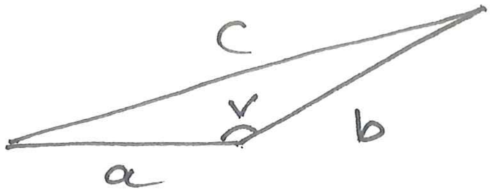
24.sep
2018

Cosinussetningen

Fausch

$0° \leq V \leq 180°$

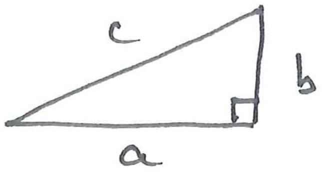
(1)



$$c^2 = a^2 + b^2 - 2ab\cos(v)$$

$V = 90°$, $\cos(90°) = 0$

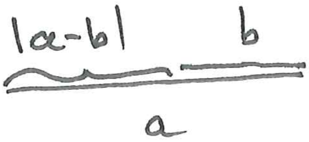
$$c^2 = a^2 + b^2$$

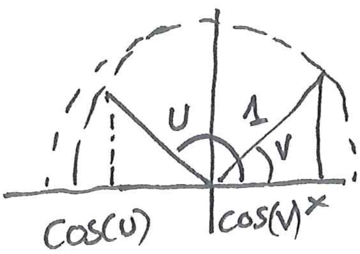


Reduseres til Pytagoras sin sats når $v = 90°$.

$V = 0°$ $\quad \cos(0°) = 1$

$$c^2 = a^2 + b^2 - 2ab \cdot 1$$
$$c^2 = (a-b)^2$$
$$c = |a-b|$$

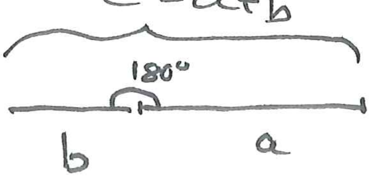




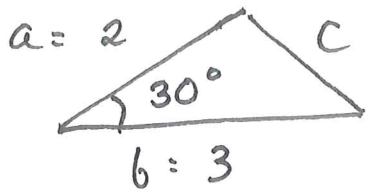
$$\cos(180° - v) = -\cos(v)$$

$V = 180°$ $\quad \cos(180°) = -1$

$$c^2 = a^2 + b^2 - 2ab(-1)$$
$$= a^2 + b^2 + 2ab = (a+b)^2$$
$$c = a + b$$



(2)



Hva er lengden til side c ?

$\cos(30°) = \sqrt{3}/2 \sim 0.866$

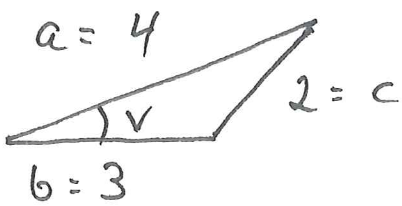
$c^2 = 2^2 + 3^2 - 2 \cdot 2 \cdot 3 \underbrace{\cos(30°)}_{\sqrt{3}/2}$

$= 13 - 6 \cdot \sqrt{3} \sim 2.6\ldots$

$c = \sqrt{13 - 6 \cdot \sqrt{3}} \sim 1.6148\ldots$

$\sim \underline{\underline{1.6}}$

Finn vinkelen V.



setter inn i

$c^2 = a^2 + b^2 - 2ab\cos(V)$

$2^2 = 4^2 + 3^2 - 2 \cdot 3 \cdot 4 \cos(V)$

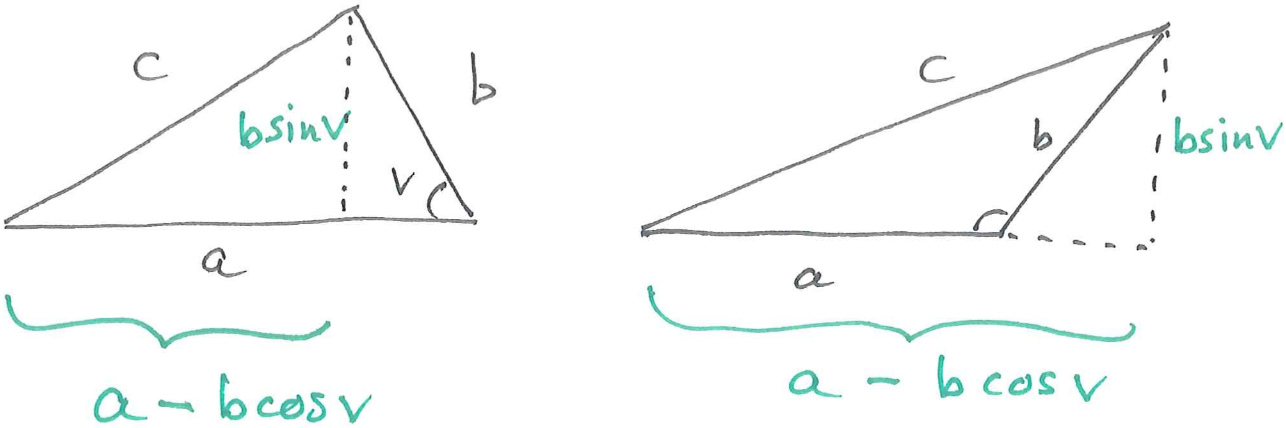
$\cos(V) = \dfrac{4^2 + 3^2 - 2^2}{2 \cdot 3 \cdot 4}$

$\cos(V) = \dfrac{16 + 9 - 4}{24} = \dfrac{21}{24} = 0.875$

$V = \arccos\left(\frac{21}{24}\right) = \underline{\underline{28.95°}}$

Bevis for cosinussetningen

(3)



Pytagoras sin sats:

$$(b\sin v)^2 + (a - b\cos v)^2 = c^2$$

$$b^2 \underbrace{\sin^2 v}_{(\sin v)^2} + a^2 + b^2\cos^2 v - 2ab\cos v = c^2$$

$$\left(\sin^2 v + \cos^2 v = 1 \right)$$
for alle v

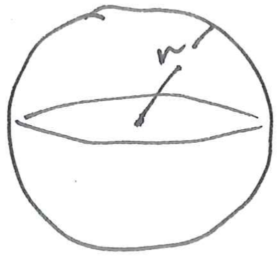
$$a^2 + b^2(\underbrace{\sin^2 v + \cos^2 v}_{1}) - 2ab\cos v = c^2$$

Dette gir cosinussetningen

$$a^2 + b^2 - 2ab\cos v = c^2$$

4

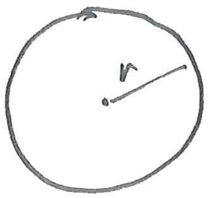
Volum og overflateareal



Volum $V = \frac{4\pi}{3} r^3$

overflatearealet $A = 4\pi r^2$

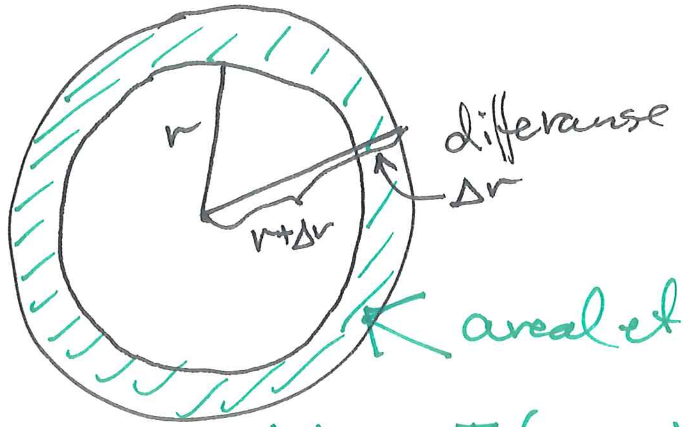
Sirkel



areal $A = \pi r^2$

omkrets $O = 2\pi \cdot r$

Vi forklarer relasjonen mellom dem.



Endringsraten til arealet til en disk er omkretsen.

arealet er

$$\Delta A = \pi(r+\Delta r)^2 - \pi r^2$$
$$= \pi(r^2 + 2r\cdot\Delta r + (\Delta r)^2) - \pi r^2$$
$$= 2\pi r\cdot\Delta r + \pi(\Delta r)^2$$

$$\frac{\Delta A}{\Delta r} = 2\pi\cdot r + \pi\cdot\Delta r$$

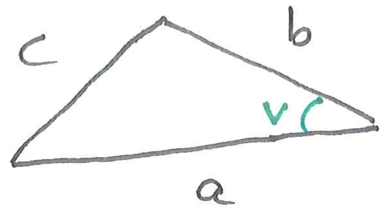
gjennomsnittlig omkrets mellom r og $r+\Delta r$

Når Δr går mot 0 blir den $\underline{\underline{2\pi r}}$

Tilsvarende for kulen...

Eksempel

(5) Uttrykk arealet til en trekant



ved a, b og c.

Vi benytter cosinussetningen til å uttrykke $\cos v$ ved hjelp av a, b og c.

Sinussetningen sier at arealet er

$$A = \frac{1}{2} a \cdot b \cdot \sin v .$$

$\sin v > 0$ for $0° < v < 180°$ så

$$\sin v = \sqrt{1 - \cos^2 v} .$$

$$c^2 = a^2 + b^2 - 2ab \cos v$$

$$\cos v = \frac{c^2 - a^2 - b^2}{2ab}$$

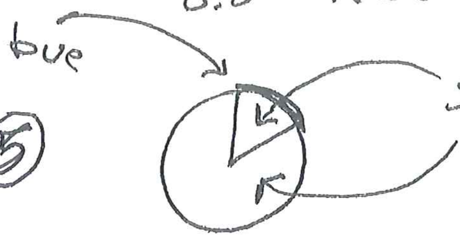
$$1 - \cos^2 v = 1 - \left(\frac{c^2 - a^2 - b^2}{2ab} \right)^2$$

$$= \frac{1}{4a^2b^2} \left[4a^2b^2 - c^4 - a^4 - b^4 + 2a^2c^2 - 2a^2b^2 + 2b^2c^2 \right]$$

$$A = \frac{1}{2} a \cdot b \cdot \sqrt{\frac{2a^2c^2 + 2a^2b^2 + 2b^2c^2 - a^4 - b^4 - c^4}{4a^2b^2}}$$

$$= \underline{\frac{1}{4} \sqrt{2a^2c^2 + 2a^2b^2 + 2b^2c^2 - a^4 - b^4 - c^4}}$$

6.8 Radianer, absolutt vinkelmål

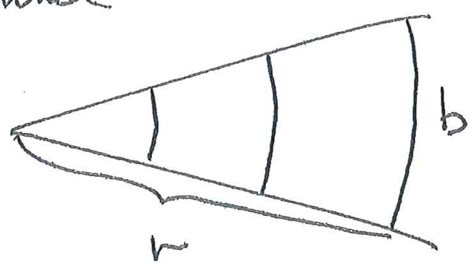


(5)

Sirkelsektorer

Del av en sirkel avgrenset av to linjer fra sentrum ut til randen.

Gitt vinkel



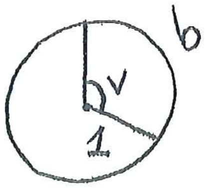
$\frac{b}{r}$ $\quad \frac{\text{buelengde}}{\text{radius}}$

er uavhengig av r
(hvilke sirkelsegment vi benytter)

$\frac{b}{r}$ er enhetsløst.

Absolutt vinkelmål er definert som $\frac{\text{buelengde}}{\text{radius}}$

som et vinkelmål gir vi det benevning (enhet) <u>radianer</u>.



Hvis radius er lik 1,
da er vinkelen lik buelengden
(med enhet rad)

Et helt omløp : $360° = \frac{2\pi \cdot r}{r} = 2\pi$ rad
(omkrets til en sirkel m. radius r er lik $2\pi r$).

$$\boxed{180° = \pi \text{ rad}}$$

$$\frac{180^\circ}{3} = \frac{\pi \text{ rad}}{3} \qquad 60^\circ = \frac{\pi}{3}\text{ rad}$$

$$\sim 1.05 \text{ rad}$$

⑦

$$90^\circ = \frac{\pi}{2} \text{ rad} \qquad 30^\circ = \frac{\pi}{6} \text{ rad}$$

$$45^\circ = \frac{\pi}{4} \text{ rad}$$

$$V_{grader} = \frac{180^\circ}{\pi \text{ rad}} V_{rad} \sim 57.3 \,{}^\circ\!/_{rad} \cdot V_{rad}$$

$$V_{rad} = \frac{\pi \text{ rad}}{180^\circ} \cdot V_{grader}$$

---

oppg.

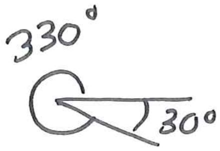
$$\frac{3}{2}\pi \text{ rad} = 1.5 \cdot \pi \text{ rad}$$

$$= \left(\pi + \frac{\pi}{2}\right) \text{ rad} = 270^\circ$$

$$2 \text{ rad} = \frac{180^\circ}{\pi \text{ rad}} \cdot 2 \text{ rad} = 114.65^\circ$$

$$(\sim 57.3 \,{}^\circ\!/_{rad} \cdot 2 \text{ rad})$$

$$330^\circ = \frac{\pi \text{ rad}}{180^\circ} \cdot 330^\circ = \dots$$



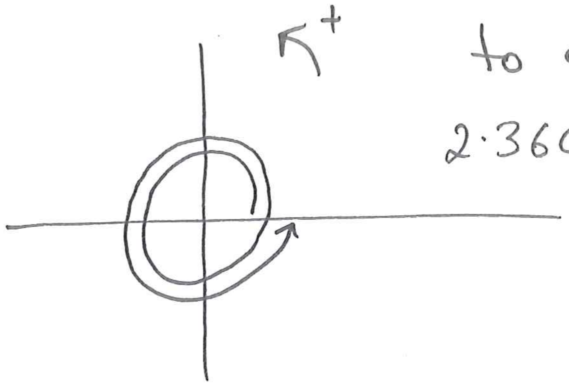
$$= 360^\circ - 30^\circ$$

$$= 2\pi \text{ rad} - \frac{\pi}{6} \text{ rad}$$

$$= \left(\frac{12\pi}{6} - \frac{\pi}{6}\right) \text{ rad} = \underline{\frac{11\pi}{6} \text{ rad}}$$

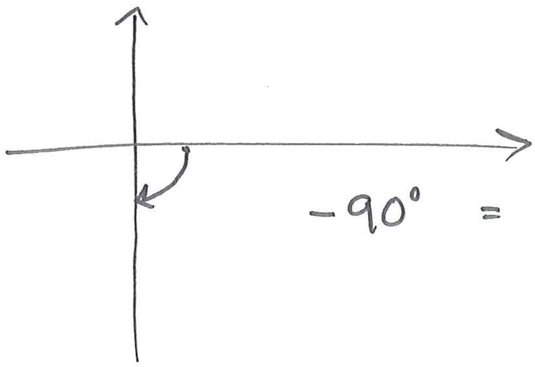
Utvider til vilkårlige vinkler

(8)

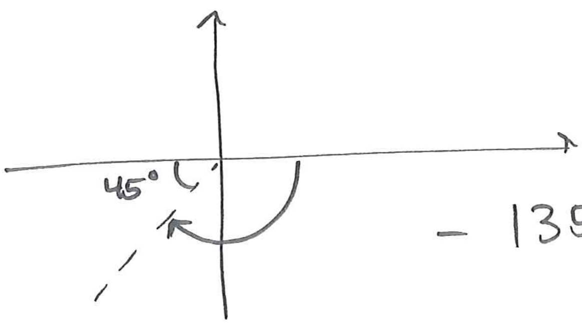


to omløp

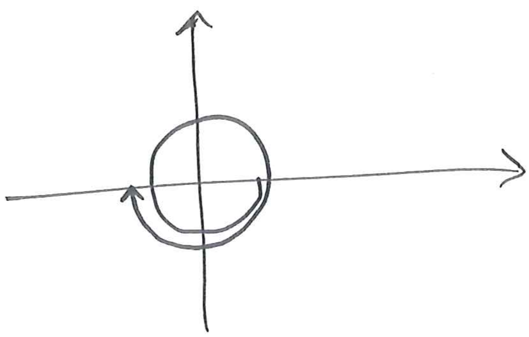
$2 \cdot 360° = 720° = 4\pi$ rad



$-90° = -\pi/2$ rad



$-135° = -\frac{3}{4}\pi$ rad.

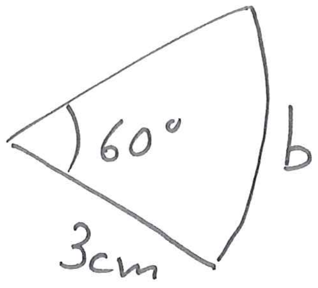


$-540° = -3\pi$ rad.

$(-360° - 180°)$

⑨

Hva er buelengden b?



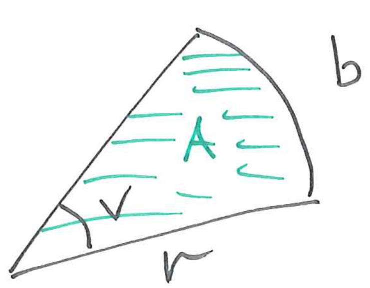
$\frac{b}{r}$ = vinkel til sirkelsegmentet i radianer

$= \pi/3$

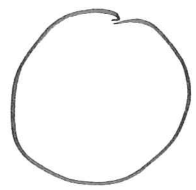
buelengden $b = \frac{\pi}{3} \cdot r = \frac{\pi}{3} \cdot 3\,\text{cm}$

$= \underline{\pi\ \text{cm}} \approx 3.14\,\text{cm}$

Areal til sirkelsegment



$V = 2\pi$ (360°)

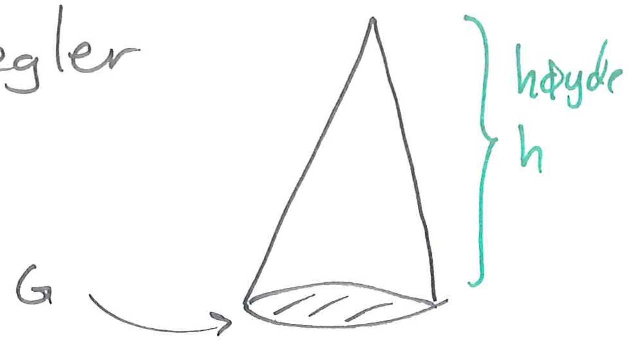


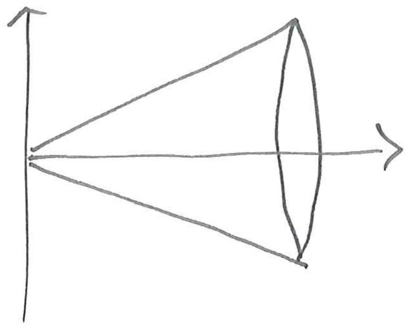
$A = \pi r^2$

$= \frac{1}{2} r \, (\overbrace{2\pi r}^{\text{omkrets}})$

$A = \frac{1}{2} r \cdot \underset{\substack{\uparrow \\ \text{buelengde}}}{b} = \frac{r^2}{2} \cdot \underset{\substack{\uparrow \\ \text{vinkel i radian}}}{V}$

Kjegler

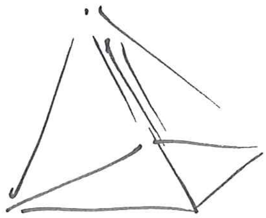




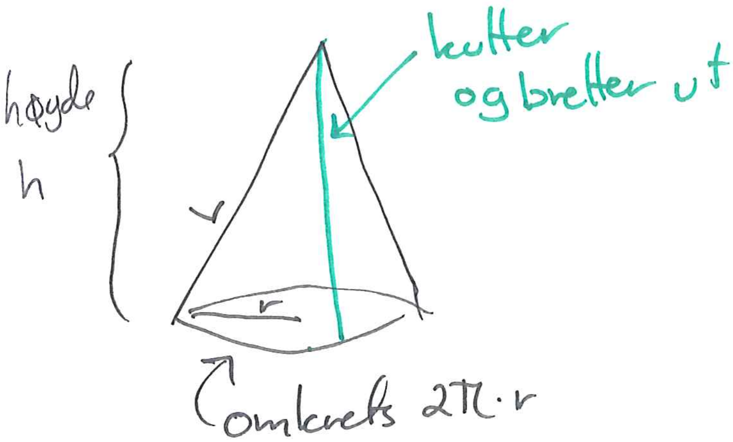
Volumet $V = \frac{G \cdot h}{3}$

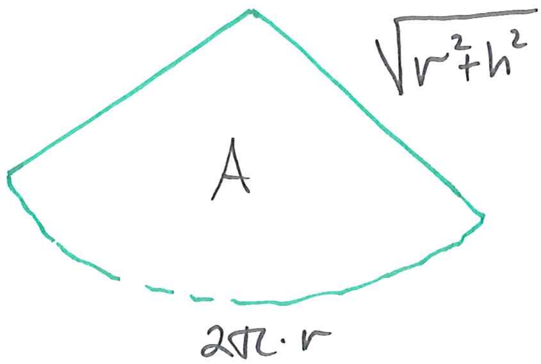
10

pyramiden

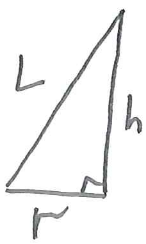


Overflatearealet til en kjegle $A = \frac{1}{2}(2\pi r)\sqrt{r^2+h^2}$





$A = \pi r \sqrt{r^2+h^2}$

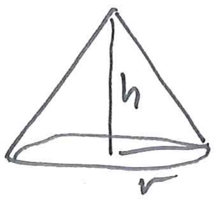


$L^2 = r^2 + h^2$

$L = \sqrt{r^2+h^2}$

En lukka kjegle (med sirkelplaten)
har overflateareal

$$\pi r^2 + \pi r\sqrt{r^2+h^2}.$$



Hva må høyden være for at overflatearealet til kjeglen (åpen) er det dobbelte av arealet til disken (πr^2)?

$$2\cdot\pi r^2 = \pi r\sqrt{r^2+h^2}$$

$$\Updownarrow$$

$$2r = \sqrt{r^2+h^2}$$

$$4r^2 = r^2+h^2 \quad \text{så}$$

$$h^2 = 3r^2$$

$$\underline{h = \sqrt{3}\,r}$$

Høyden må være $\sqrt{3}\cdot r \sim 1{,}73r$.

Til sammenlikning er arealet til en halvkule m. radius r lik $2\pi r^2$, det dobbelte av arealet til disken.