Gitt vektorer $\vec{a}, \vec{b}$ $\vec{x}$, $\vec{y}$.

slik at

$$2\vec{a} + 3\vec{b} = \vec{x}$$

$$\vec{a} - \vec{b} = \vec{y}$$

Uttrykk $\vec{a}$ og $\vec{b}$ ved hjelp av $\vec{x}$ og $\vec{y}$

() $\vec{a} = \vec{y} + \vec{b}$ ($\vec{b}$ over på høyre side)

setter inn for $\vec{a}$ i første likning

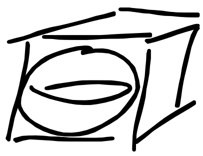
$$2(\vec{y} + \vec{b}) + 3\vec{b} = \vec{x}$$

$$2\vec{y} + 2\vec{b} + 3\vec{b} = \vec{x}$$

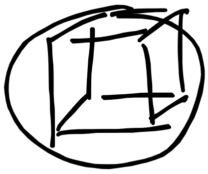
$$5\vec{b} = \vec{x} - 2\vec{y}$$

Så $\underline{\vec{b} = \frac{1}{5}(\vec{x} - 2\vec{y})}$

$$\vec{a} = \vec{y} + \vec{b} = \vec{y} + \frac{1}{5}(\vec{x} - 2\vec{y}) = \underline{\frac{1}{5}(\vec{x} + 3\vec{y})}$$



$$\frac{V_{kube}}{V_{kule}} = \frac{6}{\pi} \sim 1.91$$

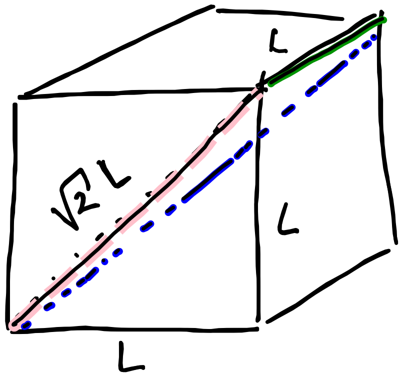


$$\frac{V_{kule}}{V_{kube}}$$ ← størst mulig kube innholdt i kulen

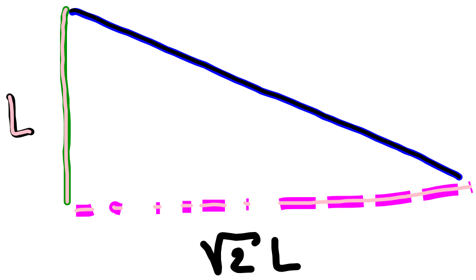
Hvilke forhold er størst?

Lengden på sidene i kuben L. $V_{kube} = L^3$.

$V_{kulen} = \frac{4\pi}{3} R^3$.



diagonalen i kuben



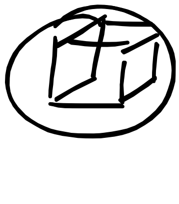
$$\text{Pyt: } diag^2 = L^2 + (\sqrt{2}L)^2 = L^2 + 2L^2 = 3L^2.$$

diagonalen i kulen har lengde $\sqrt{3}\,L$

$$R = \frac{\sqrt{3}}{2} L$$

$$V_{kule} = \frac{4}{3}\pi\left(\frac{\sqrt{3}}{2}L\right)^3 = \frac{4}{3}\pi \frac{3\cdot\sqrt{3}}{2\cdot 4} L^3 = \frac{\sqrt{3}\pi}{2} L^3$$

$$\frac{V_{kule}}{V_{kube}} = \underline{\frac{\sqrt{3}\pi}{2}} \sim 2.72$$



Løs likningene

1) $3\ln x = 7$

$\ln x = 7/3 \sim 2.33\ldots$

$x = e^{\ln x} = \underline{e^{7/3}} \sim 10.3$

2) $5^x = 10$

$x\ln 5 = \ln 5^x = \ln 10$

$x = \frac{\ln 10}{\ln 5} \sim \underline{1.43}$

3) $2^x = 8\cdot 5^x$ (husk $\ln(a\cdot b) = \ln a + \ln b$)

Anvender ln:

$\ln 2^x = \ln(8\cdot 5^x) = \ln 8 + \ln 5^x$

$x\ln 2 = \ln 8 + x\ln 5$

$x(\ln 2 - \ln 5) = \ln 8$

$x = \frac{\ln 8}{\ln 2 - \ln 5} \sim -\underline{2.26}$

Alternativt: $\frac{2^x}{5^x} = \left(\frac{2}{5}\right)^x = 8 \ldots$

4) $2\ln|x+1| - \ln(x^2-1) = -1$

($r\ln u = \ln u^r$, $-\ln u = \ln u^{-1} = \ln(\frac{1}{u}) \ldots$)

$\ln|x+1|^2 + \ln\left(\frac{1}{x^2-1}\right) = -1$

$\ln\left(\frac{(x+1)^2}{x^2-1}\right) = -1 = \ln(e^{-1})$

$\frac{(x+1)^2}{x^2-1} = \frac{1}{e}$

$\frac{(x+1)^2}{(x+1)(x-1)} = \frac{1}{e}$

$\frac{x+1}{x-1} = \frac{1}{e} \quad | \, e \text{ og } x-1$

$e(x+1) = x-1, \quad e\cdot x - e = x - 1$

$ex - x = (e-1)x = -1 - e = -(e+1)$

$x = \frac{-(e+1)}{e-1} \sim \underline{-2.18}$

Løs likningen

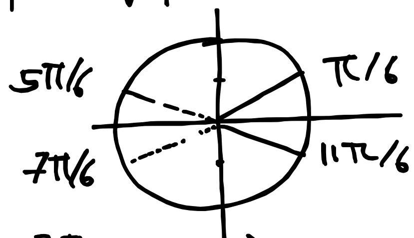
$\sin^2 x = \frac{1}{4}$ eksakt for $x \in [0, 2\pi]$.

$(\sin x)^2 = \frac{1}{4}$.

$\sqrt{u^2} = |u|$ $\qquad |\sin x| = \sqrt{\frac{1}{4}} = \sqrt{\frac{1}{2^2}} = \frac{1}{2}$

$\sin x = \frac{1}{2}$

eller $\sin x = \frac{-1}{2}$.



Løsningene er $x = \left\{ \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6} \right\}$

Anta x ligger i første kvadrant og at $\sin x = 1/3$

Regn ut $\sin(2x)$ og $\cos(2x)$ eksakt!

($\sin 2x = 2\sin x \cdot \cos x$

$\cos 2x = \cos^2 x - \sin^2 x$

Pytagoras : $\cos^2 x + \sin^2 x = 1$

så $\cos 2x = 1 - 2\sin^2 x$)

$\cos(2x) = 1 - 2 \cdot \left(\frac{1}{3}\right)^2 = 7/9 \sim 0.77\ldots$

$\cos^2 x = 1 - \sin^2 x = 1 - \frac{1}{9} = 8/9$

1. kvadrant, så $\cos x > 0$: $\cos x = \sqrt{\frac{8}{9}} = \frac{2\sqrt{2}}{3}$

$\sin(2x) = 2\sin x \cdot \cos x = 2 \cdot \frac{1}{3} \cdot \frac{2\sqrt{2}}{3} = \frac{4\sqrt{2}}{9}$

~ 0.628.

Deriver $f = (1-x)^2(1+x)^2$.

(Produktregelen $(f \cdot g)' = f' \cdot g + f \cdot g'$

Kjerneregelen $\frac{d}{dx} f(u(x)) = \frac{df}{du} \cdot \frac{du}{dx}$)

$$f' = \underbrace{\left((1-x)^2\right)'}_{2(1-x)(-1)} \cdot (1+x)^2 + (1-x)^2 \underbrace{\left((1+x)^2\right)'}_{2(1+x)\cdot 1}$$

$$= -2(1-x)(1+x)^2 + (1-x)^2 \cdot 2(1+x)$$

$$= (1-x)(1+x)[\underbrace{-2(1+x)}_{-2-2x} + \underbrace{2(1-x)}_{+2-2x}]$$

$$= -4x(1-x)(1+x) = -4x(1-x^2)$$

Alternativt: $f(x) = ((1-x)(1+x))^2 = (1-x^2)^2$

Bruk av kjerneregelen med kjerne $1-x^2$

gir $2(1-x^2)(1-x^2)' = -4x(1-x^2)$