14. feb. 2013

(vinkler har enhet radianer)

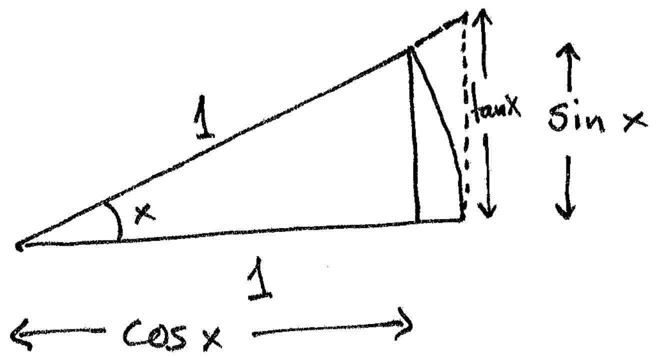
Resultat

$$\lim_{x \to 0} \frac{\sin x}{x} = 1$$

(1)

$$\lim_{x \to 0} \frac{1-\cos x}{x^2} = \frac{1}{2}$$

(dette gir at $\lim_{x \to 0} \frac{1-\cos x}{x} = 0$)



areal "liten trekant" $\leq$ areal sirkelsegment $\leq$ areal "stor trekant":

$$\frac{1}{2}\cos x \cdot \sin x \leq \frac{x}{2} \leq \frac{1}{2} \cdot 1 \cdot \tan x$$

$\frac{\pi}{2} > x > 0$: Deler med x og ganger med 2

$$\underbrace{\cos x \cdot \left(\frac{\sin x}{x}\right) \leq 1}_{\text{deler med } \cos x > 0} \quad \underbrace{1 \leq \frac{1}{\cos x} \cdot \left(\frac{\sin x}{x}\right)}_{\text{ganger med } \cos x > 0}$$

$$\cos x \leq \frac{\sin x}{x} \leq \frac{1}{\cos x}$$

$$\lim_{x \to 0^+} \frac{\sin x}{x} = 1$$

$$\frac{\sin(-x)}{-x} = \frac{-\sin(x)}{-x} = \frac{\sin x}{x}$$

Derfor er $\lim_{x \to 0^+} \frac{\sin x}{x} = \lim_{x \to 0^+} \frac{\sin(-x)}{-x} = \lim_{x \to 0^-} \frac{\sin x}{x}$

Så $\lim_{x \to 0} \frac{\sin x}{x} = 1$

(2) $$\lim_{x\to 0} \frac{1-\cos x}{x^2} = \lim_{x\to 0} \frac{(1-\cos x)(1+\cos x)}{x^2(1+\cos x)}$$

$$= \lim_{x\to 0} \frac{1-\cos^2 x}{x^2(1+\cos x)} = \lim_{x\to 0} \frac{\sin^2 x}{x^2(1+\cos x)}$$

$$= \lim_{x\to 0} \left(\frac{\sin x}{x}\right)^2 \cdot \frac{1}{1+\cos x} \overset{\text{grensesetningene}}{=} \left(\lim_{x\to 0} \frac{\sin x}{x}\right)^2 \cdot \lim_{x\to 0} \frac{1}{1+\cos x}$$

$$= 1^2 \cdot \frac{1}{2} = \frac{1}{2}.$$

Vi viser nå at $(\sin x)' = \cos x$.

$$(\sin x)' = \lim_{h\to 0} \frac{\sin(x+h) - \sin x}{h}$$ (addisjons-formel for sin)

$$= \lim_{h\to 0} \frac{\sin(x)\cdot\cos(h) + \sin(h)\cdot\cos x - \sin x}{h}$$

$$= \lim_{h\to 0} \cos x \cdot \frac{\sin(h)}{h} + \sin x \frac{\cos(h)-1}{h}$$

$$= \cos x \cdot \lim_{h\to 0} \frac{\sin h}{h} + \sin x \lim_{h\to 0} \frac{\cos(h)-1}{h}$$

$$= \cos x \cdot 1 + \sin x \cdot 0 = \underline{\cos x}$$

$$\cos x = \sin\left(\frac{\pi}{2} - x\right)$$

$$(\cos x)' = \left(\sin\left(\frac{\pi}{2} - x\right)\right)' = \underbrace{\cos\left(\frac{\pi}{2} - x\right)}_{\sin x} \cdot \underbrace{\left(\frac{\pi}{2} - x\right)'}_{(-1)}$$

$$\underline{(\cos x)' = -\sin x}$$

Alternativt bruk def. av den deriverte og addisjonsformelen for cos.

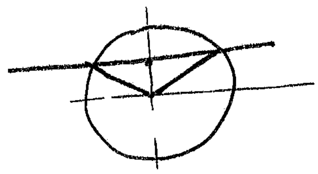
(3) y vinkel med enhet grader

$$\sin\left(\frac{\pi}{180}y\right)$$

$$\frac{d}{dy}\sin\left(\frac{\pi}{180}y\right) = \cos\left(\frac{\pi}{180}\cdot y\right)\cdot\left(\frac{\pi}{180}\cdot y\right)'$$
$$= \frac{\pi}{180}\cos\left(\frac{\pi}{180}y\right)$$

Hvis vi bruker vinkler med enhet grader blir den deriverte til sin lik $\frac{\pi}{180}$ gangetmed cos.

Trigonometriske likninger 10.1,2



$\sin(v) = \frac{1}{2}$

$v = \arcsin(1/2) = \pi/6$ (30°)

$\pi - \arcsin(1/2) = \pi - \pi/6 = \frac{5\pi}{6}$

Alle løsninger er $\frac{\pi}{6} + 2\pi\cdot n$

$\frac{5\pi}{6} + 2\pi\cdot n$ n heltall

I første omløp $[0, 2\pi\rangle$ er løsningene $\frac{\pi}{6}$ og $\frac{5\pi}{6}$

$\sin(1-2x) = \frac{1}{2}$ $x \in [0,\pi]$.

1) Løser $\sin(v) = \frac{1}{2}$ ($v = 1-2x$ vinkelen)

2) Finner x slik at $1-2x$ er lik de mulige vinklene

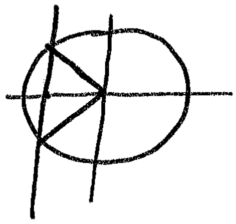
1) : $v \in [1-2\pi, 1]$ $\sin v = \frac{1}{2}$ har løsningene

$\frac{\pi}{6}$, $\frac{5\pi}{6} - 2\pi = \frac{-7\pi}{6}$

(4) $V = 1-2x$, $\quad 2x = 1-V$, $\quad x = \frac{1-V}{2}$

Løsningene er : $\underline{\underline{\frac{1-\pi/6}{2}}}$ og $\underline{\underline{\frac{1+7\pi/6}{2}}}$

$\cos V = \frac{-1}{\sqrt{2}}$



$V = \frac{3\pi}{4} + 2\pi \cdot n = \arccos\left(\frac{-1}{\sqrt{2}}\right) + 2\pi \cdot n$

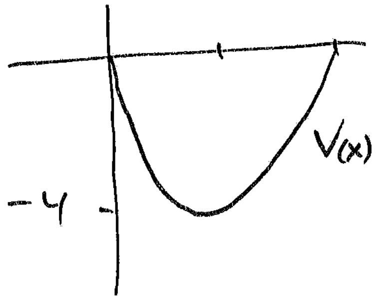
$V = \frac{-3\pi}{4} + 2\pi \cdot n = -\arccos\left(\frac{-1}{\sqrt{2}}\right) + 2\pi \cdot n$

I første omløp er løsningene $\underline{\frac{3\pi}{4}, \frac{5\pi}{4}}$

$\cos(x^2 - 4x) = \frac{-1}{\sqrt{2}}$ $\quad x \in [0,4]$

Vinkel $V = x^2 - 4x = (x-2)^2 - 4$

$V \in [-4, 0]$.



$V = \underline{\frac{-3\pi}{4}, \frac{-5\pi}{4}} \approx -3.93\ldots$

$x^2 - 4x = V$

$(x-2)^2 - 4 = V$

$(x-2)^2 = 4 + V$

$|x-2| = \sqrt{4+V}$

$\underline{x = 2 \pm \sqrt{4+V}}$ (ligger i intervallet [0,4].)

$x = \underline{\underline{2 \pm \sqrt{4 - \frac{3\pi}{4}}}}$

og $x = \underline{\underline{2 \pm \sqrt{4 - \frac{5\pi}{4}}}}$ (4 løsninger)

⑤ $\underline{\cos^2 x + \cos x = \sin^2 x}$ $(= 1 - \cos^2 x)$

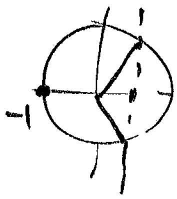
$2\cos^2 x + \cos x - 1 = 0$ 2.grads-uttrykk i cos x

$2y^2 + y - 1 = 0$ $y = \cos x$.

$(2y-1)(y+1) = 0$ (annengradsformelen ...)

løsningene er $y = -1$ og $y = \frac{1}{2}$.

$\cos x = -1$ eller $\cos x = \frac{1}{2}$



$\underline{x = \pi + 2\pi \cdot n}$ $\underline{x = \frac{\pi}{3} + 2\pi \cdot n}$

n heltall $\underline{x = \frac{-\pi}{3} + 2\pi \cdot n}$

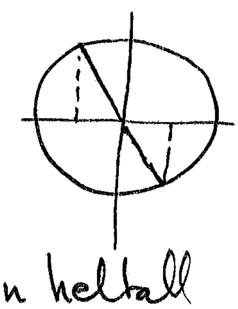
$\sin x + \sqrt{3}\cos x = 0$

$\cos x \neq 0$ $\cos x\,(\tan x + \sqrt{3}) = 0$

$\cancel{\left(\frac{\sin x}{\cos x} \cdot \cos x + \sqrt{3}\cos x\right) =}$ $\tan x + \sqrt{3} = 0$ $(\cos x \neq 0)$

(ingen løsning når $\cos x = 0$ siden $\sin x = 1$ eller -1).

$\tan x = -\sqrt{3}$



$x = \arctan(-\sqrt{3}) = -\frac{\pi}{3}$

$x = \arctan(-\sqrt{3}) + \pi \cdot n$ n heltall

$\underline{x = \frac{-\pi}{3} + \pi \cdot n}$

$\sin x + \cos x = 1/\sqrt{2}$ addisjonsformel for sin :

$\sqrt{2}\sin(x + \frac{\pi}{4}) = \sin x + \cos x = 1/\sqrt{2}$

$\sin(x + \frac{\pi}{4}) = \frac{1}{2}$ ...