Mer delbrøksoppspalting

$$\frac{1}{x^2+x-6} = \frac{1}{(x+3)(x-2)}$$
$$= \frac{A}{x+3} + \frac{B}{x-2}$$

Løser for A og B (ved å finne felles nevner)

$$\frac{1}{(x+3)(x-2)} = \frac{A(x-2)}{(x+3)(x-2)} + \frac{B(x+3)}{(x+3)(x-2)}$$

Så $1 = A(x-2) + B(x+3)$ (for $x \neq 2, -3$ og derfor like for alle x)

$$0 \cdot x + 1 = \underbrace{(A+B)}_{0} x + \underbrace{(-2A+3B)}_{1}$$

$A = -B$

setter inn: $-2A + 3B = 1$ gir $-2(-B) + 3B = 5B = 1$

Så $\underline{B = \frac{1}{5}}$ og $\underline{A = \frac{-1}{5}}$

$$\int \frac{1}{x^2+x-6}\,dx = \int \frac{1}{5}\left[\frac{-1}{x+3} + \frac{1}{x-2}\right] dx$$
$$= \frac{1}{5}\left[-1 \cdot \ln|x+3| + \ln|x-2|\right] + C$$
$$= \underline{\frac{1}{5} \ln\left|\frac{x-2}{x+3}\right| + C}$$

Oppg.

$$\int \frac{2x+1}{x^2+x-6}\,dx = \int \frac{2x+1}{(x+3)(x-2)}\,dx$$

Delbrøksoppspalting:

$$\frac{2x+1}{x^2+x-6} = \frac{A}{x+3} + \frac{B}{x-2}$$
$$= \frac{A(x-2) + B(x+3)}{(x+3)(x-2)}$$

Så $2x+1 = A(x-2) + B(x+3)$

La $x = 2$: $2 \cdot 2 + 1 = 5 = A \cdot 0 + 5 \cdot B$, $\underline{B = 1}$

$x = -3$: $2(-3) + 1 = -5 = A(-5) + 0 \cdot 5$, $A = 1$

$$\int \frac{2x+1}{x^2+x-6}\,dx = \int \frac{1}{x+3} + \frac{1}{x-2}\,dx$$
$$= \ln|x+3| + \ln|x-2| + C$$
$$= \underline{\ln|(x+3)(x-2)| + C}$$

Dette integralet kunne enklere blitt løst ved å observere at

$$(2x+1) = (\underbrace{x^2+x-6}_{u})'$$

substitusjon

$$\int \frac{2x+1}{x^2+x-6}\,dx = \int \frac{u'}{u}\,dx$$

$$= \int \frac{du}{u} = \ln|u| + C$$

$$= \underline{\ln|x^2+x-6| + C}$$

oppgave

$$\int \frac{1}{9x^2 - 4}\,dx$$

$$\frac{1}{9x^2-4} = \frac{1}{(3x-2)(3x+2)}$$

$$= \frac{A}{3x-2} + \frac{B}{3x+2}$$

$$= \frac{(3x+2)A + (3x-2)B}{(3x-2)(3x+2)}$$

$$1 = (3x+2)A + (3x-2)B = 3x(A+B) + 2(A-B)$$

$A+B=0$ og $2(A-B)=1$

Så $A = \frac{1}{4}$ og $B = \frac{-1}{4}$.

$$\int \frac{1}{9x^2-4}\,dx = \int \frac{1}{4}\left(\frac{-1}{3x+2} + \frac{1}{3x-2} \right)$$

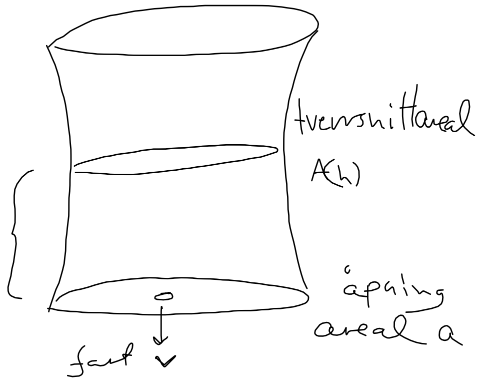
Linear substitusjon : $\frac{1}{4}\left(\frac{-1}{3}\ln|3x+2| + \frac{1}{3}\ln|3x-2| \right) + C$

$$= \frac{1}{12} \ln\left| \frac{3x-2}{3x+2} \right| + C$$

Torricellis lov

$a << A(h)$
mye mindre

ser bort fra friksjon



Ved tiden $t=0$ er høyden $H = h(0)$.
Vi ønsker å beskrive h som en funksjon av tiden.

I en liten tidsintervall Δt renner det veske med masse Δm ut av beholderen.

Tap i potensiell energi: $\Delta m \cdot h(t) \cdot g$
er lik kinetisk energi til vesken som strømmer ut: $\frac{\Delta m}{2} v^2$

Energibevaring: $\frac{\Delta m}{2} v^2 = \Delta m \, h(t) \cdot g$

$$v^2 = 2g\,h(t)$$
$$|v| = \sqrt{2g\,h(t)}$$

Volum av vesken som renner ut i en tidsintervall Δt er: $a(\cdot v \cdot \Delta t)$

Endring i høyde Δh gir en endring i volum:

$$\Delta h \cdot A(h)$$

så $$\Delta h\, A(h) = a \cdot v \cdot \Delta t$$

$$\lim_{\Delta t \to 0} \frac{\Delta h}{\Delta t} \cdot \frac{A(h)}{a} = v$$

↑ positiv retning

$$\frac{dh}{dt} \cdot \frac{A(h)}{a} = v$$

$$h' \cdot \frac{A(h)}{a} = v = -\sqrt{2g\,h(t)}$$

$$\boxed{h'\,A(h) = -a\sqrt{2g\,h(t)}}$$ Torricellis lov

$$h' A(h) = -a\sqrt{2g\,h}\,.$$

Vi studerer tilfellet med en rett sylinder
$A(h) = A$ konstant.

$$h' = \underbrace{\left(\frac{-a}{A}\sqrt{2g}\right)}_{K}\sqrt{h}$$

$$h' = K \cdot h^{1/2}$$ Separabel diff likning

$$\frac{h'}{h^{1/2}} = h^{-1/2} \cdot h' = K$$

$$\int \frac{1}{h^{1/2}} \underbrace{h'\,dt}_{dh \text{ substitusjon}} = \int K \cdot dt$$

$$\int h^{-1/2}\,dh = \int K\,dt$$

$$\frac{h^{1/2}}{1/2} = K \cdot t + C$$

$$\sqrt{h} = \frac{1}{2} K \cdot t + C$$

$$= \frac{-a}{2A}\sqrt{2g} \cdot t + C$$

$$= \frac{-a}{A}\sqrt{\frac{2g}{2^2}}\, t + C$$

$$\underline{\sqrt{h} = \frac{-a}{A}\sqrt{\frac{g}{2}}\, t + C}$$

$h(0) = H$ setter inn : $\sqrt{h(0)} = \underline{\sqrt{H}} = 0 + C = \underline{C}$

$$\underline{h(t) = \left(\sqrt{H} - \frac{a}{A}\sqrt{\frac{g}{2}} \cdot t\right)^2}$$

Tiden det tar å tømme tanken T

$h(T) = 0$: $\sqrt{H} = \frac{a}{A}\sqrt{\frac{g}{2}} \cdot T$

$$\underline{T = \left(\frac{A}{a}\sqrt{\frac{2}{g}}\right) \cdot \sqrt{H}}$$