31.08.2020

H Fauski

3.1 Lineære likningsystem

(1)

$$2x + 1 = 5 \iff 2x = 5 - 1 = 4$$

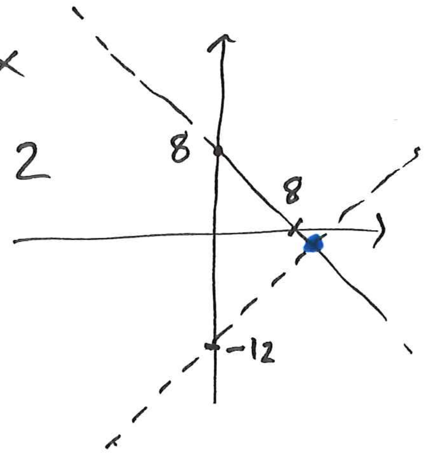
$$\iff \frac{1}{2} \cdot 2x = \frac{1}{2} \cdot 4$$

$$\underline{x = 2}$$

$$\left.\begin{aligned} x + y &= 8 \\ x - y &= 12 \end{aligned}\right\}$$

$$y = 8 - x$$

$$y = x - 12$$



* innsettingsmetoden

1 likning giv $y = 8 - x$

Setter dette inn for y i 2. likningen

$$x - (8 - x) = 12$$

$$x - 8 + x = 12 \iff 2x = 12 + 8 = 20$$

(deler med 2) $\underline{x = 10}$

$$\underline{y} = 8 - x = 8 - 10 = \underline{-2}$$

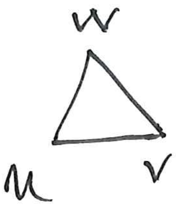
* Vi kan legge sammen og gange likninger:

$$x + y + (x - y) = 8 + 12$$

$$2x = 20 \quad \text{så} \quad x = 10\ldots$$

(2)

*



$$u + v + w = 180^\circ$$
$$u = 2 \cdot v$$
$$w = u + v$$

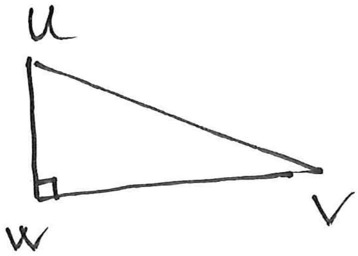
3 ukjente og 3 variabler

$$w = u + v = 2 \cdot v + v = 3v$$

Setter inn i 1. likningen

$$u + v + w = 180^\circ$$
$$2 \cdot v + v + 3 \cdot v = 180^\circ$$
$$6 \cdot v = 180^\circ$$
$$v = \frac{180^\circ}{6} = 30^\circ$$

$$u = 2 \cdot v = 60^\circ$$
$$w = 90^\circ$$



*

L1	$a + b - c = 1$	
L2	$2a - 3b + c = 2$	
L3	$3a - b + c = 2$	

"L2 − 2·L1":

$$2a - 3b + c = 2$$
$$-(2a + 2b - 2c) = -4$$
$$0 - 5b + 3c = -2$$

"L3 − 3·L1":

$$3a - b + c = 2$$
$$-3(a + b - c) = -3$$
$$-4b + 4c = -1$$

(3)

$$-5b+3c=-2 \quad | \cdot 4$$
$$-4b+4c=-1 \quad | \cdot 5$$

$$-20b+12c=-8$$
$$-20b+20c=-5$$

differansen: $12c-20c=-8-(-5)$

$$-8c=-3$$
$$\underline{c=\frac{3}{8}}$$

setter inn c-verdien $\quad -4b+4c=-1$

$$-4b+4\cdot\frac{3}{8}=-1$$
$$-4b=-1-\frac{3}{2}=\frac{-2}{2}-\frac{3}{2}=\frac{-5}{2}$$
$$\underline{b=\frac{5}{8}}$$

L1 gir $\quad a=1-b+c$
$$=1-\frac{5}{8}+\frac{3}{8}=\frac{8-5+3}{8}$$
$$\underline{a=\frac{6}{8}=\frac{3}{4}}$$

Løsningen er $\quad a=\frac{3}{4},\ b=\frac{5}{8}$ og $c=\frac{3}{8}$.

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oppg. Løs
$$3x + 2y = 12$$
$$-x + 5y = 13 \Leftrightarrow -x = 13 - 5y$$

ganger med −1 på begge sider av = ~~tegnet~~

L2 gir $x = 5y - 13$

Setter inn i L1: $3(5y-13) + 2y = 12$
$$15y + 2y - 39 = 12$$
$$17y = 12 + 39 = 51$$
$$y = \frac{51}{17} = 3.$$

$$x = 5(\cdot 3) - 13 = 15 - 13 = 2$$

Løsningen er $\underline{x = 2 \text{ og } y = 3}$

3.2 i boken

oppg. Løs likningssystem

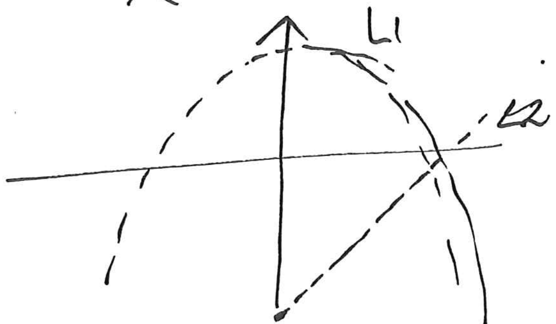
L1 $x^2 + y = 3$

L2 $2x - y = 5$.

$(y = 2x - 5)$

L1 + L2 gir $\underbrace{x^2 + 2x}_{} = 8$
$$(x+1)^2 - 1 = 8$$
$$(x+1)^2 = 8 + 1 = 9 = 3^2$$

$$x + 1 = \pm\sqrt{9} = \pm 3$$
$$x = -1 \pm 3$$



to løsninger

$x_1 = -4, \; y_1 = -13$

$x_2 = 2, \; y_2 = -1$

3.3 Ulikheter

(5) $a < b \iff b > a$ ekte ulikhet

a er ekte mindre enn b

$2 < 3$

b er ekte større enn a

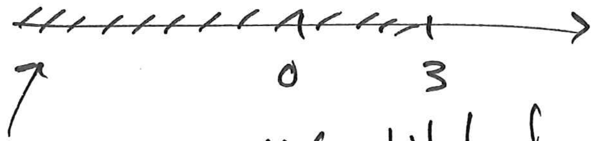
$3 > 2$

$a \leq b \iff b \geq a$

$3 \leq 3$

b er større enn eller lik a

$x < 3$



løsningen til ulikheten (påstanden) $x < 3$

$$a < b \iff a + c < b + c$$

$-x < 3$

$-3 < x$

$\iff x > -3$

legger til x og -3 på begge sider av ulikhetstegnet

$$a < b \iff \begin{array}{ll} a \cdot c < b \cdot c, & c > 0 \\ a \cdot c > b \cdot c & c < 0 \end{array}$$

$-2x + 3 > 7 \iff -2x > 7 - 3$

$-2x > 4 \quad \Big| \cdot \left(-\frac{1}{2}\right)$

snur ulikhetstegnet

$x < 4\left(-\frac{1}{2}\right)$

$\underline{x < -2}$

⑥ $-2x > 4 \iff -4 > 2x \quad \Big| \cdot \frac{1}{2}$

$$\underline{-2 > x}$$

oppg 1) $-x < 2$

2) $3x+1 > 5x+9$

1) Ganger med -1 : $\underline{x > -2}$

2) $1 > 5x - 3x + 9$

$1-9 > 2 \cdot x \iff -8 > 2x$

$\underline{-4 > x}$ (eller $x < -4$)

Forklaring på hvorfor ulikhetstegnet snus når vi ganger med $c<0$

$b > a \iff b - a > 0$

$c < 0$ da: $\iff (b-a) \cdot c < 0$

$\iff b \cdot c - a \cdot c < 0$

$\iff \underline{b \cdot c < a \cdot c}$

ulikhetstegnet er snudd.

$c > 0$ ...

3.4 Intervaller

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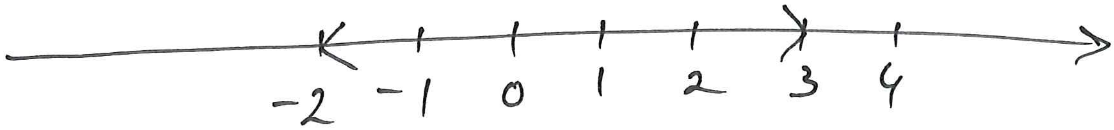


$-2 < x < 3 \iff x \in \langle -2, 3 \rangle$ åpent

$-2 \leq x < 3 \iff x \in [-2, 3\rangle$ halv-åpent

$-2 \leq x \leq 3 \iff x \in [-2, 3]$ lukket intervall

$x > 3 \iff x \in \langle 3, \infty \rangle$

$(\langle 3, \rightarrow \rangle)$ åpen

$x \leq 0 \iff x \in (\langle \leftarrow, 0])$ lukket

$\langle -\infty, 0]$

Doble ulikheter.

$$-3 < 2x+1 < -1$$

$\iff -3 < 2x+1$ og $2x+1 < -1$

$-3-1 < 2x$ $\qquad$ $2x < -1-1$

$-4 < 2x$ $\qquad$ $2x < -2$

$\underline{-2} < x$ $\qquad$ $\underline{x < -1}$

$$\underline{-2 < x < -1}$$

alternativt: $\underline{x \in \langle -2, -1 \rangle}$

Løs $0 < 3x+1 \leq 7$

8

$$0 < 3x+1 \quad \text{og} \quad 3x+1 \leq 7$$

$$-1 < 3x \qquad 3x \leq 7-1$$

$$-\frac{1}{3} < x \qquad x \leq 2$$

$$-\frac{1}{3} < x \leq 2$$

$$\underline{x \in \langle -\tfrac{1}{3}, 2]}$$

$$]-\tfrac{1}{3}, 2]$$

$$x \in [2, -\tfrac{1}{3}\rangle = \emptyset$$

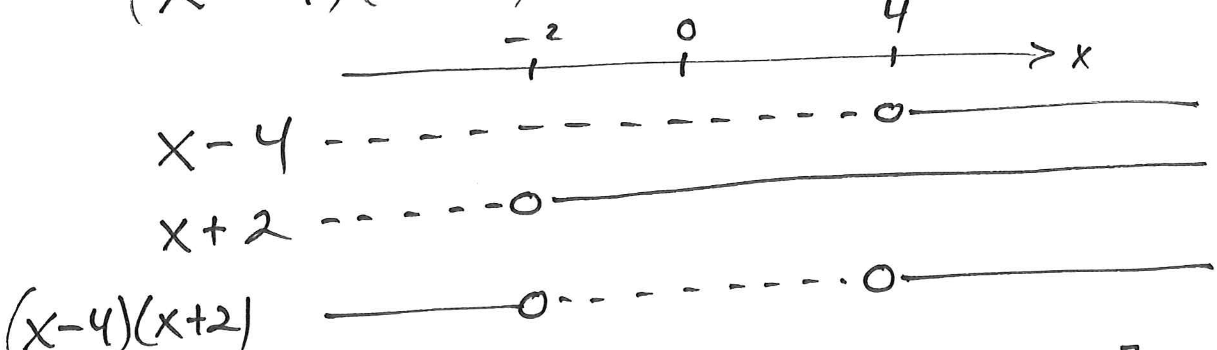
$$2 \leq x < -\frac{1}{3}$$

3.5 Andregradsulikheter

$$x^2 - 2x \geq 8$$

$$\underbrace{x^2-2x-8}_{(x-4)(x+2)} \geq 0$$

$$(x-4)(x+2) \geq 0$$



Fortegnsskjema

Løsningen bli: $\underline{x \in \langle -\infty, -2] \cup [4, \infty\rangle}$