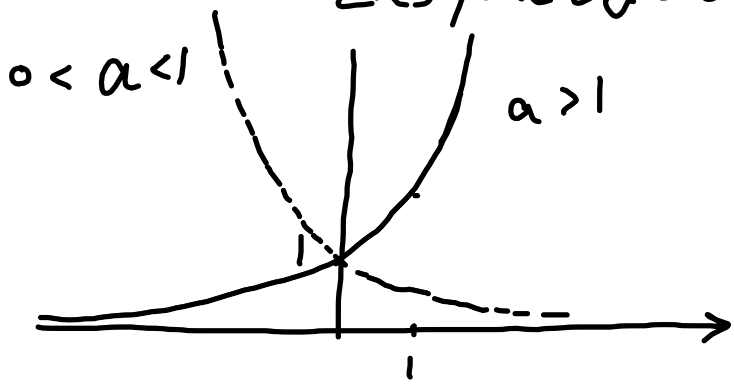
Eksponentfunksjon a^x



$a>0 \quad a \neq 1$

$$a^{-x} = (a^{-1})^x = \left(\frac{1}{a}\right)^x$$

Grafene til a^x og $\left(\frac{1}{a}\right)^x$ spegles om y-aksen.

$a>0$ så er

$$a = 10^{\text{Log}\, a}$$

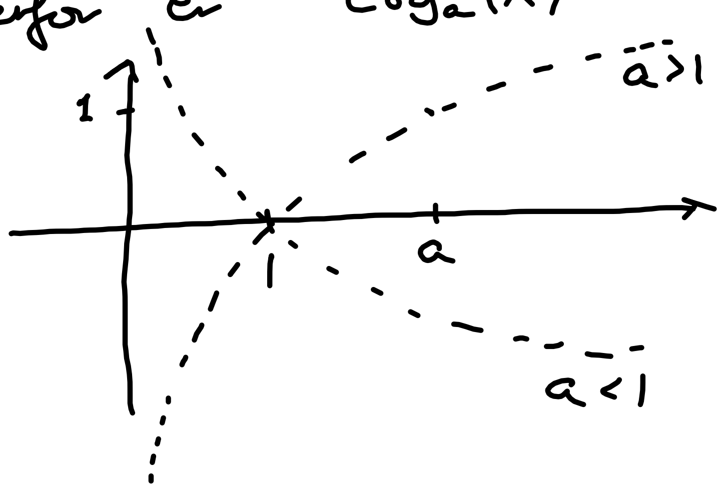
så $a^x = \left(10^{\text{Log}\, a}\right)^x = 10^{\text{Log}(a)\cdot x}$

Inversfunksjonen til a^x skrives $\text{Log}_a(x)$, logaritme med basis a.

$$\text{Log}_a\, a^r = r = \text{Log}(a^r)/\text{Log}\, a$$

$$\left(\begin{aligned} a^r &= \left(10^{\text{Log}\, a}\right)^r = 10^{\text{Log}\, a \cdot r} \\ \text{Log}\, a^r &= \text{Log}\, 10^{\text{Log}\, a \cdot r} = \text{Log}\, a \cdot r \\ \text{så} \quad r &= \frac{\text{Log}\, a^r}{\text{Log}\, a} \end{aligned} \right)$$

Derfor er $\text{Log}_a(x) = \dfrac{\text{Log}(x)}{\text{Log}(a)}$



Deriverte til eksponentfunksjoner

a^x $$\frac{d}{dx} a^x = \lim_{h \to 0} \frac{a^{x+h} - a^x}{h}$$

$$= \lim_{h \to 0} a^x \cdot \frac{a^h - 1}{h}$$ (benytter $a^{x+h} = a^x \cdot a^h$)

$$= a^x \lim_{h \to 0} \frac{a^h - 1}{h}$$

Benyttet geogebra: Grensen nærmer seg 1

når $a \sim 2.71828 \ldots$

Tallet kalles Euler tallet og skrives

e. $$e = \lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n$$

$\Bigg(\lim_{h \to 0} \frac{a^h - 1}{h} = 1 \iff \lim_{h \to 0} \frac{a^h - (1+h)}{h}$

$h = \frac{1}{n}$ $\begin{pmatrix} h \to 0 \\ n \to \infty \end{pmatrix}$ $\lim_{n \to \infty} \left[a^{1/n} - \left(1 + \frac{1}{n}\right)\right] \cdot n = 0$

$\ldots$ så $a = \lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n \Bigg)$

$$\boxed{\frac{d}{dx} e^x = e^x}$$

Logaritme med basis e kalles naturlig logaritme og skrives $\ln(x)$

$$\ln \sqrt{e} - \ln \sqrt[3]{e}$$
$$= \ln e^{1/2} - \ln e^{1/3}$$
$$\frac{1}{2} - \frac{1}{3} = \underline{\frac{1}{6}}$$

Logaritmereglene

$$\ln(a \cdot b) = \ln(a) + \ln(b)$$
$$\ln(a^r) = r \ln(a)$$

$e^x = \exp(x)$ og $\ln(x)$ er inversfunksjoner.
alternativ notasjon

$$e^{\ln x} = x \qquad x > 0$$
$$\ln(e^x) = x \qquad \text{for alle } x.$$

Eksempler

$$\frac{d}{dx}(2e^x + 3) = 2 \cdot \underbrace{\frac{d}{dx} e^x}_{e^x} + \underbrace{\frac{d}{dx}(3)}_{0}$$
$$= \underline{2e^x}$$

$$\frac{d}{dx} e^{-3x+4}$$ | sammensatt funksjon $e^{u(x)}$ hvor $u(x) = -3x+4$

kjerneregelen gir

$$= \frac{d}{du} e^u \cdot \frac{du}{dx}$$
$$= e^u \cdot (-3x+4)' = \underline{-3e^{-3x+4}}$$

Generelt: $$\frac{d}{dx} e^{u(x)} = e^{u(x)} \cdot u'(x)$$

Hvis $u(x) = ax+b$, $u'(x) = a$, så er $\frac{d}{dx} e^{ax+b} = a e^{ax+b}$

$$10^x = (e^{\ln 10})^x = e^{x \cdot \ln 10}$$

Så $\frac{d}{dx} 10^x = \ln 10 \cdot e^{x \cdot \ln 10}$

$$= \underline{\ln 10 \cdot 10^x}$$

$$\sim 2.3025\ldots \cdot 10^x$$

$$(2^x)' = ((e^{\ln 2})^x)' = (e^{x \cdot \ln 2})'$$

$$= \ln 2 \cdot 2^x \sim 0.6931\ldots \cdot 2^x$$

$$\frac{d}{dx}(a^x) = \underline{\ln a \cdot a^x}$$

$$(2^x)'' = ((2^x)')' = (\ln 2 \cdot 2^x)'$$

$$= \ln 2 \cdot (2^x)' = \ln 2 \cdot (\ln 2 \cdot 2^x)$$

$$= (\ln 2)^2 \, 2^x$$

$$\frac{d}{dx} e^{x^2} = \frac{de^u}{du} \cdot \frac{du}{dx}$$

$e^{x^2} = e^u$ hvor $u = x^2$

$$= e^u \cdot 2x$$

$$= \underline{2x e^{x^2}}$$

$$\frac{d}{dx}(x\cdot e^x) = x\cdot\underbrace{(e^x)'}_{e^x} + \underbrace{(x)'}_{1}\cdot e^x$$

$$= x\cdot e^x + e^x$$

$$= \underline{(x+1)\,e^x}$$

$$\boxed{\frac{d}{dx}\ln x = \frac{1}{x}}$$

$$e^{\ln x} = x \qquad x > 0$$

Deriverer $\quad \frac{d}{dx} e^{\ln x} = \frac{d}{dx} x = 1$

$\frac{d}{du} e^u \cdot \frac{du}{dx}$ hvor $u = \ln x$

$$e^{\ln x}\cdot\left(\frac{d}{dx}\ln x\right) = 1$$

$x\cdot \frac{d}{dx}\ln x = 1$ deler med x

Så $\frac{d}{dx}\ln x = \frac{1}{x}$.

sammensatt funksjon

Eksempler

$\frac{d}{dx}\left(\ln(3x) + \ln 5\right) = \frac{d}{dx}\ln(3x)$ $\qquad u = 3x$, $u' = 3$

$$\frac{d}{du}\ln u \cdot \frac{du}{dx} = \frac{1}{u}\cdot 3$$

$$= \frac{3}{3x} = \underline{\underline{\frac{1}{x}}}$$

Alternativt: $\ln(3x) = \ln 3 + \ln x$

↑ konstant

$$\frac{d}{dx}\ln(3x) = \frac{d}{dx}\ln x = \frac{1}{x} .$$

$$\frac{d}{dx}\ln(2x+3) = \frac{d\ln u}{du} \cdot \frac{du}{dx}$$

(hvor kjernen $u = 2x+3$
$u' = 2$)

$$= \frac{1}{u} \cdot 2 = \underline{\underline{\frac{2}{2x+3}}}$$

Generelt $\frac{d}{dx}\ln(u(x)) = \frac{u'(x)}{u(x)}$

så $\frac{d}{dx}\ln(ax+b) = \frac{a}{ax+b}$

$$\frac{d}{dx}\ln(x^4) = \frac{(x^4)'}{x^4} = \frac{4x^3}{x^4} = \underline{\underline{\frac{4}{x}}}$$

Alternativt benytt $\ln x^4 = 4\ln x$

$$\frac{d}{dx}4\ln x = 4\frac{d}{dx}\ln x = 4 \cdot \frac{1}{x} = \underline{\underline{\frac{4}{x}}}$$

Regn oppgaver fra 8.4- 8.6 og 8.8 i boka.