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15. Ubestemte integral

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$$\int \frac{1}{\sqrt{x}}\,dx = \int x^{-1/2}\,dx = \frac{x^{1/2}}{1/2} + C = \underline{2\sqrt{x} + c}$$

$$\int (1+2x)^3\,dx$$

$$= \int 1 + 3\cdot(2x) + 3\cdot 4x^2 + 8x^3\,dx$$

$$= x + 3\cdot x^2 + 4x^3 + 2x^4 + c$$ tungvint?

$$\left((1+2x)^4\right)' = 4(1+2x)^3 \cdot \underbrace{(1+2x)'}_{2} = 8(1+2x)^3$$

$$\left(\frac{1}{8}(1+2x)^4\right)' = (1+2x)^3$$

så $\frac{1}{8}(1+2x)^4$ er en antiderivert til $(1+2x)^3$.

$$\int (1+2x)^3\,dx = \underline{\frac{1}{8}(1+2x)^4 + c}$$

Mer generelt: a, b konstanter

Anta $F'(x) = f(x)$

$$\left(F(ax+b)\right)' = \underbrace{F'(ax+b)}_{f(ax+b)}\ \underbrace{(ax+b)'}_{a}$$

deler med a på begge sider

$$\left(\frac{1}{a}F(ax+b)\right)' = f(ax+b)$$

(2) $\frac{1}{a} F(ax+b)$ er en antiderivert til $f(ax+b)$.

$$\int f(ax+b)\,dx = \frac{1}{a}\int f(u)\,du \quad \text{hvor } u = ax+b$$

$$\underline{\int f(ax+b)\,dx = \frac{1}{a} \cdot \underbrace{\int f(u)\,du}_{F(ax+b)+C}}$$

oppgave

$$\int (1+2x)^{17}\,dx \qquad U(x) = 1+2x \qquad a = 2$$

$$= \frac{1}{2}\int u^{17}\,du$$

$$= \frac{1}{2} \cdot \frac{u^{18}}{18} + C \qquad \text{setter inn for } u$$

$$= \underline{\frac{(1+2x)^{18}}{36} + C}$$

oppgave

$$\int 5e^{2x-3}\,dx$$

$$= 5\int e^{2x-3}\,dx \qquad u = 2x-3 \qquad a = u' = 2$$

$$= 5 \cdot \frac{1}{2}\int e^{u}\,du$$

$$= \frac{5}{2} \cdot e^{u} + C$$

$$= \underline{\frac{5}{2} e^{2x-3} + C}$$

(test: $\left(\frac{5}{2} e^{2x-3}\right)' = \frac{5}{2} e^{2x-3} \underbrace{(2x-3)'}_{2}$

$= \underline{5e^{2x-3}}$)

(3)

$$\int 10^x \, dx$$

$ax+b$

$u = \ln 10 \cdot x$

$a = \ln 10$

$b = 0$

$$= \int e^{\ln 10 \cdot x} \, dx$$

$$= \frac{1}{\ln 10} \cdot \int e^u \, du$$

$$= \frac{1}{\ln 10} \cdot e^u + c$$

$$= \underline{\frac{10^x}{\ln 10} + c}$$

Substitusjon.

$F'(u) = f(u)$ så $F(u)$ er en antiderivert til $f(u)$

$$(F(u(x)))' \overset{\text{kjerneregelen}}{=} F'(u(x)) \cdot u'(x)$$

$$(F(u(x)))' = f(u(x)) \cdot u'(x)$$

$$\boxed{\int f(u(x)) \cdot u'(x) \, dx = \int f(u) \, du}$$

$u = x^3$

Eks.

$$\int 3x^2 e^{x^3} \, dx$$

$$= \int u'(x) \cdot e^u \, dx = \int e^u \, du$$

$$= e^u + c$$

$$= \underline{e^{x^3} + c}$$

spesialtilfelle:

$$U = ax + b$$
$$U' = a \quad \text{konstant}$$

(4)

$$\int \underbrace{f(ax+b)}_{f(U)} \cdot \underbrace{a}_{U'_{(x)}} \, dx = \int f(U) \, dU$$

Linear substitusjon

Deler med a på begge sider

$$\frac{1}{a} \int f(ax+b) \cdot a \, dx = \frac{a}{a} \int f(ax+b) \, dx$$
$$= \frac{1}{a} \int f(U) \, dU.$$

oppg. $\int \frac{1}{1-x} \, dx = \underline{-\ln|1-x| + c}$

$U = 1 - x \qquad U' = -1$

$(= ax + b \qquad a = -1, \ b = 1)$

$$\int \frac{1}{1-x} \, dx = \frac{1}{(-1)} \int \frac{1}{U} \, dU = -1 \cdot \ln|U| + c$$
$$= \underline{-\ln|1-x| + c}$$

oppg. $\int \frac{1}{\sqrt{2-3x}} \, dx \qquad (x < 2/3)$

$U = 2 - 3x \qquad a = -3 \qquad (U' = -3)$

$$\int \frac{1}{\sqrt{2-3x}} \, dx = \frac{1}{-3} \int \frac{1}{\sqrt{U}} \, dU = \frac{-1}{3}\left(2\sqrt{2-3x}\right) + c$$
$$= \underline{\frac{-2}{3}\sqrt{2-3x} + c}$$

eks. $\int \cos^2 x \, dx$

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$$\cos(2x) = \cos^2 x - \sin^2 x$$

$$= \cos^2 x - (1 - \cos^2 x)$$

$$\cos(2x) = 2\cos^2 x - 1$$

$$\cos^2 x = \frac{1}{2}(\cos(2x) + 1)$$

$$\int \cos^2 x \, dx = \frac{1}{2}\int \cos(2x) + 1 \, dx$$

$U = 2x$

$$= \frac{1}{2}\left(\frac{1}{2}\int \cos(U)\, dU + \int 1 dx \right)$$

$$= \frac{1}{4} \sin(U) + \frac{1}{2}x + c$$

$$= \underline{\frac{1}{4}\sin(2x) + \frac{x}{2} + c}$$

oppg. $\int \frac{1}{\cos^2 x} dx = \tan x + c$

oppg. $\int \sin x \cdot \cos x \, dx$

Husk at $\sin(2x) = 2\sin x \cdot \cos x$

$$\int \sin x \cdot \cos x \, dx = \int \frac{1}{2} \cdot \sin(2x) dx$$

$U = 2x$

$$= \frac{1}{2}\int \sin(2x) dx = \frac{1}{2}\left(\frac{1}{2}\int \sin(U) dU\right)$$

$$= \frac{1}{4}(-\cos(U)) + c = \underline{\frac{-1}{4}\cos(2x) + c}$$

$$= \frac{-1}{4}(\cos^2 x - \sin^2 x) + c$$

Alternativt. La $U = \cos x$

⑥ $U' = -\sin x$

$$\int \sin x \cdot \cos x \, dx = \int (-U') \cdot U \cdot dx$$

$$= - \int U \, dU$$ substitusjon

$$= - \frac{U^2}{2} + c$$

$$= - \frac{1}{2} \cos^2 x + c$$

Svarene er like

$$\frac{-1}{4}(\cos^2 x - \sin^2 x) = \frac{-1}{2}\cos^2 x + \underline{\frac{1}{4}}$$