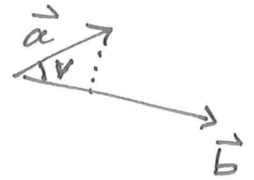
19. okt 2018

Fausk

$$\vec{a} \cdot \vec{b} = |\vec{a}| \cdot |\vec{b}| \cdot \cos(V)$$

skalar



1

$$\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$$

$\vec{a} \cdot \vec{b}$ er lineært i $\vec{a}$ og $\vec{b}$.

$$(t\vec{a}) \cdot \vec{b} = t \cdot \vec{a} \cdot \vec{b}$$

$$(\vec{a}_1 + \vec{a}_2) \cdot \vec{b} = \vec{a}_1 \cdot \vec{b} + \vec{a}_2 \cdot \vec{b}$$

På koordinatform $\quad [x_1, y_1] \cdot [x_2, y_2] = x_1 \cdot x_2 + y_1 \cdot y_2$.

13.6

Eks

Gitt $\vec{a}, \vec{b}$ slik at

$|\vec{a}| = 2 \qquad |\vec{b}| = 3 \qquad \vec{a} \cdot \vec{b} = 5$

Hva er vinkelen mellom $\vec{a}$ og $\vec{b}$?

$$\cos(V) = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|} = \frac{5}{2 \cdot 3} = \frac{5}{6}$$

$$V = \arccos\left(\frac{5}{6}\right) \approx \underline{33.5^\circ}$$

$|\vec{a}|, |\vec{b}|, \vec{a} \cdot \vec{b}$ som ovenfor

Hva er $(2\vec{a} + \vec{b}) \cdot \vec{a}$?

$$(2\vec{a}) \cdot \vec{a} + \vec{b} \cdot \vec{a} = 2|\vec{a}|^2 + \vec{b} \cdot \vec{a}$$

$$= 2 \cdot 2^2 + 5 = 8 + 5 = \underline{13}$$

Hva er $|2\vec{a} + \vec{b}|$?

linearitet gir

$$|2\vec{a} + \vec{b}|^2 = (2\vec{a} + \vec{b}) \cdot (2\vec{a} + \vec{b})$$

$$2\vec{a} \cdot (2\vec{a} + \vec{b}) + \vec{b} \cdot (2\vec{a} + \vec{b})$$

$$2\vec{a} \cdot \ldots + 2\vec{b} \cdot \vec{a} + \vec{b} \cdot \vec{b}$$

$$= 4\vec{a} \cdot \vec{a} + 2\vec{a} \cdot \vec{b}$$

2

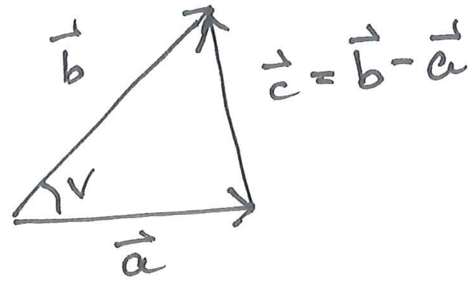
$$= 4|\vec{a}|^2 + 4\vec{a}\cdot\vec{b} + |\vec{b}|^2$$

$$= 4\cdot 2^2 + 4\cdot 5 + 3^2$$

$$= 45$$

$$|2\vec{a}+\vec{b}| = \underline{\sqrt{45}} \approx 6.708$$

Kosinussetningen og skalarproduktet:



$$|\vec{c}|^2 = |\vec{b}-\vec{a}|^2$$

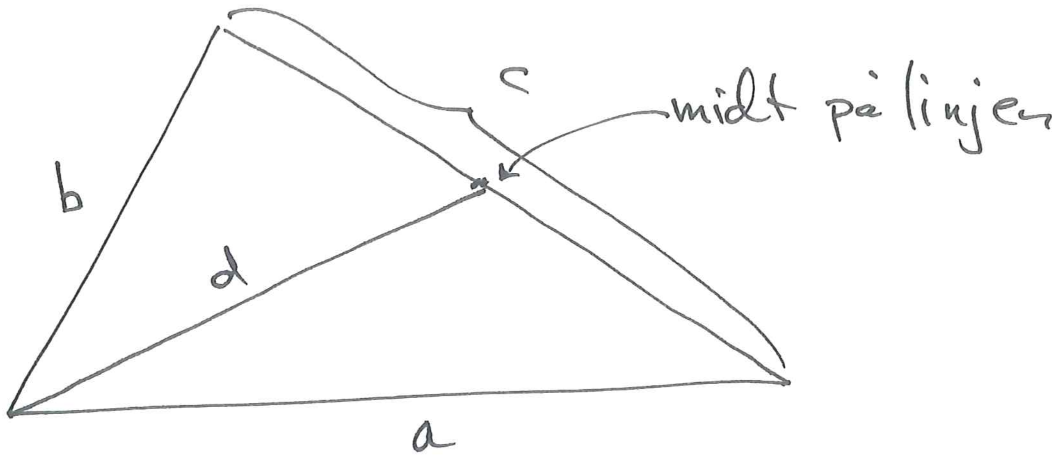
$$= (\vec{b}-\vec{a})\cdot(\vec{b}-\vec{a})$$ linearitet gir

$$= |\vec{b}|^2 + |\vec{a}|^2 - 2\vec{a}\cdot\vec{b}$$

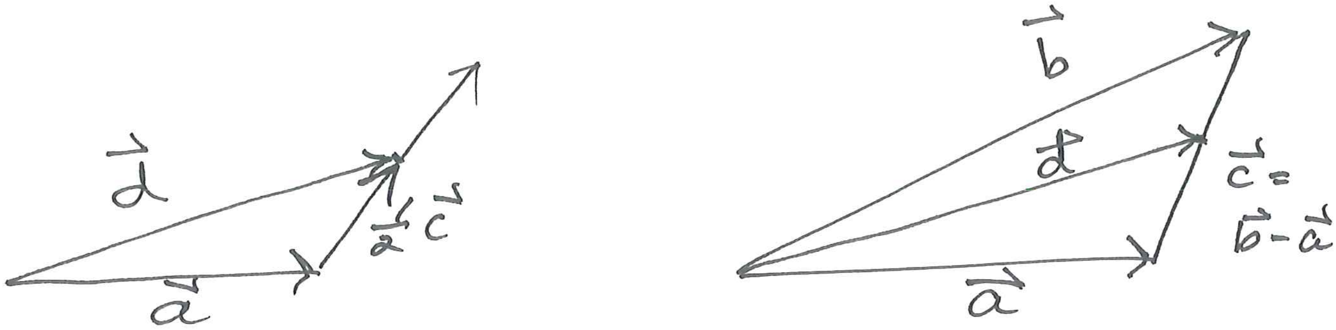
Dette er kosinussetningen

$$c^2 = a^2 + b^2 - 2a\cdot b\cos(v)$$

③ Eksempel



Apollonius identitet: $a^2+b^2 = \frac{c^2}{2} + 2d^2$
for alle trekanter



$$d^2 = |\vec{a} + \tfrac{1}{2}\vec{c}|^2 = |\vec{a} + \tfrac{1}{2}(\vec{b}-\vec{a})|^2 = |\tfrac{1}{2}(\vec{b}+\vec{a})|^2$$

$$d^2 = \tfrac{1}{2}(\vec{b}+\vec{a}) \cdot \tfrac{1}{2}(\vec{b}+\vec{a}) = \tfrac{1}{4}\left[|\vec{b}|^2 + |\vec{a}|^2 + 2\vec{a}\cdot\vec{b}\right]$$

$$c^2 = |\vec{b}-\vec{a}|^2 = (\vec{b}-\vec{a})(\vec{b}-\vec{a})$$
$$= |\vec{b}|^2 + |\vec{a}|^2 - 2\vec{a}\cdot\vec{b}$$

Så $$\frac{c^2}{2} + 2d^2 = \frac{a^2+b^2}{2} \underbrace{- \vec{a}\cdot\vec{b} + \frac{a^2+b^2}{2} + \vec{a}\cdot\vec{b}}_{\text{kanselerer}}$$

$$= a^2+b^2$$

Dette viser Apollonius' identitet.

Determinanter 13.7

(4) $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

2×2-matrise

vektorer

$$\begin{vmatrix} [a, b] \\ [c, d] \end{vmatrix} = \left| \begin{bmatrix} a, & b \\ c & d \end{bmatrix} \right| = ad - bc$$

Eks:

$$\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = \begin{vmatrix} \vec{e}_1 \\ \vec{e}_2 \end{vmatrix} = 1 \cdot 1 - 0 \cdot 0 = 1$$

$$\begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} = \begin{vmatrix} \vec{e}_2 \\ \vec{e}_1 \end{vmatrix} = 0 \cdot 0 - 1 \cdot 1 = -1$$

$$\begin{vmatrix} 2 & 5 \\ -1 & 3 \end{vmatrix} = 2 \cdot 3 - (-1) \cdot 5 = 6 + 5 = 11$$

$$\begin{vmatrix} 2 & 5 \\ 1 & 3 \end{vmatrix} = 6 - 5 = 1$$

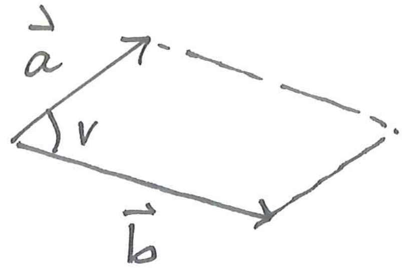
$$\begin{vmatrix} -1 & 2 \\ 3 & 6 \end{vmatrix} = (-1)6 - 2 \cdot 3 = -6 - 6 = -12$$

$$\begin{vmatrix} 1 & 2 \\ 3 & 6 \end{vmatrix} = 6 - 6 = 0.$$

$$\left| \begin{bmatrix} c & a \\ b & d \end{bmatrix} \right| = cd - ab$$

(det : determinant
abs : absoluttverdien)

5



arealet til parallellogrammet utspent av $\vec{a}$ og $\vec{b}$ er lik $\left|\det\begin{bmatrix}\vec{a}\\ \vec{b}\end{bmatrix}\right|$

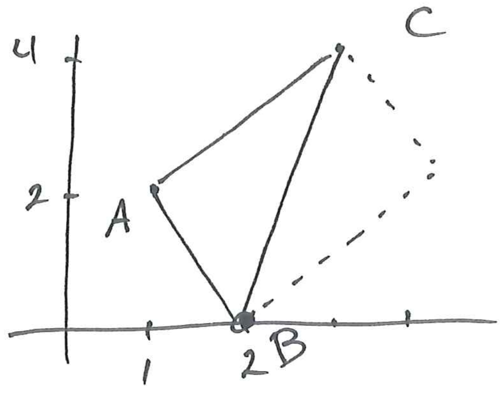
$$= \text{abs}\begin{vmatrix}\vec{a}\\ \vec{b}\end{vmatrix}$$

$$\underline{\text{abs}\begin{vmatrix}\vec{a}\\ \vec{b}\end{vmatrix} = |\vec{a}|\cdot|\vec{b}|\cdot\sin(v)}$$

Eks $A(1,2)$, $B(2,0)$ $C(3,4)$

Hva er arealet til trekanten ABC?

Arealet er $\frac{1}{2}\cdot$ arealet til parallellogrammet utspent av $\overrightarrow{AB}$ og $\overrightarrow{AC}$



$$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$$
$$= [2,0] - [1,2] = [1,-2]$$

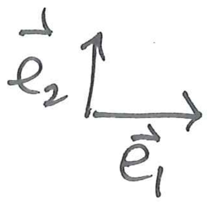
$$\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA} = [3,4] - [1,2] = [2,2]$$

⑥ $\det\begin{bmatrix}\vec{AB}\\ \vec{AC}\end{bmatrix} = \det\begin{bmatrix}1, & -2\\ 2 & 2\end{bmatrix} = 1\cdot 2-(-2)\cdot 2$

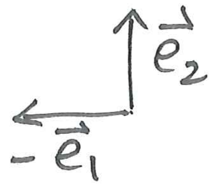
$= 2+4 = \underline{6}$

Arealet til trekanten ABC er derfor $\frac{6}{2} = \underline{\underline{3}}$

Fortegnet til determinanten $\det\begin{bmatrix}\vec{a}\\ \vec{b}\end{bmatrix}$ gir orienteringen til vektorene.



$\det\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} = 1$



$\det\begin{bmatrix}-1 & 0\\ 0 & 1\end{bmatrix} = -1$

$\det\begin{bmatrix}\vec{a}\\ \vec{b}\end{bmatrix}$ er 1) positiv hvis "tommelen peker $\boxed{\text{utover}}$ når vi vrir fra $\vec{a}$ til $\vec{b}$ med høyre hand"

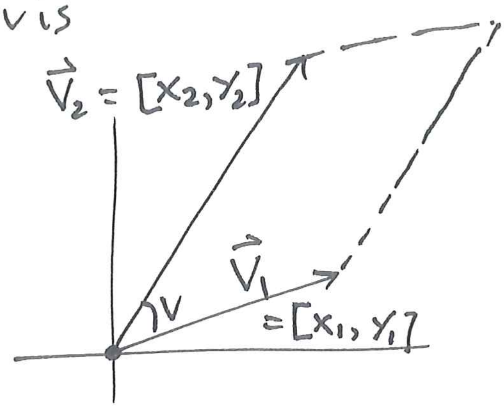
2) negativ hvis "tommelen peker $\boxed{\text{innover}}$ når vi vrir fra $\vec{a}$ til $\vec{b}$ med høyre hand.

I 13.7 i boken står et geometrisk bevis for resultatet

(7)

$$\text{abs}\begin{vmatrix}\vec{a}\\\vec{b}\end{vmatrix} = \text{arealet av parallellogrammet utspent av } \vec{a} \text{ og } \vec{b}.$$

Vi gir et alternativt bevis



Arealet A er lik

$$A = |\vec{V}_1||\vec{V}_2|\sin(v)$$

Vi benytter at $\sin^2(v) = 1 - \cos^2(v)$ og at

$$\vec{V}_1 \cdot \vec{V}_2 = |\vec{V}_1||\vec{V}_2|\cos(v).$$

$$A^2 = |\vec{V}_1||\vec{V}_2|^2 \sin^2(v) = |\vec{V}_1|^2|\vec{V}_2|^2 - |\vec{V}_1\cdot\vec{V}_2|^2$$

$$= (x_1^2 + y_1^2)(x_2^2 + y_2^2) - (x_1\cdot x_2 + y_1\cdot y_2)^2$$

$$= x_1^2 x_2^2 + x_1^2 y_2^2 + y_1^2 x_2^2 + y_1^2 y_2^2$$

$$- x_1^2 x_2^2 - 2x_1x_2y_1y_2 - y_1^2y_2^2$$

$$= (x_1y_2)^2 + (x_2y_1)^2 - 2(x_1y_2)(x_2y_1)$$

$$= (x_1y_2 - x_2y_1)^2$$

Derfor er A (≥ 0) lik

$$A = |x_1y_2 - x_2y_1|$$

$$= \text{abs}\begin{vmatrix} x_1, y_1 \\ x_2, y_2 \end{vmatrix}$$