18 april Luftmotstanden viser seg å være proporsjonal til fart kvadrert : $\ell \cdot V^2$.

ℓ konstant.

x

$$x' = V \quad \text{fart}$$

$$x'' = V' = a \quad \text{akselerasjon}$$

Newtons andre lov $\quad m \cdot a = \Sigma$ krefter

$$m \cdot V' = m \cdot g - \ell \cdot V^2.$$

Når legemet faller vil farten stabilisere seg.

$V' = 0$: $\quad m \cdot g - \ell \cdot V^2 = 0$

så stabil fart er: $\quad V_{st} = \sqrt{\frac{m \cdot g}{\ell}}$.

Hvis $V_{st} = 60\,m/s$ så er $\frac{\ell}{m} = \underline{0.0027\ m^{-1}}$

(~ 220 km/t)

Vi løser differensiallikningen

29 april
$$mV' = mg - \ell \cdot V^2$$

$$V' = g - \left(\frac{\ell}{m}\right) \cdot V^2$$

$$\frac{V'}{g - \frac{\ell}{m} \cdot V^2} = 1$$

alternativt: $\quad \dfrac{\frac{m}{\ell} \cdot V'}{\frac{mg}{\ell} - V^2} = 1$

Delbrøksoppspalting : $\quad \frac{m}{\ell} V' \dfrac{1}{\left(\sqrt{\frac{mg}{\ell}} - V\right)\left(\sqrt{\frac{mg}{\ell}} + V\right)} = 1$

$$\frac{m}{l} V' \left(\frac{1}{\left(\sqrt{\frac{mg}{l}} - V\right)} + \frac{1}{\sqrt{\frac{mg}{l}} + V} \right) \frac{1}{2\sqrt{\frac{mg}{l}}} = 1$$

integrerer

$$\frac{m}{l} \frac{1}{2\sqrt{\frac{mg}{l}}} \int \frac{1}{\sqrt{\frac{mg}{l}} - V} + \frac{1}{\sqrt{\frac{mg}{l}} + V} \, dV = \int 1 \, dt$$

$$= \frac{1}{2\sqrt{\frac{m \cdot g}{l} \cdot \frac{l}{m}}} \left[-\ln\left|\sqrt{\frac{mg}{l}} - V\right| + \ln\left|\sqrt{\frac{mg}{l}} + V\right| \right] = t + c$$

$$= \underbrace{\frac{1}{2\sqrt{\frac{m \cdot g}{l} \cdot \frac{l^2}{m^2}}}}_{2\sqrt{\frac{gl}{m}}} \ln\left| \frac{\sqrt{\frac{mg}{l}} + V}{\sqrt{\frac{mg}{l}} - V} \right| = t + c$$

$$\left| \frac{\sqrt{\frac{mg}{l}} + V}{\sqrt{\frac{mg}{l}} - V} \right| = e^{2\sqrt{\frac{gl}{m}} \cdot t + c}$$

$$= e^{c} \cdot e^{2\sqrt{\frac{gl}{m}} t}$$

Lineær likning i variabel V

$$\frac{\sqrt{\frac{mg}{l}} + V}{\sqrt{\frac{mg}{l}} - V} = K \cdot e^{2\sqrt{\frac{gl}{m}} \cdot t}$$

$$\sqrt{\frac{mg}{l}} + V = \left(\sqrt{\frac{mg}{l}} - V\right) K e^{2\sqrt{\frac{gl}{m}} \cdot t}$$

$$V\left(1 + K e^{2\sqrt{\frac{gl}{m}} t}\right) = \sqrt{\frac{mg}{l}} \left(K e^{2\sqrt{\frac{gl}{m}} \cdot t} - 1\right)$$

Så $$V(t) = \sqrt{\frac{mg}{l}} \cdot \frac{K e^{2\sqrt{\frac{gl}{m}} \cdot t} - 1}{K e^{2\sqrt{\frac{gl}{m}} \cdot t} + 1}$$

Randbetingelsen $V(0) = 0$ gir : $K = 1$.

Da er løsningen

$$V(t) = \sqrt{\frac{mg}{l}} \cdot \frac{e^{2\sqrt{\frac{gl}{m}}t} - 1}{e^{2\sqrt{\frac{gl}{m}}t} + 1} = \underline{\sqrt{\frac{mg}{l}}\left(1 - \frac{2}{e^{2\sqrt{\frac{gl}{m}}\cdot t} + 1}\right)}$$

Hvor lang tid tar det før vi oppnår 90% av stabil fart? Da ~~må~~ $1 - \frac{2}{e^{2\sqrt{\frac{gl}{m}}t} + 1} = \frac{9}{10} = 90\%$

så $\frac{2}{e^{2\sqrt{\frac{gl}{m}}} + 1} = \frac{1}{10}$

så $e^{2\sqrt{\frac{gl}{m}}\cdot t} = 20 - 1 = 19.$

$$t_{90\%\,fart} = \frac{\ln 19}{2\sqrt{\frac{g\cdot l}{m}}}$$

$\frac{l}{m}$ som tidligere

$$\approx \frac{2.94}{2\sqrt{9.8\,m/s^2 \cdot 0.027\,m^{-1}}} \approx \underline{9\,sekund}$$

Posisjonen $S(t)$ er gitt ved $S'(t) = V(t)$

$$S(t) = \int V(t)\,dt$$
$$= \sqrt{\frac{mg}{l}}\left(t + \sqrt{\frac{m}{gl}}\ln\left(1 + e^{-2\sqrt{\frac{gl}{m}}t}\right)\right) + C$$

Rand betingelsen $S(0) = H$ gir

$$S(t) = \sqrt{\frac{mg}{l}} \cdot t + \frac{m}{l}\left(\ln\left(1 + e^{-2\sqrt{\frac{gl}{m}}t}\right) - \ln 2\right) + H$$