi}{2} + 2\pi \cdot n}}$

oppgave Løs likningen $2\sin v = \sqrt{2}$

$v \in [-\pi, 3\pi]$.



$$\sin v = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}} \sim 0.707$$

$$v = \frac{\pi}{4} \; (45^\circ) + 2\pi \cdot n$$

$$v = \pi - \frac{\pi}{4} = \frac{3\pi}{4}.$$

$$v = \frac{3\pi}{4} + 2\pi \cdot n$$

Løsningsmengden er $\underline{\left\{ \frac{\pi}{4}, \frac{3\pi}{4}, \frac{\pi}{4} + 2\pi, \frac{3\pi}{4} + 2\pi \right\}}$

Eks

10.8

$$\sin(\underbrace{x+1}_{v}) = \frac{1}{\sqrt{2}} \qquad x \in [-\pi, \pi]$$

1. Løs for v

2. Løs for x

1 $\quad v = \frac{\pi}{4} + 2\pi \cdot n$

og $v = \frac{3\pi}{4} + 2\pi \cdot n$

2 $\quad v = x + 1$

7

$x = v - 1$

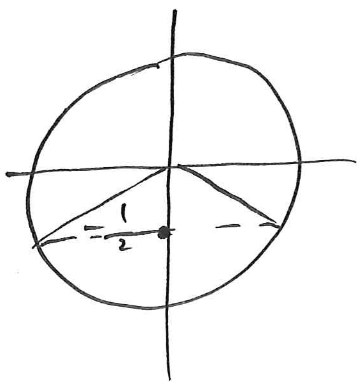
så $\quad x = \frac{\pi}{4} - 1 + 2\pi n \qquad n = 0$

og $\quad x = \frac{3\pi}{4} - 1 + 2\pi \cdot n. \qquad n = 0$

Løsningsmengden er $\quad x = \left\{ \frac{\pi}{4} - 1, \frac{3\pi}{4} - 1 \right\}$

oppg $\quad$ Løs $\quad \sin(2x) = \frac{-1}{2}$

for $\quad x \in [0, \pi]$.



La $v = 2x$: $\sin(v) = -\frac{1}{2}$

$v = -\frac{\pi}{6} \quad (-30°) \quad + 2\pi \cdot n$

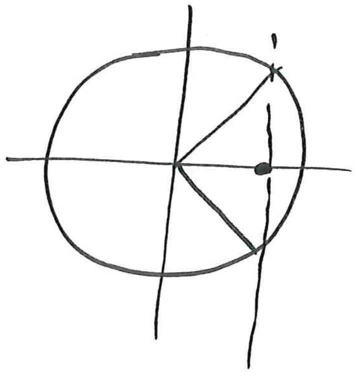
$v = \pi - (-\frac{\pi}{6}) + 2\pi \cdot n = \frac{7\pi}{6} + 2\pi \cdot n$

$x = \frac{v}{2} \quad = -\frac{\pi}{12} + \pi \cdot n \quad$ og $\quad \frac{7\pi}{12} + \pi \cdot n$

i $[0, \pi]$ har vi løsningene $\quad x = \frac{11}{12}\pi$ og $\frac{7\pi}{12}$

$v \in [0, 2\pi]$

10.4 $\cos(v) = \frac{1}{\sqrt{2}}$



$v = \arccos\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$

Løsningene er $v = \frac{\pi}{4} + 2\pi \cdot n$

og $v = -\frac{\pi}{4} + 2\pi \cdot n$

i intervallet $[0, 2\pi]$ er løsningene

$v = \left\{\frac{\pi}{4}, \frac{7\pi}{4}\right\}$

8

$\sin x = 2\cos x$

$\frac{\sin x}{\cos x} = 2$, $\tan(x) = 2$

$x = \arctan(2) + \pi \cdot n$

halvt omløp

10.5.