3.02.2020

Kap 8

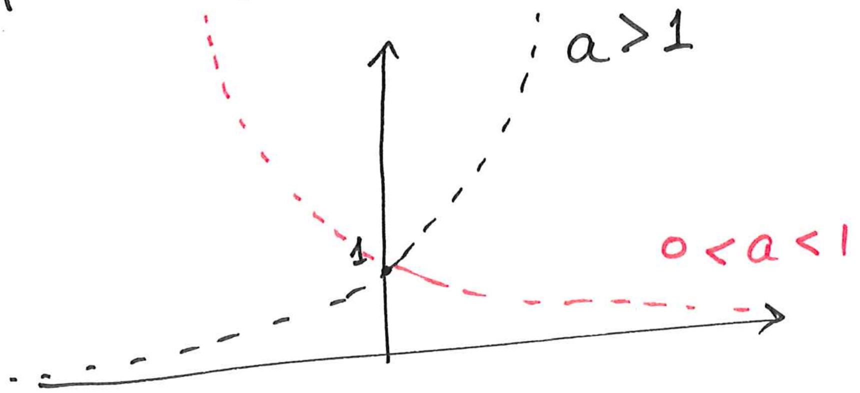
(1) Eksponentfunksjoner og logaritmefunksjoner

a^x ← eksponent, grunntall, potens

x^r ← fast, potensfunksjoner

a^x eksponentfunksjon, a fast

$a > 0$, $a \neq 1$ (fordi $1^x = 1$ alle x)



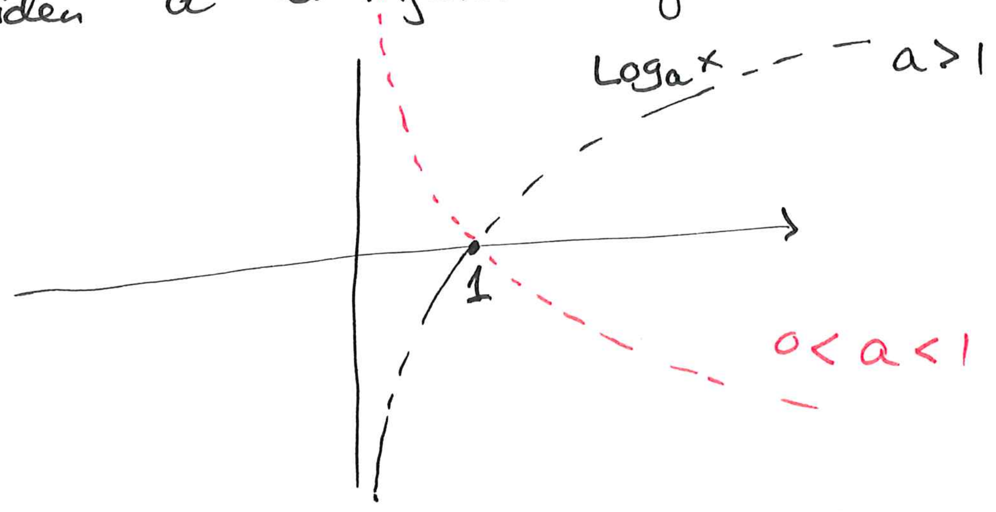
$\left(\frac{1}{a}\right)^x = (a^{-1})^x = a^{-x}$

grafen til $\left(\frac{1}{a}\right)^x$ er lik grafen til a^x reflektert om y-aksen.

Inversfunksjonen til a^x er $\text{Log}_a x$, $x > 0$

Logaritme med basis a.

(eksisterer siden a^x er injektiv for $0 < a$, $a \neq 1$)



$$\log_{(\frac{1}{a})} x = -\log_a x$$

(2) $$a^{\log_a x} = x$$

$$\left(\frac{1}{a}\right)^{-1} = a \quad \text{så} \quad a^{\log_a x} = \left(\frac{1}{a}\right)^{-\log_a x} = x$$

$$\left(\frac{1}{a}\right)^{\log_{(\frac{1}{a})} x} = x$$ eksponenten må

være like: $\log_{(\frac{1}{a})} x = -\log_a x$

kalles også lg

$\log_{10}(x) = \log(x)$ Briggsk logaritme

$$\log_a(x) = \frac{\log(x)}{\log(a)} = \left(\frac{1}{\log(a)}\right) \log(x)$$

konstant

Eks $3^x = 27$ ser: $x = 3$

$$3^x = 10$$

Anvender Log på begge sider

$$\log(3^x) = \log 10$$

$$x \log(3) = \log 10$$

$$x = \frac{\log 10}{\log 3} \sim 2.0959$$

OPPGAVER

a) $9^{2x-3} = 24$

b) $2^x 7 = 3^x$

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c) $\text{Log}(x+1) = 2 + \text{Log}(x)$

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a)
$$\text{Log}\, 9^{(2x-3)} = \text{Log}\, 24$$
$$(2x-3)\,\text{Log}\, 9 = \text{Log}\, 24$$
$$2x-3 = \frac{\text{Log}\, 24}{\text{Log}\, 9}$$
$$x = \frac{1}{2}\left(\frac{\text{Log}\, 24}{\text{Log}\, 9} + 3\right)$$
$$\sim 2.223$$

b)
$$2^x \cdot 7 = 3^x \iff 7 = \frac{3^x}{2^x} = \left(\frac{3}{2}\right)^x$$
$$\text{Log}(2^x \cdot 7) = \text{Log}\, 3^x$$
$$\text{Log}(2^x) + \text{Log}\, 7 = \text{Log}\, 3^x$$
$$x\,\text{Log}\, 2 + \text{Log}\, 7 = x\,\text{Log}\, 3$$
$$\text{Log}\, 7 = x\,\text{Log}\, 3 - x\,\text{Log}\, 2$$
$$= x(\text{Log}\, 3 - \text{Log}\, 2)$$

så
$$x = \frac{\text{Log}\, 7}{\text{Log}\, 3 - \text{Log}\, 2} \sim 4.799$$

$$a^x \cdot a^y = a^{x+y} \qquad a^0 = 1$$

$$(a^x)^y = a^{x \cdot y}$$

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$$\mathrm{Log}(a \cdot b) = \mathrm{Log}\ a + \mathrm{Log}\ b$$

$$\mathrm{Log}(a^r) = r\, \mathrm{Log}\ a$$

$$\mathrm{Log}\ 1 = 0$$

c) $\text{Log}(x+1) = 2 + \text{Log}\, x \quad , \quad x > 0$

5 $\text{Log}(x+1) \underbrace{- \text{Log}\, x}_{\text{Log}\, x^{-1}} = 2$

$\text{Log}\left(\frac{x+1}{x}\right) = 2 = \text{Log}(10^2)$

så $\frac{x+1}{x} = 10^2 = 100$

$x+1 = 100x$

$1 = 100x - x = 99x$

Så $\underline{x = 1/99}$

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8.8 Derivert til a^x

Fra def. av den deriverte:

$$(a^x)' = \lim_{h\to 0} \frac{a^{x+h} - a^x}{h}$$

$$a^{x+h} = a^x \cdot a^h$$

så $$(a^x)' = \lim_{h\to 0} a^x \cdot \frac{(a^h - 1)}{h}$$

$$= a^x \cdot \underbrace{\left(\lim_{h\to 0} \frac{a^h-1}{h}\right)}_{\text{konstant.}}$$

Ønsker å brube en a slik at konstanten blir enklest mulig: 1.

Konstanten $\lim_{h\to 0} \frac{a^h-1}{h}$ blir 1

når $a = e$ Euler tallet

$e \approx 2{,}71828\ldots$ irrasjonalt tall (ikkje en brøk)

$$e = \lim_{n\to\infty} \left(1+\frac{1}{n}\right)^n$$

$\ln x = \log_e x$
naturlig logaritme

så $$\boxed{(e^x)' = e^x}$$

$$a^x = \left(e^{\ln a}\right)^x = e^{(\ln a)\cdot x}$$

$$(a^x)' = e^{(\ln a \cdot x)} \underbrace{(\ln a \cdot x)'}_{\ln a}$$

⑦ $(a^x)' = (\ln a)\, a^x$

$(2^x)' = \ln 2 \cdot 2^x \sim (0.6931\ldots)\, 2^x$

$(10^x)' = \ln(10) \cdot 10^x \sim (2.30258\ldots)\, 10^x$

Eks $\quad (e^{3x-7})' = (e^{u(x)})' \quad$ hvor $u(x) = 3x-7$, $u' = 3$

kjerneregelen

$= e^{u(x)} \cdot u'(x) = \underline{3e^{3x-7}}$

$(e^{x^2})' = e^{x^2} \cdot (x^2)' = \underline{2x e^{x^2}}$

$((e^x)^2)' = (e^{2x})' = e^{2x} \cdot (2x)'$

$= \underline{2e^{2x}}$

$(e^{x^2} \cdot e^{-x})' = (e^{x^2-x})'$

$= e^{x^2-x} \cdot (x^2-x)'$

$= (2x-1)\, e^{x^2-x}$

Oppg. $\dfrac{7(e^x)^5}{e^{2x+1}}$ Deriver uttrykket

$= \dfrac{7e^{5x}}{e^{(2x+1)}} = 7e^{5x}(e^{2x+1})^{-1}$

$= 7e^{5x} e^{-2x-1} = 7e^{5x-2x-1} = 7e^{3x-1}$

Den deriverte er $\quad (7e^{3x-1})' = 7(e^{3x-1})'$

$= 7e^{3x-1}(3x-1)' = \underline{21e^{3x-1}}$

oppg. Deriver

8 a) $e^{\sqrt{x}}$ b) $5^x \cdot 7^x$

a) $e^{\sqrt{x}} \cdot (\sqrt{x})' = \dfrac{1}{2\sqrt{x}} e^{\sqrt{x}}$

$\left(\sqrt{x} = x^{1/2} \text{ og } (x^{1/2})' = \frac{1}{2} x^{\frac{1}{2}-1} = \frac{1}{2} x^{-1/2} = \frac{1}{2\sqrt{x}} \right)$

b) $5^x \cdot 7^x = (5 \cdot 7)^x = 35^x$

$\left((35)^x\right)' = \left(e^{\ln(35) \cdot x}\right)' = \underline{\ln(35)\, 35^x}$

Eks $5^x \cdot 7^{x^2} = e^{(\ln 5) \cdot x} \cdot e^{(\ln 7) \cdot x^2}$

$= e^{(\ln 5) x + (\ln 7) \cdot x^2}$

$(5^x \cdot 7^{x^2})' = \underline{5^x 7^{x^2} (\ln 5 + 2x \cdot \ln 7)}$

Eks $f(x) = x e^{-x}$

$f'(x) = (x)' \cdot e^{-x} + x \cdot (e^{-x})'$ produkt regelen

$= 1 \cdot e^{-x} + x(-e^{-x})$

$= (1-x) e^{-x}$ stiger for $x < 1$, synker for $x > 1$

toppunkt: $(1, \frac{1}{e})$.

$f''(x) = (f'(x))' = (1-x)' \cdot e^{-x} + (1-x) \cdot (e^{-x})'$

$= -e^{-x} + (1-x)(-e^{-x})$

$= -e^{-x} + (x-1) e^{-x}$

$= (x-2) e^{-x}$

Naturlig logaritme

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$\ln x$ inversfunksjonen til e^x

Logaritmen med grunntall e.

$e^{\ln x} = x$

$\ln e^x = x$

$\ln e = 1.$

$\ln x$ def for $x > 0$

$$(e^{\ln x})' = (x)' = 1$$

$$e^{\ln x} \cdot (\ln x)' = 1$$

$$x \cdot (\ln x)' = 1$$

$$\boxed{(\ln x)' = \frac{1}{x}}$$

Eks

$$(\ln(3x+7))' = \frac{1}{3x+7} \cdot (3x+7)'$$

$$= \frac{3}{3x+7}$$

$\ln(-x)$ def. for $x < 0$

$$(\ln(-x))' = \frac{1}{-x}(-x)' = \frac{-1}{-x} = \frac{1}{x}$$

$$|x| = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$$

$$\boxed{(\ln|x|)' = \frac{1}{x} \quad x \neq 0}$$

Opp 9 Deriver: $\ln(5x)$

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$$\left(\ln(5x)\right)' = \frac{1}{5x}(5x)' = \frac{5}{5x} = \frac{1}{x}.$$

Alternativt $\ln 5x = \ln 5 + \ln x$

$$(\ln 5x)' = (\ln 5)' + (\ln x) = \frac{1}{x}$$

Eks $f(x) = \ln\left|\frac{x+1}{x^2+1}\right| = \ln|x+1| - \ln|x^2+1|$

$$f'(x) = \underline{\frac{1}{x+1} - \frac{2x}{x^2+1}} = \frac{x^2+1 - 2x(x+1)}{(x+1)(x^2+1)}$$

$$= \frac{-x^2-2x+1}{(x+1)(x^2+1)}$$

Eks $g(x) = x\ln|x|$

$$g'(x) = (x\ln|x|)' = (x)'\ln|x| + x(\ln|x|)'$$

$$= \ln|x| + x\cdot\frac{1}{x} = \underline{\ln|x| + 1}$$

(Så $(x\ln|x| - x)' = \ln|x|$.)