5. sep 2018

Faush

Intervaller

(1)

$x < 3$

$x \geq 2$

"x større enn eller lik 2 og mindre enn 3"

$x \geq 2$ og $x < 3$

$(2 \leq x)$

Dette skrivest kortere (mer oversiktlig og kompakt)

som $2 \leq x < 3$

dobbel ulikhet

En alternativ skrivemåte for tallene større enn eller lik 2 og mindre enn 3

er $[2, 3\rangle$

(alternativt $[2, 3)$)

$x \in [-3, 4]$ $\Leftrightarrow$ $-3 \leq x \leq 4$

element i

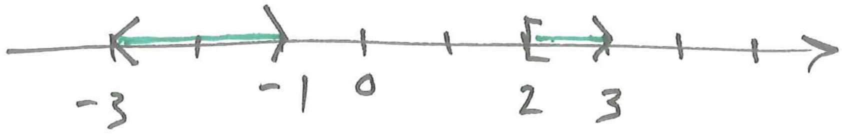
Lukka intervaller $[-3, 4]$, $[-2, \infty\rangle$

(alt: $[-2, \rightarrow\rangle$)

Åpne intervaller $\langle 2, 3\rangle$ (eller $(2, 3)$) $\langle -\infty, 5\rangle$

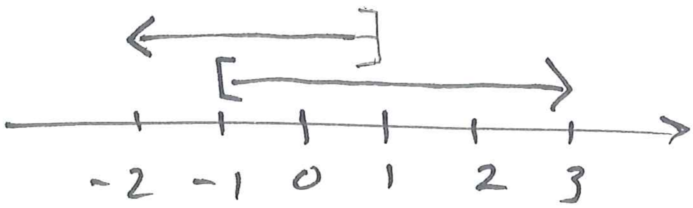
Halvåpne intervaller $[2, 3\rangle$, $\langle -\infty, -2]$

2



$x \in \langle -3, -1 \rangle \cup [2, 3\rangle$

union



$\langle -2, 1] \cup [-1, 3\rangle = \langle -2, 3 \rangle$

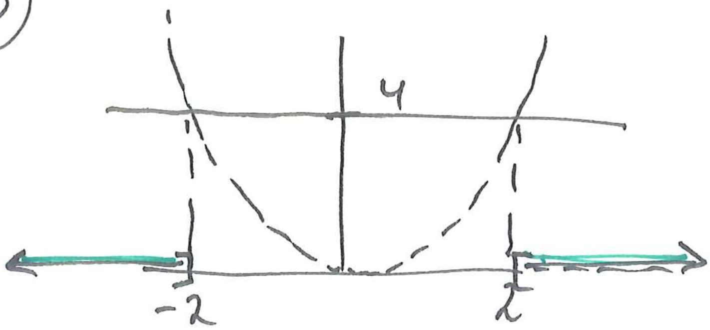
Et punkt kan beskrives om $[2,2]$

alternativt: $\{2\}$

listen av elementet 2.

35 Andregradsulikheter

(3) $x^2 \geq 4 \iff x^2 - 4 \geq 0$



grafisk løsning

Løsningen er $x \in \langle -\infty, -2] \cup [2, \infty\rangle$

systematisk metode

$x^2 - 4 \geq 0$ Faktorisere uttrykket

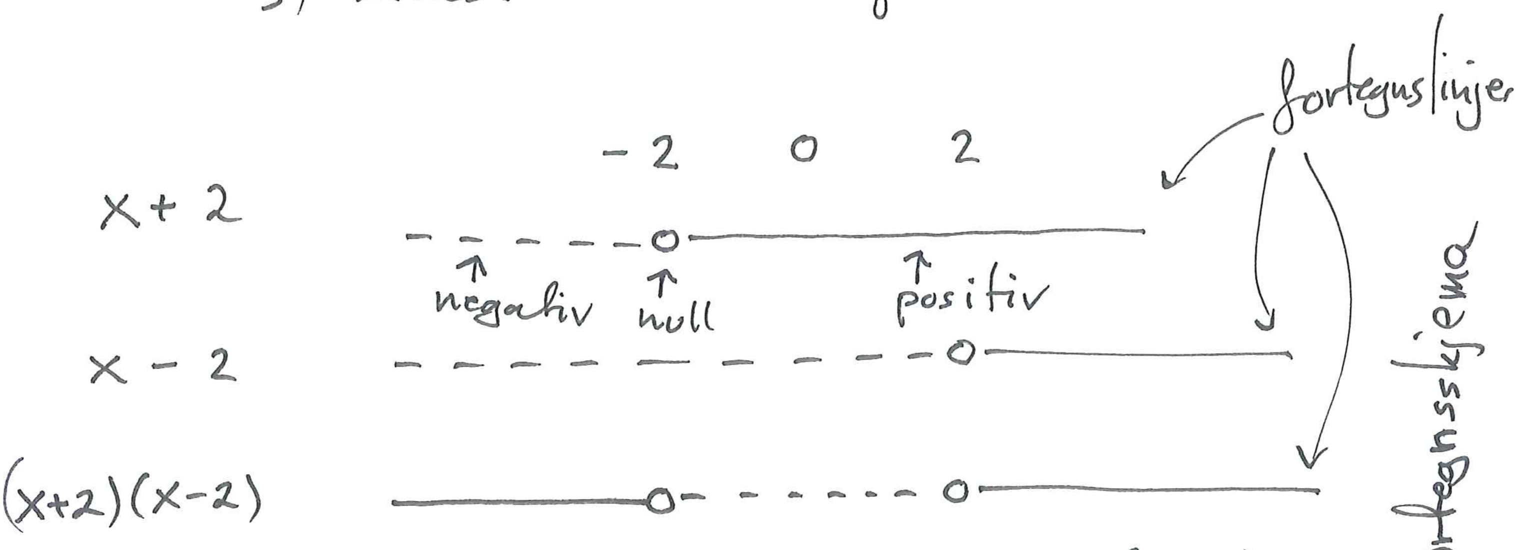
$x^2 - 4 = (x+2)(x-2)$

$(x+2)(x-2) \geq 0$

Når 1) begge faktorene er positive

2) ——— negative

3) minst en av faktorene er null.



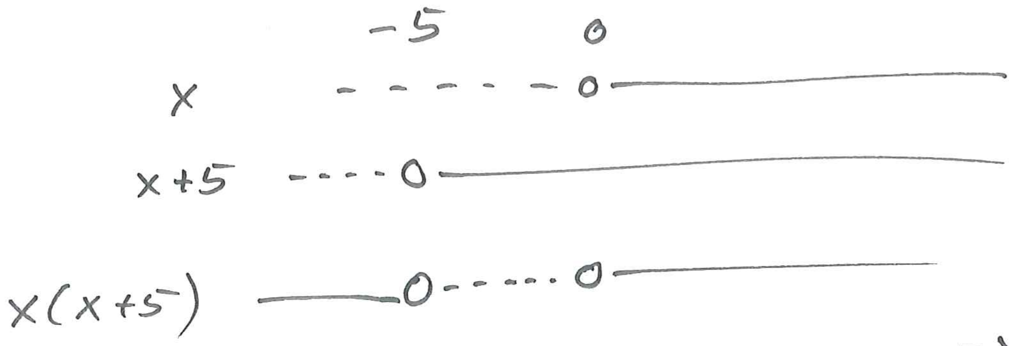
Vi leser av løsningen til $x^2 - 4 \geq 0$

som $\langle -\infty, -2] \cup [2, \infty\rangle$

4)

$x^2 + 5x > 0$

$x(x+5) > 0$

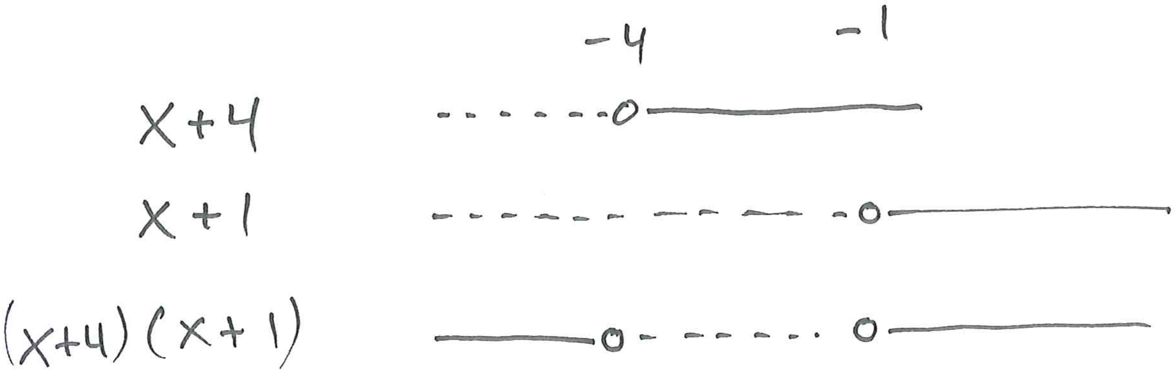


Løsningen er $\underline{\langle -\infty, -5\rangle \cup \langle 0, \infty\rangle}$

Alternativt kan vi skrive: $x < -5$ eller $x > 0$.

$x^2 + 5x + 4 < 0$

$(x+4)(x+1) < 0$



Løsningene er $\underline{\langle -4, -1\rangle}$

Løs ulikheten

(5) $$4y^2 - 6y > 1$$

$$\Leftrightarrow \quad 4y^2 - 6y - 1 > 0$$

Finner nullpunktene til $4y^2 - 6y - 1$

$$4y^2 - 6y - 1 = 0$$

$$\sqrt{2^2\cdot(-3)^2 - 2^2(-4)}$$

abc-formelen $$y = \frac{-(-6) \pm \sqrt{(-6)^2 - 4\cdot 4(-1)}}{2\cdot 4}$$

$$y = \frac{+6 \pm \sqrt{4}\sqrt{9+4}}{2\cdot 4} = \frac{\cancel{2}}{\cancel{2}} \cdot \frac{3 \pm \sqrt{13}}{4}$$

$$4y^2 - 6y - 1 = 4\left(y - \frac{3-\sqrt{13}}{4}\right)\left(y - \frac{3+\sqrt{13}}{4}\right)$$

Fortegnsskjema

$\frac{3-\sqrt{13}}{4}$

$\left(y - \frac{3-\sqrt{13}}{4}\right)$ - - - - - - - - - 0 ————————

$\frac{3+\sqrt{13}}{4}$

$\left(y - \frac{3+\sqrt{13}}{4}\right)$ - - - - - - - - - - - 0 ————————

4 er alltid positiv

$4y^2 - 6y - 1$ ———————— 0 - - - - 0 ————————

Løsningen er $\left\langle -\infty, \frac{3-\sqrt{13}}{4} \right\rangle \cup \left\langle \frac{3+\sqrt{3}}{4}, \infty \right\rangle$

oppgave Løs ulikheten

(6)

$$3x^2 - x \le x + 1$$

$$\Leftrightarrow \quad 3x^2 - 2x - 1 \le 0$$

Faktoriserer $3x^2 - 2x - 1$:

Røttene til (nullpunktene) $3x^2 - 2x - 1$ er

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4 \cdot 3(-1)}}{2 \cdot 3}$$

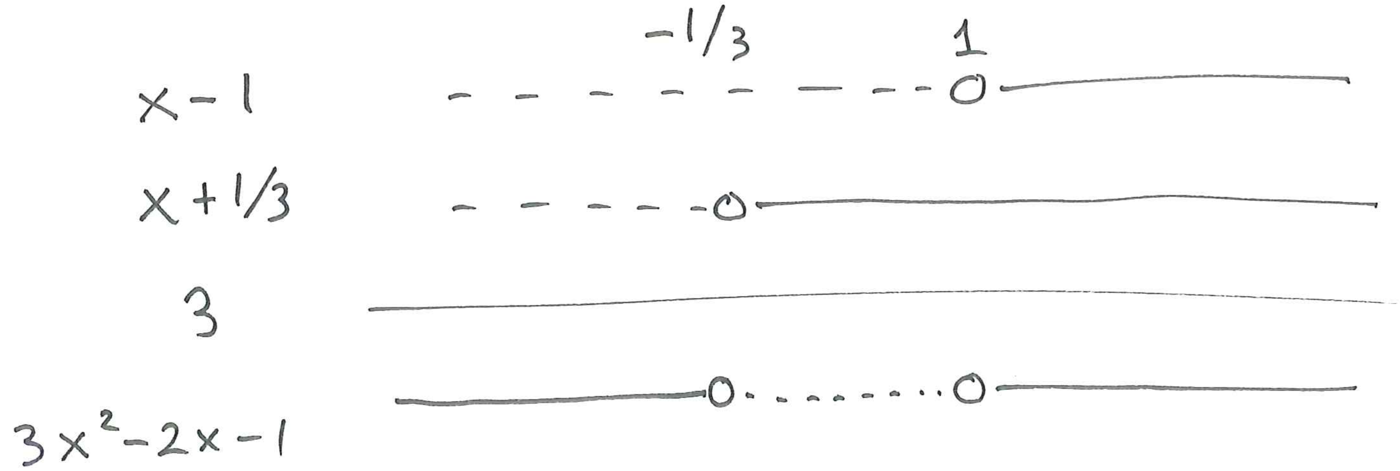
$$= \frac{2 \pm \sqrt{4 + 12}}{2 \cdot 3} = \frac{2 \pm \sqrt{16}}{2 \cdot 3}$$

$$x = \frac{2 \pm 4}{2 \cdot 3} = \frac{\cancel{2}}{\cancel{2}} \frac{1 \pm 2}{3}$$

$x = 1$ eller $x = -1/3$

$$3x^2 - 2x - 1 = 3(x - 1)(x + 1/3)$$

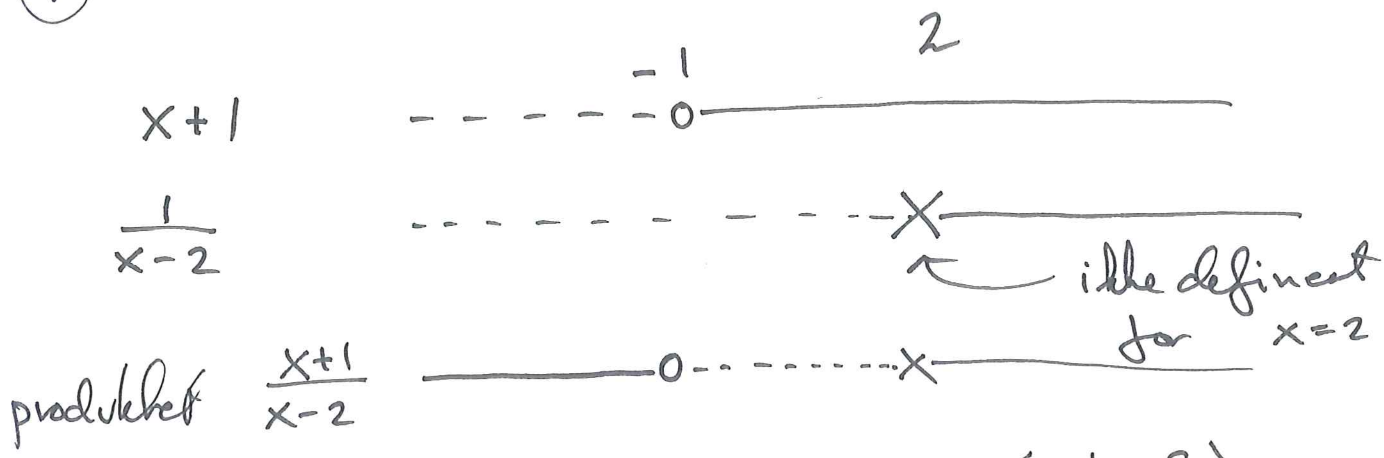
$$= (x - 1)(3x + 1)$$



Løsningene er $[-1/3, 1]$

Rasjonale ulikheter

(7) Løs ulikheten $\dfrac{x+1}{x-2} < 0$



Løsningene er $\langle -1, 2 \rangle$

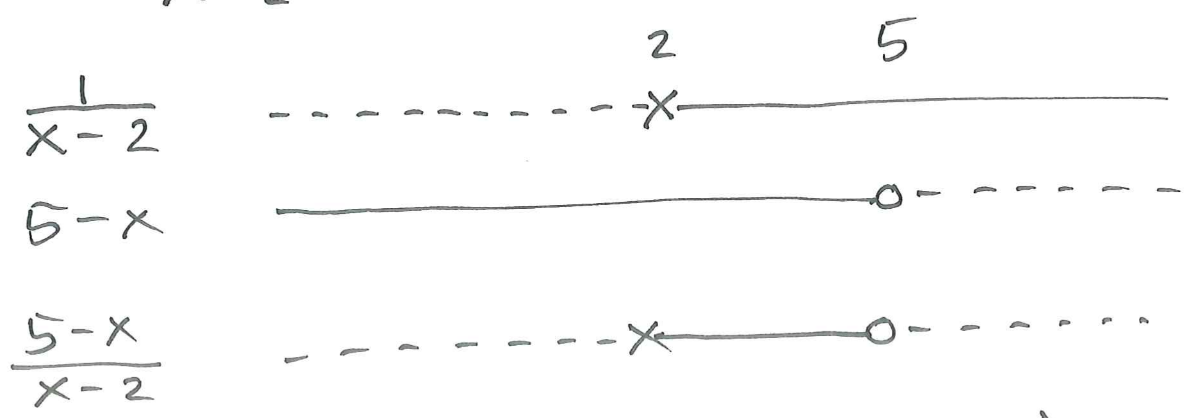
Løs ulikheten $\dfrac{x+1}{x-2} > 2 \iff \dfrac{x+1}{x-2} - 2 > 0$

felles nevner

$$\frac{x+1}{x-2} - 2\,\frac{x-2}{x-2} > 0$$

$$\frac{(x+1) - 2(x-2)}{x-2} > 0 \iff \frac{x+1-2x+4}{x-2} > 0$$

$$\frac{5-x}{x-2} > 0$$



Løsningene er $\langle 2, 5 \rangle$

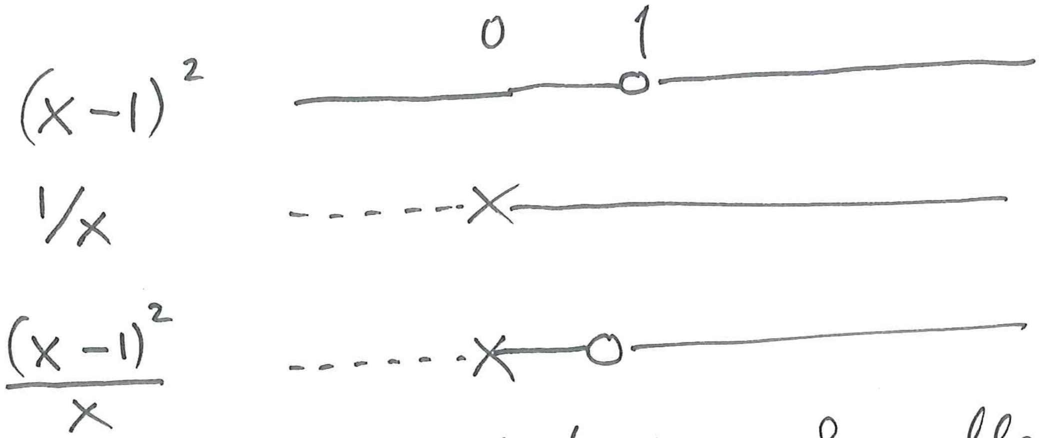
Vis at $$x + \frac{1}{x} \geq 2 \quad \text{for alle} \quad x > 0$$
(det er likhet bare for $x = 1$)

(8)

$$x + \frac{1}{x} \geq 2 \quad \Leftrightarrow \quad x + \frac{1}{x} - 2 \geq 0$$

$$\Leftrightarrow \quad \frac{x^2 + 1 - 2x}{x} \geq 0$$

$$\Leftrightarrow \quad \frac{(x-1)^2}{x} \geq 0$$



så $x + \frac{1}{x} \geq 2$ for alle $x > 0$

og $x + \frac{1}{x} = 2$ når $x = 1$

Løs ulikheten

9

$$\frac{(x-2)(x+1)}{x+3} < x+2$$

$$\Leftrightarrow \quad \frac{(x-2)(x+1)}{x+3} - (x+2) < 0$$

Felles nevner

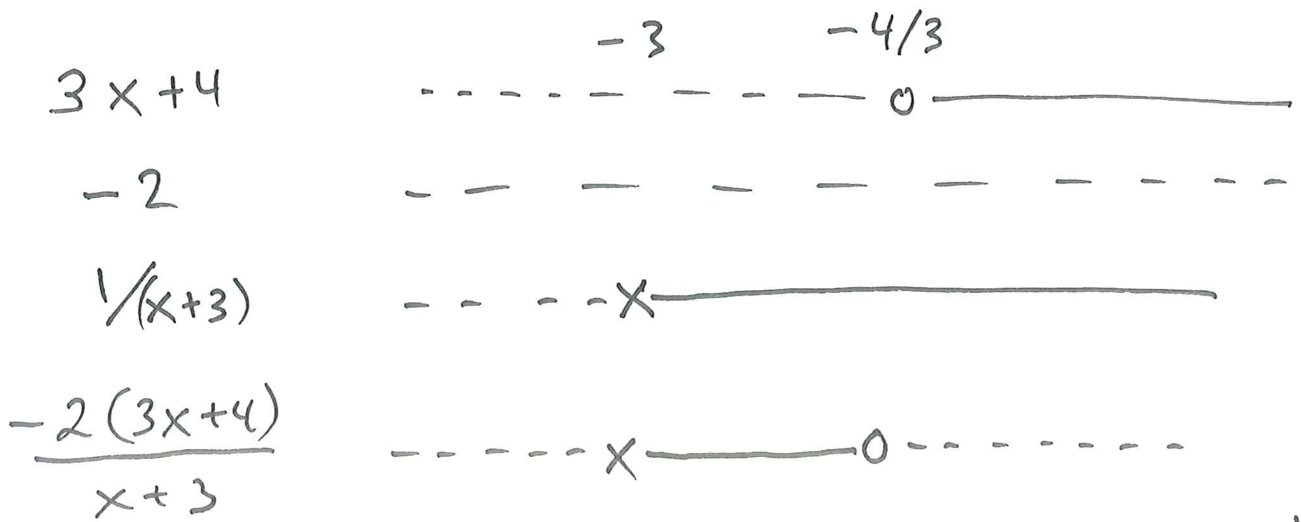
$$\frac{(x-2)(x+1)}{x+3} - \frac{(x+2)(x+3)}{x+3} < 0$$

$$\frac{x^2 - x - 2 - (x^2+5x+6)}{x+3} < 0$$

$$\frac{-x-5x-2-6}{x+3} < 0$$

$$\frac{-6x-8}{x+3} < 0$$

$$\frac{-2(3x+4)}{x+3} < 0$$



Løsningene er $\langle \infty, -3 \rangle \cup \langle -4/3, \infty \rangle$