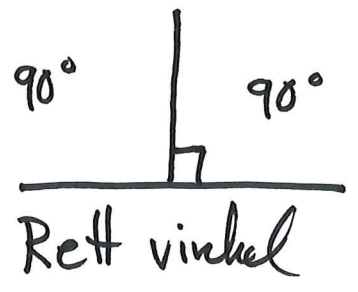
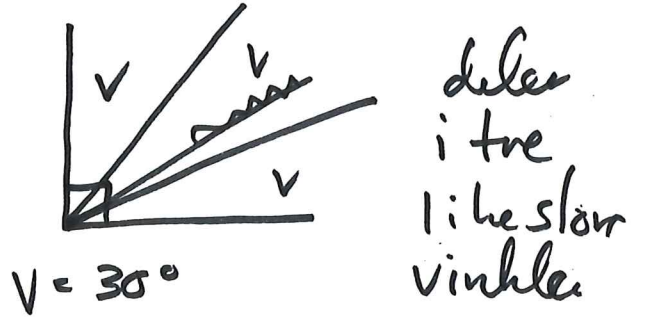
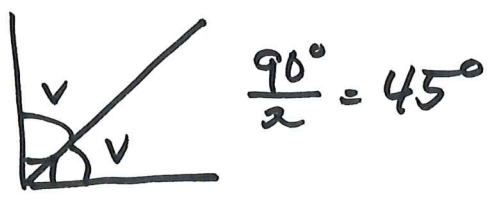


21.09
2020
Fæusk

9 Trigonometri

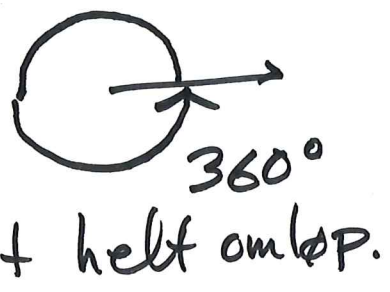
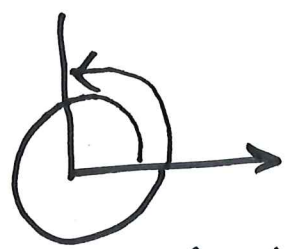
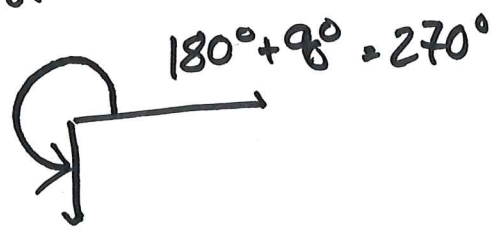


9.1



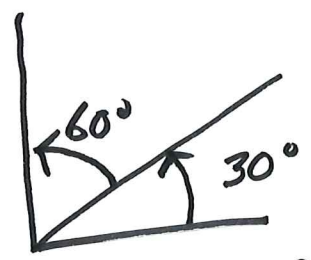
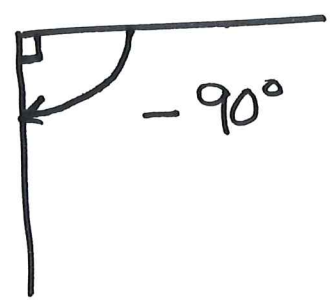
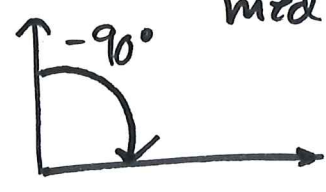
①

"mot klokken"

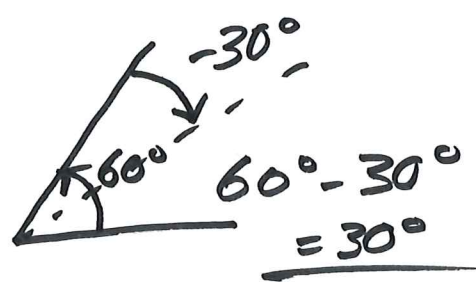


$360^\circ + 90^\circ = 450^\circ$

"med klokken"



$30^\circ + 60^\circ = 90^\circ$



Grader (deg) Helt omkøp 360°
 Gon (gradianer, grad) ————— 400°

Radianer (rad) ————— 2π

b buelengden



(2)



omkretsen
er $2\pi r$

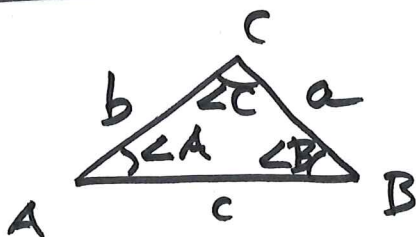
vinkel i radianer

$$\frac{b}{r} = \frac{\text{buelengd}}{\text{radius}}$$

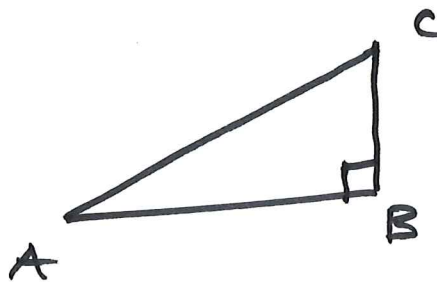
til sirkelsegmentet

(kap 10)

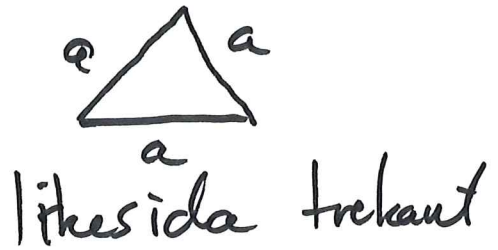
3 - kanter



Rettvinklede trekanter

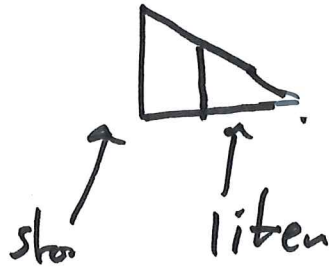


En vilkårlig Δ
består av to rettvinklede Δ -er



(3)

Formlikhet



□ formlik



samme form, men muligens ulik størrelse

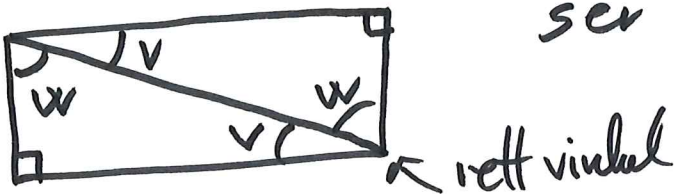
Kongruens



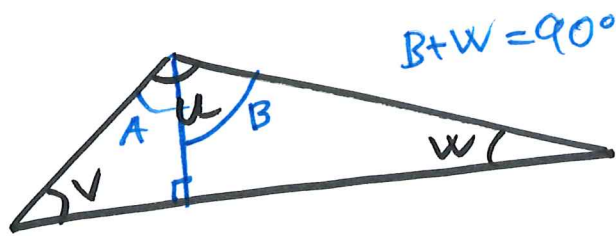
samme form og størrelse.

Summen av vinklene i en trekant er 180° .

~ ser at $v + w = 90^\circ$



Summen av vinklene i den rettvinklede Δ er er $v + w + 90^\circ = 90^\circ + 90^\circ = 180^\circ$



$$A + V = 90^\circ$$

$$B + W = 90^\circ$$

$$A + B = U$$

Så

$$A + V + B + W = 90^\circ + 90^\circ = 180^\circ$$

$$\underline{U + V + W = 180^\circ}$$

(4)

n-kant

$$n \geq 3$$

$$n=3$$



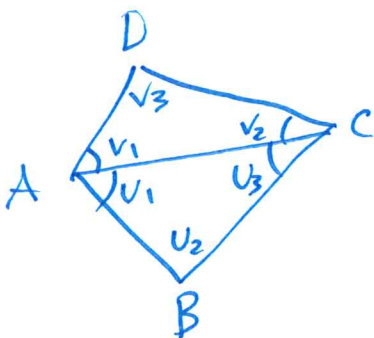
n=4 firkant



$$n=5$$



summen av vinklene i en
n-kant er $180^\circ(n-2)$



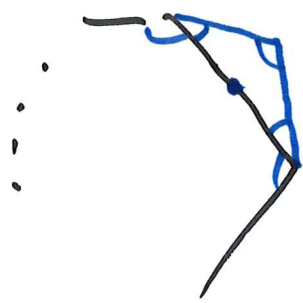
$$A + B + C + D$$

$$(U_1 + V_1) + U_2 + (V_2 + U_3) + V_3$$

$$= (U_1 + U_2 + U_3) + (V_1 + V_2 + V_3) = 180^\circ + 180^\circ$$

$$= 360^\circ$$

Fra n -kant til $n+1$ -kant



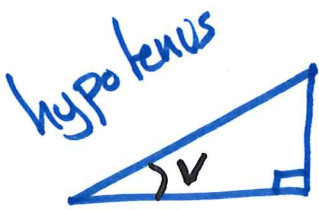
må legge til 180° for summen av vinklene.

gjør en rett side om til 2 sider. ~~der~~ summen av vinklene med 180°

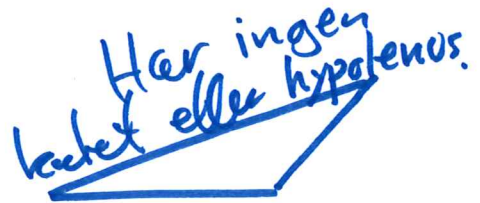
5

9.2

høstiggende katet

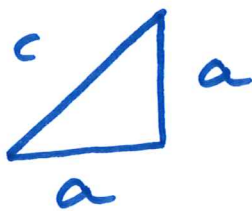
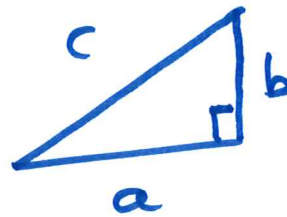


motstående katet



Pytagoras sin setz

$$c^2 = a^2 + b^2$$



Pyt gi: $a^2 + a^2 = c^2$

$$2a^2 = c^2$$

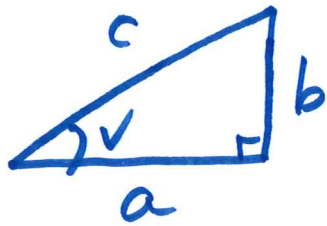
så $\frac{c}{a} = \sqrt{2}$

$a, c > 0$:

$$\frac{c^2}{a^2} = 2$$

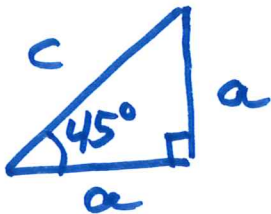
$$\frac{a}{c} = \frac{1}{\sqrt{2}}$$

Sinus til en vinkel



$$\sin(V) = \frac{\text{motsænde katet}}{\text{hypotenus}}$$
$$= \frac{b}{c}$$

⑥



$$\sin(45^\circ) = \frac{1}{\sqrt{2}}$$

$$\sin(90^\circ) = \underline{1}$$



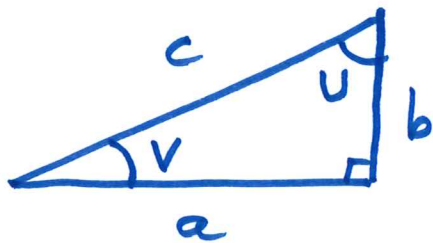
$$\sin(0^\circ) = \underline{0}$$



cosinus til vinkel

$$\cos(V) = \frac{\text{hosliggende katet}}{\text{hypotenus}}$$
$$= \frac{a}{c}$$

Så på noen eksempler på \mathbb{R} .

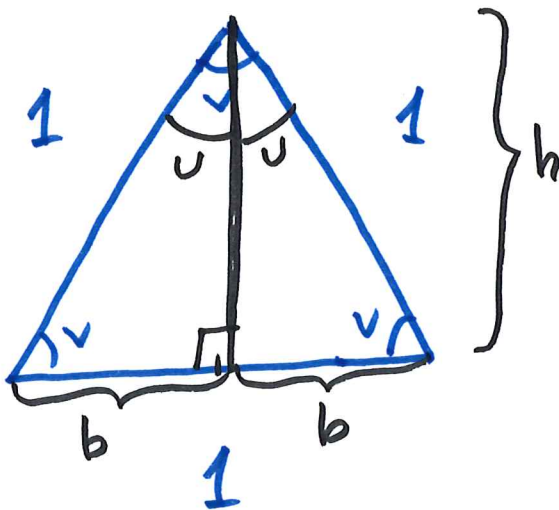


$$U = 90 - V$$

$$\sin V = \frac{b}{c} = \cos(U) = \cos(90^\circ - V)$$

(Tilsvarende $\sin U = \cos(90^\circ - U)$)

⑦ $\sin(90^\circ - V) = \cos(V)$



Likesida

$$3 \cdot V = 180^\circ$$

$$V = \frac{180^\circ}{3} = 60^\circ$$

$$2U = 60^\circ = V$$

$$U = 30^\circ$$

$$b = \frac{1}{2}$$

Pytagoras

$$b^2 + h^2 = 1^2$$

$$\left(\frac{1}{2}\right)^2 + h^2 = 1$$

$$h^2 = 1 - \frac{1}{4} = \frac{3}{4}, \quad h > 0$$

$$h = \frac{\sqrt{3}}{\sqrt{4}} = \frac{\sqrt{3}}{2} \sim \underline{\underline{0.866\dots}}$$

(10.6
i boka)

$$\cos(90^\circ) = \sin(0^\circ) = 0$$

$$\cos(60^\circ) = \sin(30^\circ) = \frac{1}{2} = 0.5$$

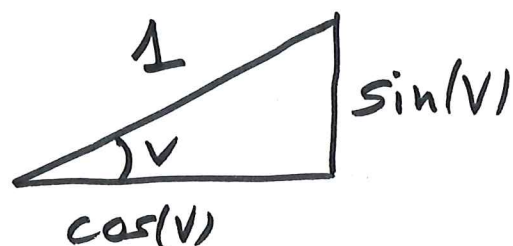
$$\cos(45^\circ) = \sin(45^\circ) = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \sim 0.707\dots$$

$$\cos(30^\circ) = \sin(60^\circ) = \frac{\sqrt{3}}{2} \sim 0.866$$

$$\cos(0^\circ) = \sin(90^\circ) = 1$$

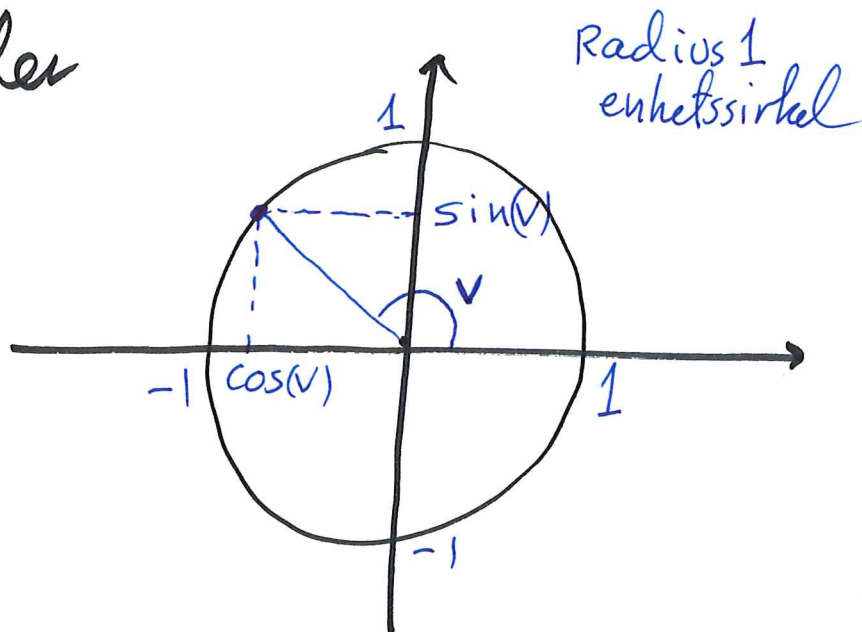
Lær utenatt

⑧



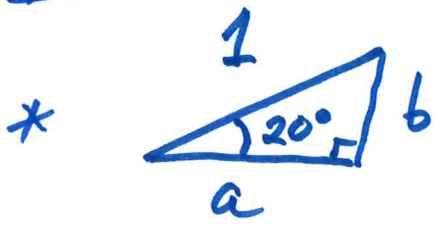
Sin & cos kan utvides til
Vilkaarlige vinkler

10.2



cos og sin tar verdier
i $[-1, 1]$.

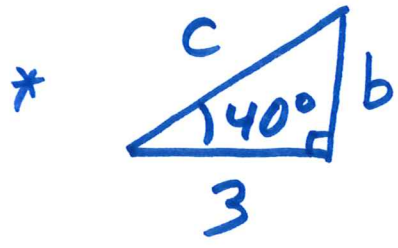
off



Finn: $a = 0.939...$
 $b = 0.342...$

(alternativt:

Pyt: $b = \sqrt{1^2 - a^2}$



Finn c og b.

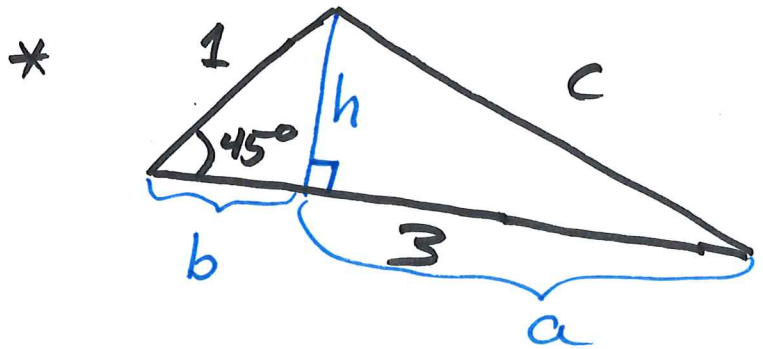
$\cos(40^\circ) = \frac{3}{c}$

$c = \frac{3}{\cos(40^\circ)} \approx 3.916...$

9

$b = c \cdot \sin(40^\circ)$
 $= 2.918...$

(alternativt Pytagoras)



Hva er c?

$h = 1 \cdot \sin(45^\circ) = \frac{1}{\sqrt{2}}$

$b = 1 \cdot \cos(45^\circ) = \frac{1}{\sqrt{2}}$

$a = 3 - b = (3 - \frac{1}{\sqrt{2}})$

Pytagoras: $c^2 = a^2 + h^2 = (3 - \frac{1}{\sqrt{2}})^2 + (\frac{1}{\sqrt{2}})^2$

$c^2 = 9 + \frac{1}{2} + \frac{1}{2} - \frac{3 \cdot 2}{\sqrt{2}} = 10 - 3\sqrt{2}$
 så $c = \sqrt{10 - 3\sqrt{2}}$