Løsningsforslag innlevering 2

1

a) $f(x) = x^3 + 3x^2 - x - 3$ $P(1,0)$

$$\begin{aligned}
&x^3 + 3x^2 - x - 3 : x - 1 = \underline{x^2 + 4x + 3} \\
&-(x^3 - x^2) \\
&\quad 4x^2 - x \\
&\quad -(4x^2 - 4x) \\
&\qquad 3x - 3 \\
&\qquad 3x - 3 \\
&\qquad\quad 0
\end{aligned}$$

$$x^2 + 4x + 3 = 0$$

$$(x+1)(x+3) = 0$$

$$\underline{x = -1} \lor \underline{x = -3}$$

Grafen skjærer x-aksen i
$\underline{\underline{x = -3, \quad x = -1 \quad \text{og} \quad x = 1}}$

b) $\frac{x^3 + 3x^2 - x - 3}{x^2 + 2x - 3} = \frac{(x-1)(x+1)(x+3)}{(x-1)(x+3)} = \underline{\underline{x+1}}$

2

a) $(-4)^3 + 4 \cdot 4^2 - (-4) - 4 = -4^3 + 4^3 + 4 - 4 = \underline{\underline{0}}$

b) $P(2) = 0$

$$\begin{aligned}
2^3 - a \cdot 2^2 + 3 \cdot 2 + 2 &= 0 \\
-4a &= -16 \quad | : -4 \\
\underline{a} &\underline{= 4}
\end{aligned}$$

③

a) $\sqrt{x+2} = x$

$\Downarrow$

$x+2 = x^2$

$\Updownarrow$

$x^2 - x - 2 = 0$

$\Updownarrow$

$\underline{\underline{x = 2 \lor x = -1}}$

sætter prøve

$x = 2$ V.S : $\sqrt{x+2} = \sqrt{4} = 2$
H.S : $x = 2$
$\Big\}$ VS = HS

$x = -1$ V.S. : $\sqrt{-1+2} = \sqrt{1} = 1$
HS $x = -1$
$\Big\}$ VS ≠ HS

$\underline{\underline{x = 2}}$

b) $1 = \sqrt{2x+5} - x$

$\Updownarrow$

$\sqrt{2x+5} = 1 + x$

$\Downarrow$

$2x+5 = (1+x)^2$

$\Updownarrow$

$2x+5 = 1 + 2x + x^2$

$\Updownarrow$

$x^2 - 4 = 0$

$\Updownarrow$

$\underline{x = \pm 2}$

Prøve

$x = 2$ V.S 1
H.S : $\sqrt{2x+5} - x = \sqrt{4+5} - 2 = 3 - 2 = 1$
$\Big\}$ HS = VS

$x = -2$ VS 1
HS : $\sqrt{2(-2)+5} - (-2) = 1 + 2 = 3$
$\Big\}$ HS ≠ VS

$\underline{x = 2}$

4) a) $\lim_{x\to 2} \frac{x^2-4}{x-2} = \lim_{x\to 2} \frac{(x+2)(x-2)}{(x-2)} = \lim_{x\to 2} x+2 = \underline{\underline{4}}$

b) $\lim_{x\to 4} f(x) = 2$ og $\lim_{x\to 4} g(x) = -1$

$\lim_{x\to 4} \left(f(x)\, g(x) + \frac{f(x)}{g(x)} \right) = 2(-1) + \frac{2}{-1}$

$= -2 - 2 = \underline{\underline{-4}}$

5) a) $f(x) = x+5$, $D_f = \mathbb{R}$

$\lim_{x\to a} f(a) = a+5 = f(a)$ $\underline{\underline{\text{kontinuerlig}}}$

b) $g(x) = \begin{cases} x+5, & x \leq 1 \\ 3x+3, & x > 1 \end{cases}$

$\lim_{x\to 1^-} g(x) = \lim_{x\to 1^-} x+5 = \underline{6}$

$\lim_{x\to 1^+} g(x) = \lim_{x\to 1^+} 3x+3 = \underline{6}$

$g(1) = 6$

$\lim_{x\to 1^-} g(x) = \lim_{x\to 1^+} g(x) = g(1)$ $\underline{\underline{\text{kontinuerlig}}}$

c) $h(x) = \begin{cases} x+5 & , x < 1 \\ 0 & , x = 1 \\ 3x+3 & , x > 1 \end{cases}$

$\lim_{x \to 1^-} h(x) = \lim_{x \to 1^+} h(x) = 6$

$h(1) = 0$

ettersom grenseverdien ikke er lik funksjonsverdien er funksjonen ikke kontinuerlig

5

$f(x) = \frac{4x - 2}{x^2 - 16}$

V.A $x^2 - 16 = 0$

$x = \pm 4$

Når $x \to \pm 4$ vil $|f(x)| \to \infty$

Horisontal:

$f(x) = \frac{\frac{4x}{x^2} - \frac{2}{x^2}}{\frac{x^2}{x^2} - \frac{16}{x^2}} = \frac{\frac{4}{x} - \frac{2}{x^2}}{1 - \frac{16}{x^2}}$

Når $x \to \pm \infty$ vil $f(x) \to 0$

$y = 0$ er horisontal asymptote

og $x = \pm 4$ er vertikale asymptoter

b) $g(x) = \frac{x^2 + 3x + 1}{x+1}$

Vertikal $x + 1 = 0$

$x = -1$

Når $x \to -1$ vil $|f(x)| \to \infty$

$\underline{\underline{x = -1 \text{ er vertikal asymptote}}}$

$$x^2 + 3x + 1 : x+1 = x + 2 - \frac{1}{x+1}$$
$$\underline{-(x^2 + x)}$$
$$2x + 1$$
$$\underline{-(2x + 2)}$$
$$-1$$

Når $x \to \pm\infty$ vil $\frac{1}{x+1} \to 0$ og vi

har en skrå asymptote i $\underline{\underline{y = x+2}}$

7

a) $g(x) = x^3 - x + 4 \qquad a = 2$

$g'(x) = 3x^2 - 1$

$3x^2 - 1 = 2$

$3x^2 = 3$

$x^2 = 1$

$x = \pm 1$

$g(1) = 1^3 - 1 + 4 = 4$ $g(-1) = (-1)^3 - (-1) + 4 = 4$

$y - y_1 = a(x - x_1)$

$y - 4 = 2(x - 1)$ $y - 4 = 2(x - (-1))$

$y = 2x - 2 + 4$ $y = 2x + 2 + 4$

$\underline{\underline{y = 2x + 2}}$ $\underline{\underline{y = 2x + 6}}$

b) stigningstall til normal $= -\frac{1}{2}$

$P(1, 4)$

$y - 4 = -\frac{1}{2}(x - 1)$

$y = -\frac{1}{2}x + \frac{1}{2} + 4$

$\underline{\underline{y = -\frac{1}{2}x + \frac{9}{2}}}$

8 a) $h'(t) = -0,0025 \cdot 3t^2 + 0,075 \cdot 2t$

$= -0,0075t^2 + 0,15t$

$h''(t) = -0,0075 \cdot 2t + 0,15$

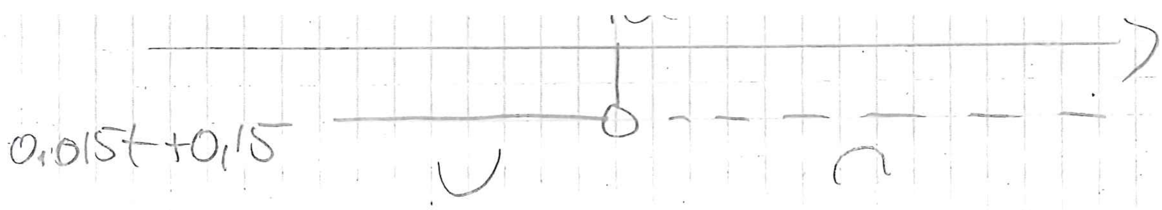
$= -0,015t + 0,15$

$h''(t) = 0$

$-0,015t + 0,15 = 0$

$-0,015t = -0,15$

$\underline{t = 10}$



funksjonen vokser raskest når $t = 10$ år

b) $h(10) = -0{,}0025 \cdot 10^3 + 0{,}075 \cdot 10^2 + 0{,}50$

$= -2{,}5 + 7{,}5 + 0{,}5$

$= \underline{5{,}5\ \text{m}}$

c) $h'(10) = -0{,}0075 \cdot 10^2 + 0{,}15 \cdot 10$

$= -0{,}75 + 1{,}5$

$= \underline{0{,}75}$

Det vokser 0,75 m/år

9