26.02.2020 Fauck

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En antiderivert $F(x)$ til $f(x)$ er en funksjon slik at $F'(x) = f(x)$

Samlingen av alle antideriverte til $f(x)$ er det ubestemte integralet

$$\int f(x)\,dx = F(x) + C$$

$$\frac{d}{dx}\left(\frac{x^{r+1}}{r+1}\right) = \frac{r+1}{r+1}x^{r+1-1} = x^r \qquad r \neq -1$$

$$\frac{d}{dx}\left(\ln|x|\right) = \frac{1}{x} = x^{-1}$$

$$\int x^r\,dx = \begin{cases} \frac{x^{r+1}}{r+1} + C & r \neq -1 \\ \ln|x| + C \end{cases}$$

Eks $\int x^7\,dx = \underline{\frac{x^8}{8} + C}$

$$\int \frac{1}{x^5}\,dx = \int x^{-5}\,dx = \frac{x^{-4}}{-4} + C$$

$$= \underline{\frac{-1}{4x^4} + C}$$

$$\int \frac{\sqrt{x}}{\sqrt[3]{x}}\,dx = \int \frac{x^{1/2}}{x^{1/3}}\,dx = \int x^{1/2}\left(x^{1/3}\right)^{-1}dx$$

$$= \int x^{1/2}\cdot x^{-1/3}\,dx = \int x^{\frac{1}{2}-\frac{1}{3}}\,dx = \int x^{\frac{1}{6}}\,dx = \frac{x^{7/6}}{7/6} + C$$

$$= \underline{\frac{6}{7}\cdot x\sqrt[6]{x} + C}$$

(2) $F'(x) = f(x)$ k konstant

$$(kF(x))' = k \cdot F'(x) = k \cdot f(x)$$

$$\int k f(x)\,dx = k \cdot \int f(x)\,dx$$

$$F'(x) = f(x) \text{ og } G'(x) = g(x)$$

$$(F(x) + G(x))' = F'(x) + G'(x) = f(x) + g(x)$$

$$\int f(x) + g(x)\,dx = \int f(x)\,dx + \int g(x)\,dx$$

$$\left(\int \underbrace{f(x) - g(x)}\,dx \right.$$

$$\left. \int f(x) + (-1)g(x)\,dx = \int f(x)\,dx + \int (-1)g(x)\,dx = \int f(x)\,dx - \int g(x)\,dx \right)$$

$\int - \,dx$ er lineært

Ex. $$\int 3x^5 - 7/x^2\,dx$$

$$= \int 3x^5 dx - \int 7 \cdot \frac{1}{x^2}\,dx$$

$$= 3\int x^5 dx - 7\int x^{-2} dx$$

$$= 3\,\frac{x^6}{6} - 7\,\frac{x^{-1}}{-1} + C$$

$$= \frac{1}{2}x^6 + \frac{7}{x} + C$$

③ oppg. $\int \frac{2+\sqrt{x}}{x\sqrt{x}}\,dx$

$$= \int \frac{2}{x\sqrt{x}} + \frac{\sqrt{x}}{x\sqrt{x}}\,dx$$

$$= \int \frac{2}{x\sqrt{x}}\,dx + \int \frac{\sqrt{x}}{x\sqrt{x}}\,dx$$

$$= \int \frac{2}{x \cdot x^{1/2}}\,dx + \int \frac{1}{x}\,dx$$

$$= \int 2x^{-3/2}\,dx + \int x^{-1}\,dx$$

$$= 2\,\frac{x^{-1/2}}{-1/2} + \ln|x| + C$$

$$= \underline{\frac{-4}{\sqrt{x}} + \ln|x| + C}$$

Hvis $F'(x) = f(x)$, da er

$$\left(\frac{1}{a}F(ax+b)\right)' = f(ax+b)$$

$$\int f(ax+b)\,dx = \frac{1}{a}F(ax+b) + C$$

Lineær substitusjon.

eks $\int \sin(3x+1)\,dx = \underline{-\frac{1}{3}\cos(3x+1) + C}$

$$(\cos x)' = -\sin(x)$$

$$(-\cos x)' = \sin(x)$$

oppg

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$$\int x^4 + \sin(3x) - e^{2x}\,dx$$

$$= \int x^4\,dx + \int \sin(3x)\,dx - \int e^{2x}\,dx$$

$$= \frac{x^5}{5} + \frac{-\cos(3x)}{3} - \frac{e^{2x}}{2} + C$$

eks

$$\int \frac{1}{3x+2}\,dx$$

$$f(u) = \frac{1}{u} \quad , \quad \frac{1}{3x+2} = f(3x+2)$$

$$F(u) = \ln|u|$$

$$\int \frac{1}{3x+2}\,dx = \frac{1}{3}\ln|3x+2| + c$$

$$\int \frac{1}{\sqrt{5-2x}}\,dx$$

$$f(u) = \frac{1}{\sqrt{u}} = u^{-1/2} \quad , \quad \frac{1}{\sqrt{5-2x}} = f(5-2x)$$

$$F(u) = \frac{u^{1/2}}{1/2} = 2u^{1/2}$$

$$\int \frac{1}{\sqrt{5-2x}}\,dx = \frac{2(5-2x)^{1/2}}{-2} + c$$

$$= -\sqrt{5-2x} + c$$

Oppg

(5) Finn $\int \frac{4x}{x-4}\,dx$ hint: benytt polynomdivisjon

$$4x : x-4 = 4 + \frac{16}{x-4}$$
$$\frac{4x-16}{16}$$

$$\int \frac{4x}{x-4}\,dx = \int 4 + \frac{16}{x-4}\,dx$$
$$= \int 4\,dx + 16\int \frac{1}{x-4}\,dx$$
$$= 4x + 16\,\frac{\ln|x-4|}{1} + c$$
$$= \underline{4x + 16\ln|x-4| + c}$$

Mer substitusjon.

$$\left(\ln|u(x)|\right)' = \frac{1}{u(x)}\cdot u'(x) = \frac{u'(x)}{u(x)}$$
$$\int \frac{u'(x)}{u(x)}\,dx = \ln|u(x)| + c$$

Eks $\int \frac{x-1}{(x+1)(x-3)}\,dx = \int \frac{x-1}{x^2-2x-3}\,dx$

$u(x) = x^2-2x-3$ $u' = 2x-2 = 2(x-1)$

Så integralet er lik $\int \frac{\frac{1}{2}u'}{u}\,dx$

$$= \frac{1}{2}\int \frac{u'}{u}\,dx = \frac{1}{2}\ln|u(x)| + c$$
$$= \underline{\frac{1}{2}\ln|x^2-2x-3| + c}$$

Oppg. Finn $\int \frac{4x}{x^2-4}\,dx$

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$u = x^2-4$ $\qquad u' = 2x$

$4x = 2u'$

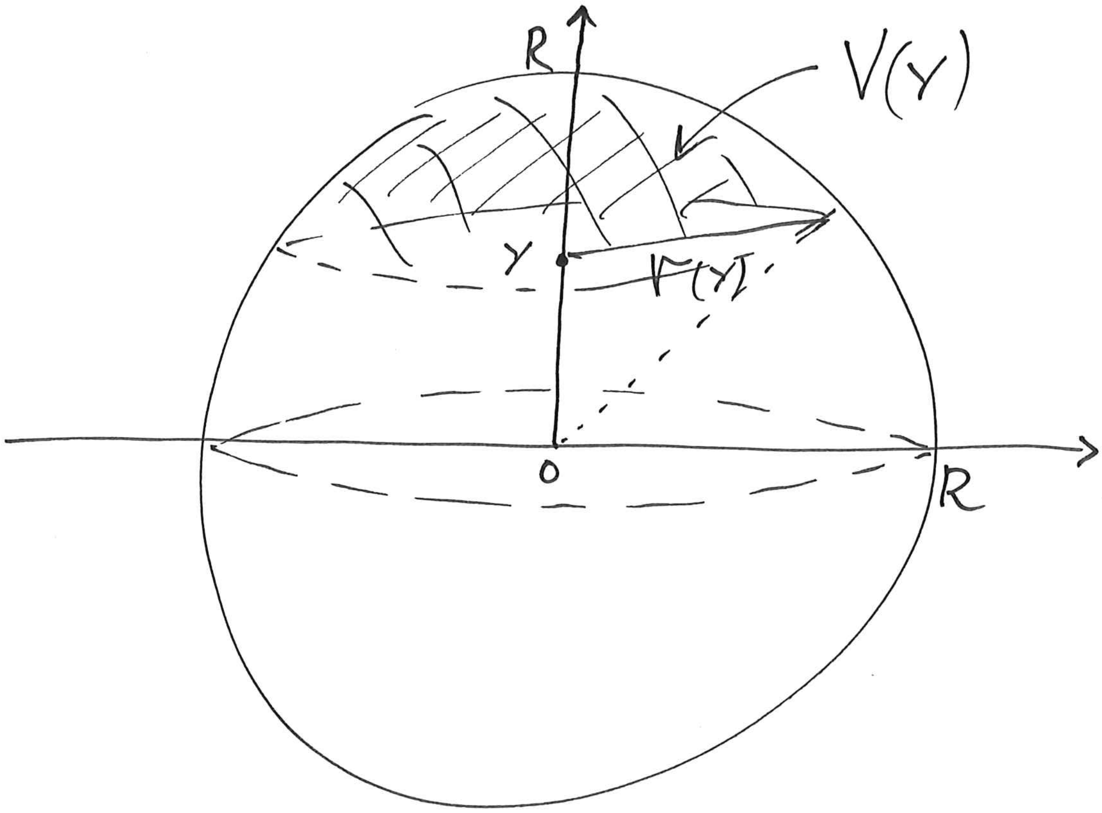
$$\int \frac{4x}{x^2-4}\,dx = \int \frac{2u'}{u}\,dx$$

$$= 2\int \frac{u'}{u}\,dx = 2\ln|u(x)| + c$$

$$= \underline{2\ln|x^2-4| + c}$$

16.1
i boka

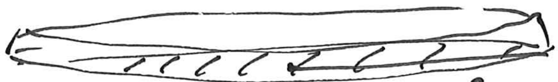
7



$V(0)$ = Volum til halve kula.

$V(R) = 0$

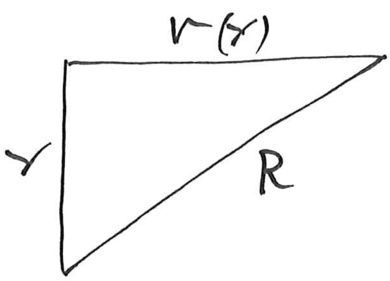
$$V'(y) = \lim_{\Delta y} \frac{\overbrace{V(y+\Delta y) - V(y)}^{\Delta V}}{\Delta y}$$



arealet e $\pi r^2_{(y)}$

$$\frac{\Delta V}{\Delta y} \sim -\frac{\pi r^2_{(y)} \cdot \Delta y}{\Delta y} = -\pi r^2_{(y)}$$

$$V'(y) = -\pi r^2_{(y)}$$



Pytagoras

$R^2 = y^2 + r_{(y)}^2$

$r^2_{(y)} = R^2 - y^2.$

(8) $V'(y) = -\pi r^2(y) = -\pi(R^2 - y^2)$

$$V(y) = -\pi\left(R^2 \cdot y - \frac{y^3}{3}\right) + C$$

Vi har at $V(R) = 0$

Setter inn: $V(R) = C - \pi\left(R^3 - \frac{R^3}{3}\right)$

$$= C - \pi \cdot R^3\left(1 - \frac{1}{3}\right)$$

$$= C - \frac{2\pi}{3}R^3 = 0$$

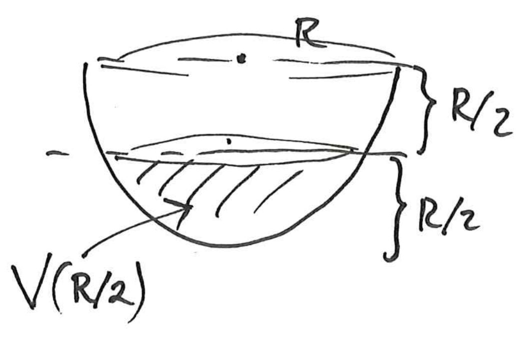
Så $C = \frac{2\pi}{3}R^3$.

$$V(y) = \frac{2\pi}{3}R^3 - \pi\left(R^2 \cdot y - \frac{y^3}{3}\right) \quad 0 \le y \le R$$

$V(0) = \frac{2\pi}{3}R^3$ er volumet til en halv kule.

Så volumet til en kule er lik 2 × volumet til en hav kule

$$\frac{4\pi}{3}R^3 \sim 4.1888\, R^3$$



$$V\left(\frac{R}{2}\right) = \frac{2\pi}{3}R^3 - \pi\left(\frac{R^3}{2} - \frac{(R/2)^3}{3}\right)$$

$$= \pi R^3\left[\frac{2}{3} - \frac{1}{2} + \frac{1}{24}\right]$$

$$= \pi R^3\left[\frac{16}{24} - \frac{12}{24} + \frac{1}{24}\right]$$

$$V\left(\frac{R}{2}\right) = \frac{5}{24}\pi R^3$$

Forholdet $\frac{V(R/2)}{V(R)} = \frac{5/24}{2/3} = \frac{5}{16} = 0.3125$