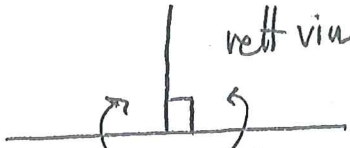
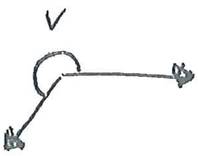
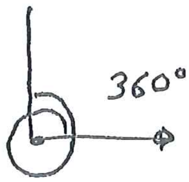
① 6 Trigonometri og geometri



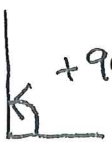
En rett vinkel er $\frac{180°}{2} = 90°$

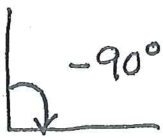


Vinkler kan utvides fra $[0°, 360°]$ (et helt omløp) til alle reelle tall.

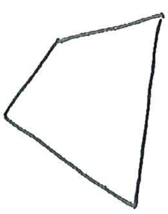


Velge positiv retning til en vinkel mot klokken





4-kant

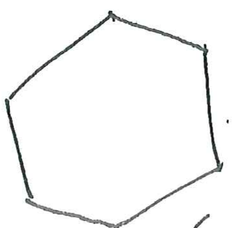


4 rette sider

3-kant

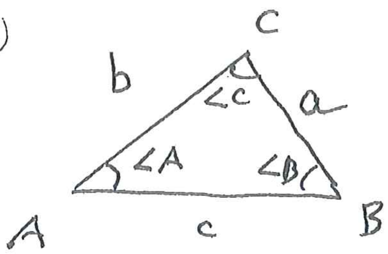
n kant

$n \geq 3$



6-kant.

3

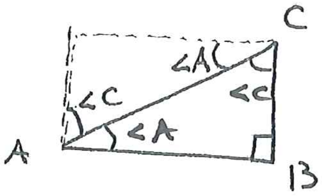


- vinkelen i hjørne A : $\angle A$ eller bare A.
- side a . vi bruker også a om lengden til siden
(motsatt hjørne A)

Summen av vinklene i en trekant er alltid <u>180 grader</u>

$$\angle A + \angle B + \angle C = 180°.$$

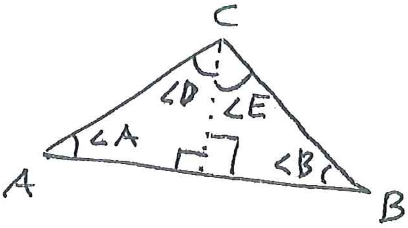
Forklaring:



$\angle A + \angle C = 90°$

$\angle B = 90°$

så $\angle A + \angle B + \angle C = 180°$ i en rettvinklet △



$\angle A + \angle D = 90°$

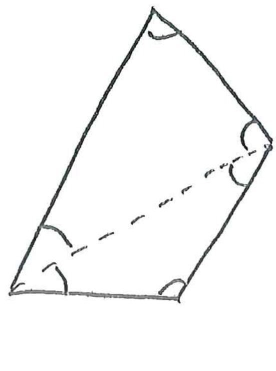
$\angle B + \angle E = 90°$

(siden rettvinkla trekanter)

Legger samman $\angle A + \angle B + \underbrace{(\angle D + \angle E)} = 180°$

$$\underline{\angle A + \angle B + \angle C = 180°}.$$

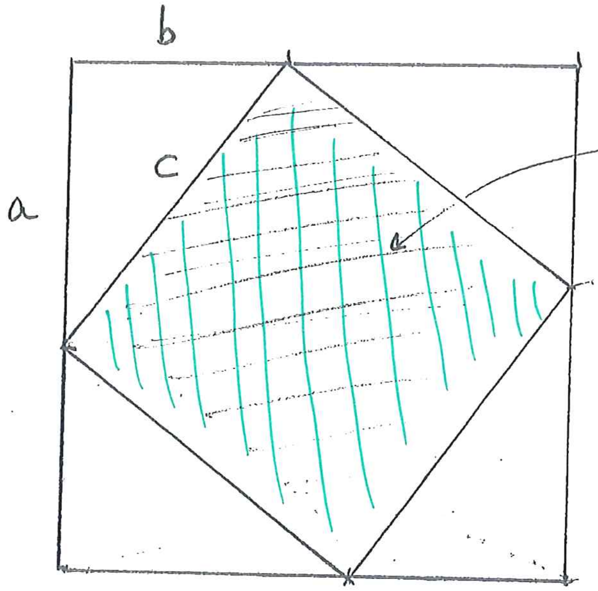
Summen av vinklene i en 4-kant er 360 grader



Summen av vinklene i firkanten er summen av vinklene i de to hjelpetrekantene.

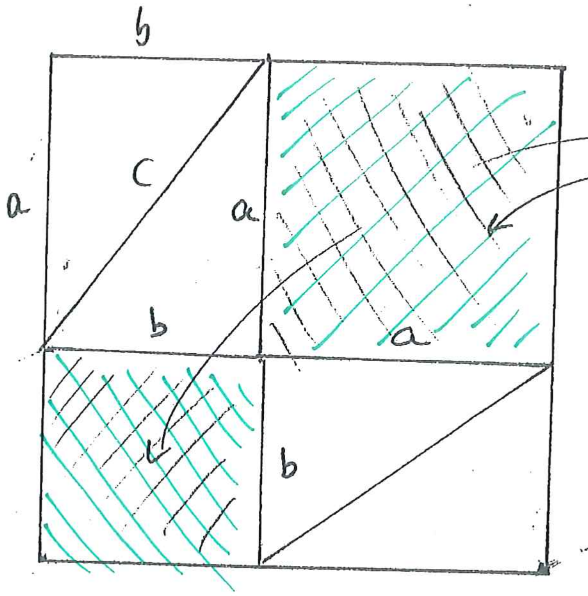
Summen er $180° + 180° = 360°$.

⑤ Geometrisk bevis for Pytagoras sin sats.



arealet er c^2

Flytter på de fire identiske trekantene inni kvadratet.



summen av arealene (utenfor trekantene) er $a^2 + b^2$

Derfor er $c^2 = a^2 + b^2$

Dette argumentet er gyldig for alle rettvinkla trekanter.

$\sin(36.8°) = 0.6 \quad (0.59902\ldots)$

Tilsvarende $\cos^{-1}$ eller arccosinus

"inverscosinus" "arcus cosinus"

$\arccos\left(\frac{4}{5}\right) = \arccos(0.8) = 36.8$

$\overset{\shortparallel}{\cos^{-1}\left(\frac{4}{5}\right)}$ (men $\left(\cos\frac{4}{5}\right)^{-1} = \frac{1}{\cos(4/5)} \approx 1$)

$\sin^{-1}(0.8) = \arcsin(0.8) = 53.13$

(vinkel u.) $\approx 90° - 36.87..$

$\arccos(0.6) = 53.13$

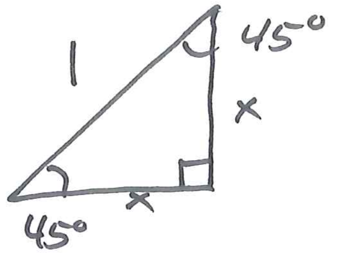
$\sin^{-1} y$ er vinkelen v (mellom $-90°$ og $90°$)

slik at $\sin(v) = y$

Dette må ikkje forveksles med $\frac{1}{\sin y}$.

Eksakte verdier til sin og cos

45°



likebeina trekant
katetene er like store

Pytagoras :
$$x^2 + x^2 = 1$$
$$2x^2 = 1$$
$$x^2 = \frac{1}{2}$$

(x er positiv)

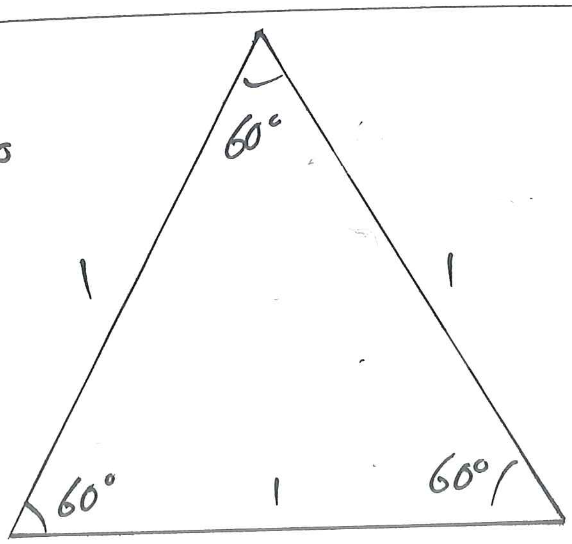
$$x = \sqrt{\frac{1}{2}} = \underline{\frac{1}{\sqrt{2}}}$$

$$\left(\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{\sqrt{2}\cdot\sqrt{2}} = \frac{\sqrt{2}}{2} \right)$$

$$\sin(45°) = \cos(45°) = \underline{\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \approx 0.707}$$

$$\tan(45°) = \frac{1/\sqrt{2}}{1/\sqrt{2}} = \underline{1}$$

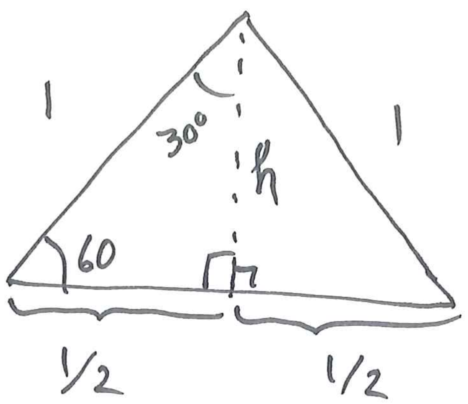
30° og 60°



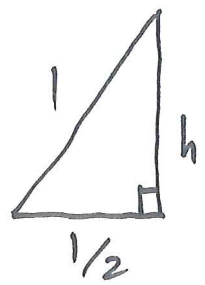
Alle sidene har lengde 1

Alle tre vinklene er like store.

De er derfor $\frac{180°}{3} = 60°$



$$\cos(60°) = \sin(30°) = 1/2$$



Pytagoras : $\left(\frac{1}{2}\right)^2 + h^2 = 1^2$

$$h^2 = 1 - \frac{1}{4} = \frac{3}{4}$$

$$h = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{\sqrt{4}} = \frac{\sqrt{3}}{2} \quad (h > 0)$$

$$\sin(60°) = \cos(30°) = \underline{h = \frac{\sqrt{3}}{2} \sim 0{,}866}$$

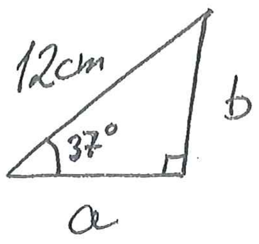
$$\tan(30°) = \frac{\sin(30°)}{\cos(30°)} = \frac{1/2}{\sqrt{3}/2} = \frac{1}{\sqrt{3}}$$

$$\tan(60°) = \frac{\sin(60°)}{\cos(60°)} = \frac{\sqrt{3}/2}{1/2} = \sqrt{3}$$

Vis at $\tan(V) \cdot \tan(90-V) = 1$

for $0 < V < 90°$

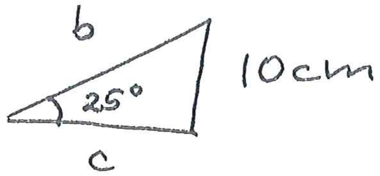
1



Finn a og b.
(lengden til a og b)

$b = 12\text{cm} \cdot \sin(37°) \approx 7.22\text{cm}$

$a = 12\text{cm} \cdot \cos(37°) \approx 9.58\text{cm}$



Finn b og c.

$\sin(25°) = \dfrac{10\text{ cm}}{b}$

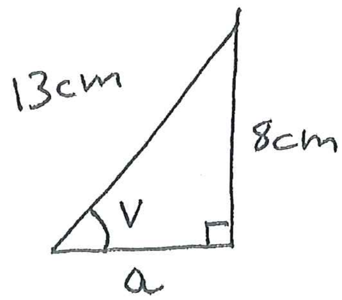
Så $b = \dfrac{10\text{ cm}}{\sin(25°)} \approx 23{,}66\text{cm}$

$c = b \cdot \cos(25°) \approx 21.44\text{cm}$

Alternativt kunne vi benytte Pytagoras:

$b^2 = (10\text{cm})^2 + c^2$

$c = \sqrt{b^2 - (10\text{cm})^2}$ ...



Hva er vinkelen v?
Hva er lengden til side a?

$\sin(v) = \dfrac{8\text{cm}}{13\text{cm}} = \dfrac{8}{13}$

$v = \arcsin\left(\frac{8}{13}\right)$ (eller $\sin^{-1}\left(\frac{8}{13}\right)$)

$= 37.979... \approx \underline{38°}$

* $a = 13\text{cm} \cdot \cos(v)$

* Eller Pytagoras: $a^2 = (13\text{cm})^2 - (8\text{cm})^2 = (169 - 64)\text{cm}^2$
$= 105\text{cm}^2$

$a \approx 10.25\text{cm}$