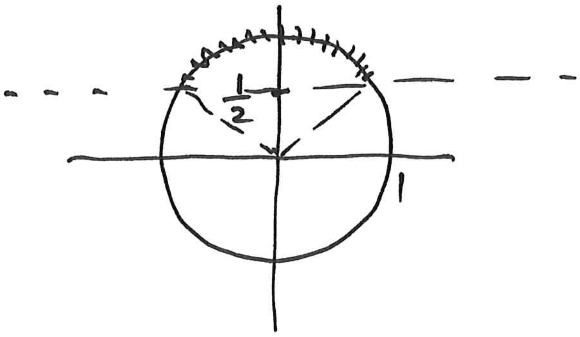
12.02.2020

11.7 Trigonometriske ulikheter

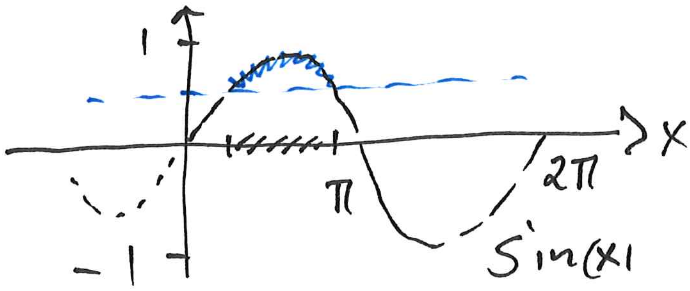
①

$\sin x > \frac{1}{2}$ $\qquad x \in [0, 2\pi]$

1) 

$$\left(\begin{array}{l} \sin x = \frac{1}{2} \\ x = \frac{\pi}{6}, \frac{5\pi}{6} \end{array} \right)$$

Løsningene er $\qquad \underline{x \in \langle \frac{\pi}{6}, \frac{5\pi}{6} \rangle}$

2) 

$\sin(\underbrace{2x-1}_{u(x)}) > \frac{1}{2}$ $\qquad x \in [0, \pi]$

Deler opp oppgaven 1) $\sin(u) > \frac{1}{2}$

2) Løs for x uttrykt ved u.

$u = 2x - 1$ $\qquad u \in [-1, 2\pi - 1]$

Fra foregående oppg $\quad \frac{\pi}{6} < u < \frac{5\pi}{6}$

$\frac{\pi}{6} < 2x - 1 < \frac{5\pi}{6} \iff \frac{\pi}{6} + 1 < 2x < \frac{5\pi}{6} + 1$

$\iff \underline{\frac{1}{2}\left(\frac{\pi}{6} + 1\right) < x < \frac{1}{2}\left(\frac{5\pi}{6} + 1\right)}$

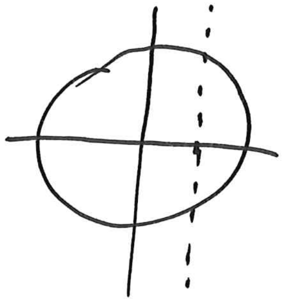
d) $\cos x < 2\cos^2 x \qquad x \in [0, \pi]$

Flytter $\cos x$ over til andre siden

$$2\cos^2 x - \cos x > 0$$

$$\cos x\,(2\cos x - 1) > 0$$

$$2\cos x\left(\cos x - \tfrac{1}{2}\right) > 0$$



Fortegn til $\cos x$:

$\cos x = 0$ for $x = \frac{\pi}{2} \sim 1.57$

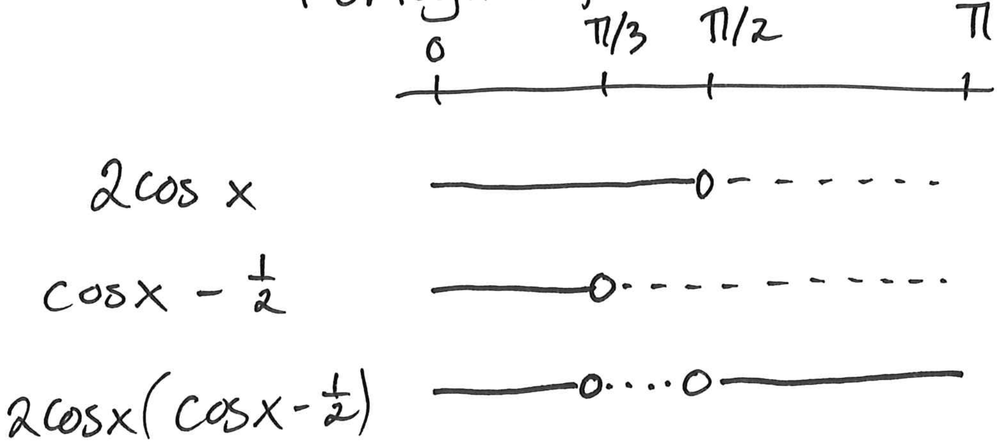
$\cos x > 0 \qquad 0 \leq x < \frac{\pi}{2}$

Fortegn til $f(x)$ $\cos x - \frac{1}{2}$ $\qquad \left(\begin{array}{l} \cos x - \frac{1}{2} > 0 \\ \Leftrightarrow \cos x > \frac{1}{2} \end{array}\right)$

$f(x) = 0 \qquad x = \pi/3 \sim 1$

$f(x) > 0 \qquad 0 \leq x < \frac{\pi}{3}$

Fortegnskjema



Løsningsmengden er $\underline{[0, \frac{\pi}{3}\rangle \cup \langle\frac{\pi}{2}, \pi]}$

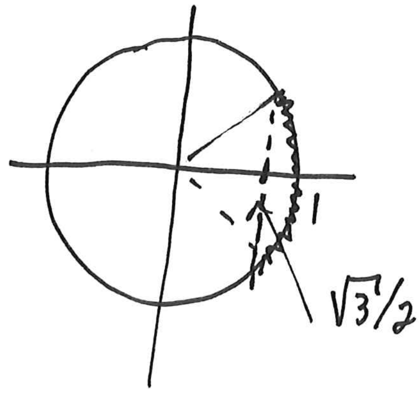
Ex oppg. juni 2013

(3) $2\cos(\underbrace{2x}_{u}) \geq \sqrt{3}$ $\qquad x \in [0, \pi]$

$u = 2x$

$u \in [0, 2\pi]$

$\cos(u) \geq \frac{\sqrt{3}}{2}$



$\cos(u) = \frac{\sqrt{3}}{2}$ $\qquad u = \pm \pi/6$

$\cos(u) \geq \frac{\sqrt{3}}{2}$ har løsning

$u \in [0, \pi/6] \cup [\frac{11\pi}{6}, 2\pi]$

$x = \frac{u}{2}$ $\qquad$ Løsningen er $\underline{x \in [0, \frac{\pi}{12}] \cup [\frac{11\pi}{12}, \pi]}$

—

$\cos(2x) > \cos(x) - 1$

Benytter trig. identiteter.

$\cos(2x) = \cos^2 x - \underbrace{\sin^2 x}_{(1-\cos^2 x)} = 2\cos^2 x - 1$

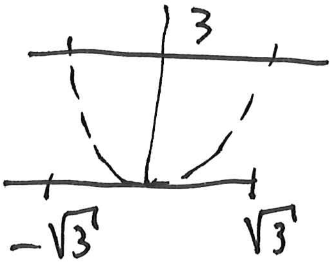
Ulikheten er ekvivalent til $\qquad 2\cos^2 x - 1 > \cos x - 1$

$\Leftrightarrow \quad 2\cos^2 x > \cos x$ $\qquad$ Løst tidligere.

4. $(\tan x)^2 < 3$ $\qquad$ $x \in [0, \pi]$

(ikke def for $x = \frac{\pi}{2}$)

$u^2 < 3$



Løsning $-\sqrt{3} < u < \sqrt{3}$.

Uliketheten $\Leftrightarrow$ $-\sqrt{3} < \tan x < \sqrt{3}$.

$\tan x = \cancel{\sqrt{x}}\ \sqrt{3}$ $\qquad$ $x = \pi/3$

$\tan x = -\sqrt{3}$ $\qquad$ $x = 2\pi/3$.

Løsning $\qquad$ $x \in [0, \frac{\pi}{3}\rangle \cup \langle \frac{2\pi}{3}, \pi]$.

(5) 11.9 Funksjonsdrøfting

$f(x) = 2\cos x + x$

$f'(x) = 2(-\sin x) + 1 = 1 - 2\sin x$

stasjonære punkt $\quad f'(x) = 0$

$\Leftrightarrow \quad \sin x = \frac{1}{2}$

$\Leftrightarrow \quad x = \frac{\pi}{6} + 2\pi \cdot n$

$x = \frac{5\pi}{6} + 2\pi \cdot n$

f økende hvor $\quad f'(x) > 0$

$f'(x) > 0 \quad \Leftrightarrow \quad 1 - 2\sin x > 0$

$\Leftrightarrow \quad \frac{1}{2} > \sin(x)$

(sett på tidligere ...)

$f''(x) = -2(\cos x) = \underline{-2\cos x}$

(6)

$$g(x) = e^{-x}\cos x$$

$$g'(x) = \left((e^{-x}) \cdot \cos(x) \right)'$$

$$= (e^{-x})' \cos x + (e^{-x})(\cos x)'$$

$$= -e^{-x}\cos x + e^{-x}(-\sin x)$$

$$= -e^{-x}(\cos x + \sin x)$$

stasjonære punkt $\qquad \cos x + \sin x = 0$

må ha

$\sin x = -\cos x \qquad (\cos x \neq 0)$

$\Leftrightarrow \quad \tan x = -1 \qquad x = \frac{3\pi}{4} + \pi \cdot n$

$g(x)$ stigende når $\quad \cos x + \sin x < 0$

$\cos x (1 + \tan x) < 0 \qquad$ (når $\cos x \neq 0$)

så i tillegg løsninger

til $\cos x = 0, \sin x = -1$

7

Eks $f(x) = \begin{cases} \dfrac{\sin x}{x} & x \neq 0 \\ 1 & x = 0 \end{cases}$

kontinuerlig siden $\lim_{x \to 0} \dfrac{\sin x}{x} = 1$.

$$f'(x) = \left(\frac{\sin x}{x}\right)' = \left(\sin x \cdot x^{-1}\right)'$$

(kvotientregelen $(f/g)' = (f' \cdot g - f \cdot g')/g^2$)

$$= \frac{(\cos x) \cdot x - \sin x \cdot 1}{x^2}$$

$$= \frac{x \cos x - \sin x}{x^2} \qquad x \neq 0$$

$f'(0) = 0$

$$f'(0) = \lim_{h \to 0} \frac{f(h) - f(0)}{h} = \lim_{h \to 0} \frac{\frac{\sin(h)}{h} - 1}{h}$$

$$= \lim_{h \to 0} \frac{\sin(h) - h}{h^2} = 0$$

(dette er ikke helt opplagt. Det viser seg at for h liten er $\sin(h) \sim h - \dfrac{h^3}{6}$)

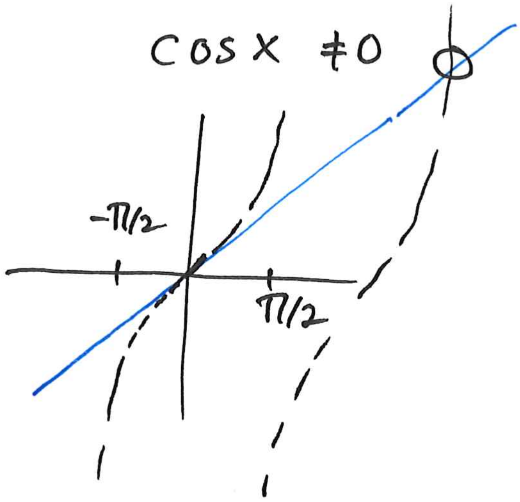
(8) stasjonære punkt $\qquad x\cos x = \sin x$

f stigende $\qquad x\cos x > \sin x$

$$x\cos x = \sin x$$

$$x = \tan x$$

$\cos x \neq 0$



numeriske estimater
(ingen analytisk beskrivelse
av løsningene annet enn $x=0$)

⑨ Hvorfor er en oktav delt i 12 toner?

kvint $\frac{3}{2} = 1.5$ C – G

kvart $\frac{4}{3} = 1.33...$ C – F

ters $\frac{5}{4} = 1.25$ C – E

oktav $\frac{2}{1} = 2$

Temperert toneskala

Deler en oktav i n toner slik at

$$\frac{\text{frekvens neste tone}}{\text{frekvens tone}} = \text{konstant } k.$$

$$\overbrace{k \cdots k}^{n} = 2 \qquad k = 2^{1/n} = \sqrt[n]{2}$$

Forhold mellom tonefrekvenser er $(2^{1/n})^m$

$$= \underline{2^{m/n}}$$

Ønsker å finne n (og m) slik at

$$\underline{1.5 \sim 2^{m/n}}$$

$$\text{Log}(1.5) = \text{Log } 2^{m/n} = \frac{m}{n} \text{Log } 2$$

$$\frac{\text{Log}(1.5)}{\text{Log}(2)} \sim \frac{m}{n}.$$

En god tilnerming er $\frac{7}{12}$

(bedre $\frac{24}{41}$ og $\frac{31}{53}$)

Kjedebrøker kan benyttes til å finne tilnerminger av rasjonale tall med brøker.

(10)

$$(\sin(x))' = \cos(x)$$

vinkel i radianer

$$(\cos(x))' = -\sin(x)$$

$$(\tan(x))' = \left(\frac{\sin(x)}{\cos(x)}\right)' = 1 + \tan^2 x = \frac{1}{\cos^2 x}$$

produktregelen $(f \cdot g)' = f' \cdot g + f \cdot g'$

$$(x \sin(3x))' = (x)' \sin(3x) + x(\sin(3x))'$$
$$= \sin(3x) + 3x\cos(3x)$$