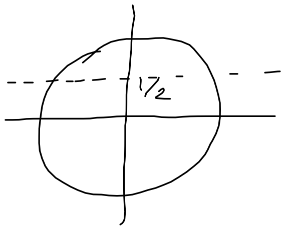
11.7 Trigonometriske ulikheter

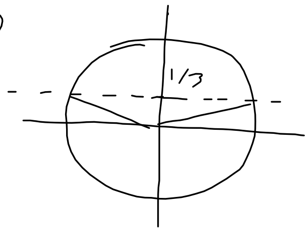
$\sin x \geq \frac{1}{2}$ $\quad -\pi \leq x < \pi$



$\sin x = \frac{1}{2}$ for $x = \frac{\pi}{6}, \frac{5\pi}{6}$

Løsningen til likningen er $\underline{\frac{\pi}{6} \leq x \leq \frac{5\pi}{6}}$

$\sin x < 1/3$ $\quad x \in [-\pi, \pi\rangle$



$\sin x = \frac{1}{3}$: $x = \arcsin\left(\frac{1}{3}\right) \sim 0.339\,\text{rad}$
$19.5°$

$x = \pi - 0.339$
$\sim 2.80\,\text{rad}$

Løsningen til ulikheten er $\underline{[-\pi, 0.339\rangle \cup \langle 2.80, \pi\rangle}$

$2\sin x \cos x \geq \cos x$ $\quad x \in [-\pi, \pi\rangle$

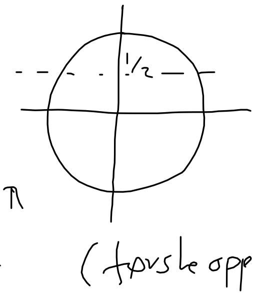
(merk at $2\sin x \cdot \cos x = \sin 2x$, så ulikheten er lik $\sin 2x \geq \cos x$.)

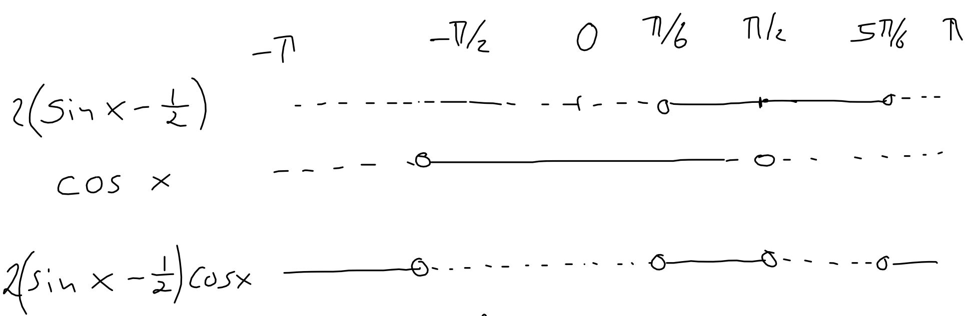
Flytter alle uttrykk over på en side

$2\sin x \cos x - \cos x \geq 0$

$(2\sin x - 1)\cos x \geq 0$

setter opp fortegnsskjema.



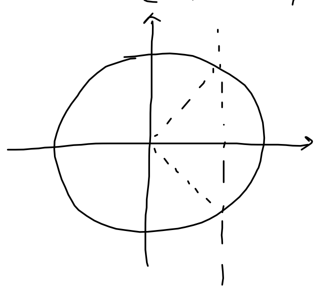


(første oppg.

Løsningen til ulikheten er

$\underline{[-\pi, -\frac{\pi}{2}] \cup [\frac{\pi}{6}, \frac{\pi}{2}] \cup [\frac{5\pi}{6}, \pi\rangle}$

oppg $\cos x < \frac{1}{\sqrt{2}}$ $x \in [0, 2\pi\rangle$

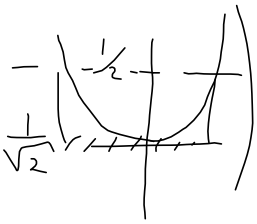


$\cos x = \frac{1}{\sqrt{2}}$ har

$x = \frac{\pi}{4}$ og $\frac{7\pi}{4}$

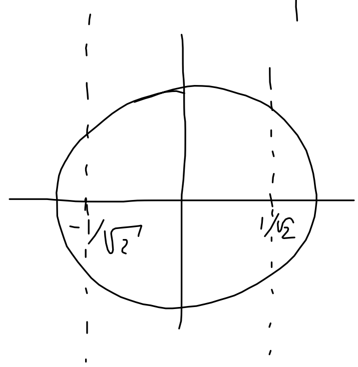
Løsningen til ulikheten er $\underline{\langle \frac{\pi}{4}, \frac{7\pi}{4} \rangle}$

oppg. $\cos^2 x < \frac{1}{2}$ $x \in [0, 2\pi\rangle$



(vi har at $u^2 < \frac{1}{2} \iff \frac{-1}{\sqrt{2}} < u < \frac{1}{\sqrt{2}}$

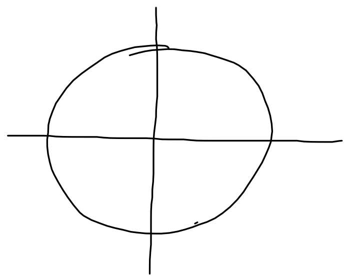
Ulikheten er ekvivalent til
$\frac{-1}{\sqrt{2}} < \cos x < \frac{1}{\sqrt{2}}$



Løsningene til ulikheten er
$\underline{\langle \frac{\pi}{4}, \frac{3\pi}{4} \rangle \cup \langle \frac{5\pi}{4}, \frac{7\pi}{4} \rangle}$

Løs ulikheten $\sin x < \sqrt{3}\cos x \quad , x \in [-\pi, \pi\rangle$

$\cos x > 0$ deler med $\cos x$



1) $\tan x < \sqrt{3}$

2) $\cos x < 0$

$\tan x > \sqrt{3}$

3) $\cos x = 0$: $\sin x < 0$: $x = -\pi/2$.

1) $-\frac{\pi}{2} < x < \frac{\pi}{2}$ og $\tan x < \sqrt{3}$ $\quad \underline{-\frac{\pi}{2} < x < \frac{\pi}{3}}$

2) $\pi > x > \frac{\pi}{2}$ og $-\pi < x < -\frac{\pi}{2}$ $(\cos x < 0)$

$\tan x > \sqrt{3}$ $\quad -\frac{2\pi}{3} < x < -\frac{\pi}{2}$

Løsningsmengden er $-\frac{2\pi}{3} < x < \frac{\pi}{3}$

11.9 Funksjonsdrøfting

La $f(x) = \sin x + \frac{1}{2}\sin(2x)$, $x \in [0, 2\pi]$

Bestem topp bunnpunkt.
vendepunkt.

Deriverer $f(x)$: $f'(x) = \cos(x) + \frac{1}{2}\left(\sin(\overset{u}{2x})\right)'$

$= \cos x + \frac{1}{2}\cdot\cos(2x)\cdot 2$

$= \cos x + \cos(2x)$

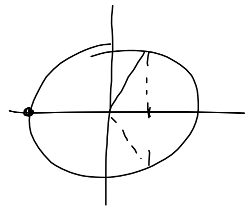
Benytter formel for dobling av vinkel

$f'(x) = \cos(x) + \cos^2 x - \underbrace{\sin^2 x}_{1-\cos^2 x}$ Pytagoras

$= 2\cos^2 x + \cos x - 1$

2. grads uttrykk i $\cos x$.

$= (2\cos x - 1)(\cos x + 1)$



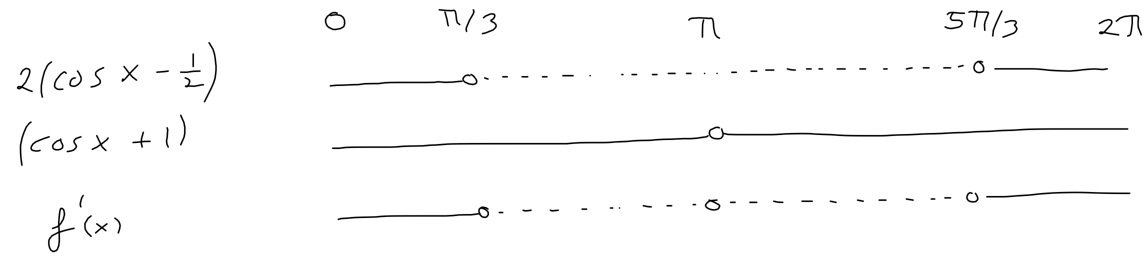
Ekstremalverdier :

$f'(x) = 0 \iff 2\cos x - 1 = 0$ eller $\cos x + 1 = 0$

$\cos x = \frac{1}{2}$ $\qquad$ $\cos x = -1$

$x \in \frac{\pi}{3}, \frac{5\pi}{3}$ $\qquad$ $x = \pi$

Fortegnsskjema for $f'(x)$ $\qquad (f'(x) > 0)$



Bunnpunkt i $(0, f(0)) = (0, 0)$

toppunkt i $\left(\frac{\pi}{3}, f\left(\frac{\pi}{3}\right)\right) = \left(\frac{\pi}{3}, \sin\left(\frac{\pi}{3}\right) + \frac{1}{2}\sin\left(\frac{2\pi}{3}\right)\right)$

$= \left(\frac{\pi}{3}, \frac{3\pi}{4}\right)$

Bunnpunkt $\left(\frac{5\pi}{3}, f\left(\frac{5\pi}{3}\right)\right) = \left(\frac{5\pi}{3}, -\frac{3\pi}{4}\right)$

toppunkt $(2\pi, f(2\pi)) = (2\pi, 0)$

Vi har et terrassepunkt i $(\pi, 0)$.

Vendepunkt.

$$f''(x) = (f')' = (\cos x + \cos(2x))'$$

$$= -\sin x + -2\sin(2x)$$

$$= -(\sin x + 2\underbrace{\sin(2x)}_{2\sin x \cdot \cos x})$$

$$= -\sin x(1 + 4\cos x)$$

$$f''(x) = 0 \iff \sin x = 0 \quad \text{eller} \quad 1 + 4\cos x = 0$$

$$\cos x = \frac{-1}{4}$$

etc...