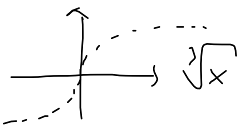
oblig 6



1.4 Finn største def. mengde og deriver $f(x) = \ln\left(5\sqrt[3]{x^3 - x^2 + 6x}\right)$

$\ln a$ er bare def. for $a > 0$

Så $f(x)$ er def. presis når $x^3 - x^2 + 6x > 0$

$$x\underbrace{(x^2 - x + 6)}_{\underbrace{(x-\frac{1}{2})^2 - (\frac{1}{2})^2 + 6}_{>0}} > 0 \iff x > 0$$

Definisjonsmengden til $f(x)$ er $\langle 0, \infty \rangle$ eller $x > 0$.

$$f(x) = \ln\left(5\sqrt[3]{x^3 - x^2 + 6x}\right)$$
$$= \ln(5) + \ln\left((x^3 - x^2 + 6x)^{1/3}\right)$$
$$= \underbrace{\ln(5)}_{\text{konstant}} + \frac{1}{3}\ln(x^3 - x^2 + 6x)$$

$$f'(x) = 0 + \frac{1}{3}\frac{(x^3 - x^2 + 6x)'}{x^3 - x^2 + 6x}$$

$$= \underline{\frac{3x^2 - 2x + 6}{3(x^3 - x^2 + 6x)}}$$

$$y' = -y^2$$

1. ordens
ikkje-lineær diff likning
separabel

$$\int \frac{y'}{y^2}\,dx = \int -1\,dx$$

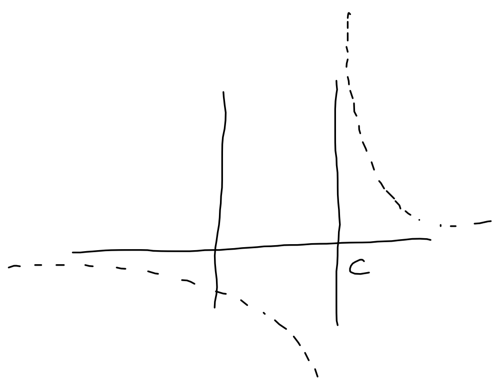
$$\int \frac{dy}{y^2} = -\int 1\,dx$$

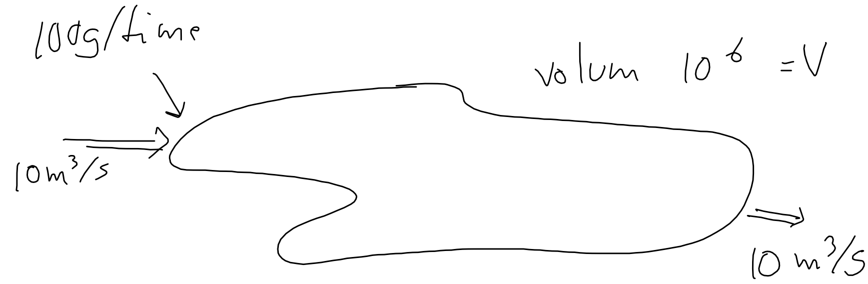
$$\int y^{-2}\,dy = -x + C$$

$$-y^{-1} = -x + C$$

$$\frac{1}{y} = x - C$$

$$y(x) = \frac{1}{x-C}$$





$Y(t)$ konsentrasjonen av giften i vannet (kg/m^3).

Massen av giften i vannet er
$$Y(t) \cdot V$$

Endringsraten
$$\underbrace{\Delta Y(t)}_{Y(t+\Delta t) - Y(t)} \cdot V = (0.1\ kg/t) \cdot \Delta t - \underbrace{\left[(10 m^3/s)(3600\ s/t) \cdot \Delta t\right]}_{\text{volum som renner ut}} \cdot Y$$

$$\lim_{\Delta t \to 0} \frac{\Delta Y}{\Delta t} = Y' = \frac{0.1\ kg/timer}{V} - \frac{36\,000\ m^3/timer}{V} \cdot Y$$

$$Y' = \frac{1}{V}\left(0.1\ kg/timer - 36 \cdot 10^3\ m^3/timer \cdot Y\right)$$

Hvilke konsentrasjon vil Y nærme seg når $t \to \infty$. Det svarer til $Y' = 0$

$$0.1\ kg/timer - 36 \cdot 10^3 \frac{m^3}{timer} \cdot Y = 0$$

$$Y_{stabil} = \frac{0.1\ kg/time}{36 \cdot 10^3\ m^3/timer} = \frac{1}{36 \cdot 10^4}\ kg/m^3 = \underline{\frac{1}{360}\ \frac{gram}{m^3}}$$

$$\int \frac{dY}{0.1 - 36 \cdot 10^3 Y} = \int \frac{1}{V}\, dt$$

$$\frac{-1}{36 \cdot 10^3} \ln|0.1 - 36 \cdot 10^3 Y| = \frac{t}{V} + C$$

$$0.1 - 36 \cdot 10^3 Y = k e^{-\frac{36 \cdot 10^3}{V} t}$$

$Y(0) = 0$ giv $k = 0.1\ kg/timer$.

$$\underline{Y(t) = \frac{0.1}{36 \cdot 10^3}\left(1 - e^{-\frac{36}{10^3} \cdot t}\right)}$$

$$\int \tan^2(3x)\,dx$$

$$= \int \frac{\sin^2(3x)}{\cos^2(3x)}\,dx$$

$$\left(\text{hint: } (\tan x)' = \left(\frac{\sin x}{\cos x}\right)' = \frac{1}{\cos^2 x} = 1 + \tan^2 x\right)$$

$$(\tan 3x)' = 3\left(1 + \tan^2(3x)\right)$$

$$\tan^2(3x) = \frac{1}{3}(\tan 3x)' - 1$$

så $\int \tan^2(3x)\,dx = \int \frac{1}{3}(\tan(3x))' - 1\,dx$

$$= \underline{\frac{1}{3}\tan(3x) - x + C}$$

$$\int \frac{x^2}{x^2-1}\,dx$$

- polynomdivisjon
- Delbrøksoppspalting.

$$\frac{x^2}{x^2-1} = \frac{x^2-1+1}{x^2-1} = 1 + \frac{1}{x^2-1}$$

$$= 1 + \frac{1}{(x+1)(x-1)}$$

$$= 1 + \left(\frac{-1/2}{x+1} + \frac{1/2}{x-1}\right)$$

$$\int \frac{x^2}{x^2-1}\,dx = \int 1 + \frac{1}{2}\left(\frac{1}{x-1} - \frac{1}{x+1}\right)dx$$

$$= x + \frac{1}{2}\left(\ln|x-1| - \ln|x+1|\right) + C$$

$$= \underline{x + \frac{1}{2}\ln\left|\frac{x-1}{x+1}\right| + C}$$

3e

$$2\log_2(x+1) = \log_2(x) + 2$$

$$2\log_2(x+1) - \log_2 x = 2$$

$$\log_2 (x+1)^2 + \log_2(x^{-1}) = \log_2 2^2$$

$$\log_2 \frac{(x+1)^2}{x} = \log_2 4$$

$$\Leftrightarrow \quad \frac{(x+1)^2}{x} = 4$$

$$(x+1)^2 = 4 \cdot x$$

$$x^2 + 2x + 1 = 4x$$

$$x^2 - 2x + 1 = 0$$

$$(x-1)^2 = 0$$

$$x = 1 \quad \text{eneste løsning}$$