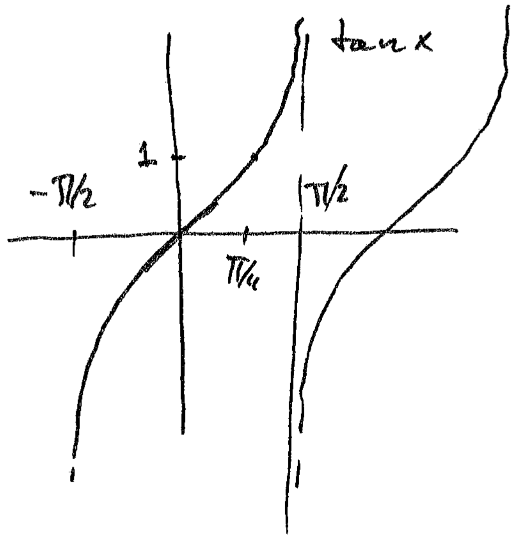
13.02.2012

10.9 Kurvedrøfting.

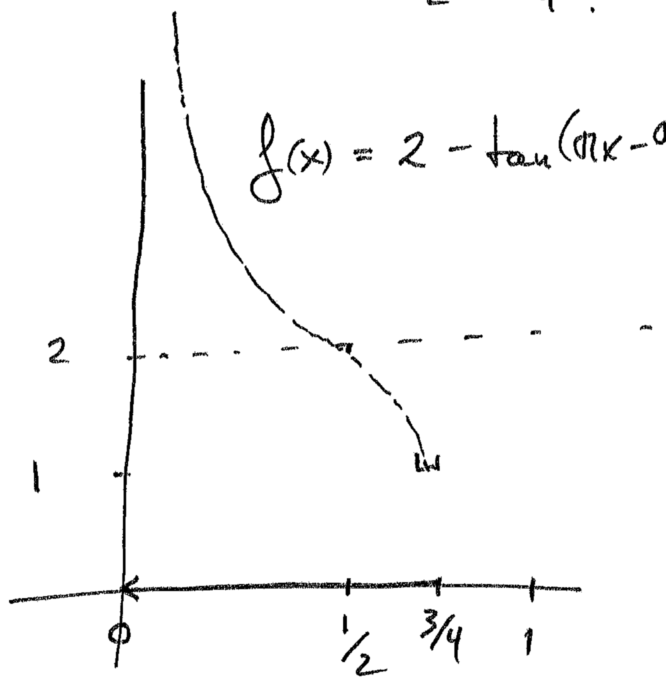
(1) $f(x) = 2 - \tan(\pi x - \frac{\pi}{2})$ $\quad 0 < x \le \frac{3}{4}$

Finn ekstremalpunkt. Bestem konkavitet.

Lag en skisse av grafen til $f(x)$.



$-\frac{\pi}{2} < \pi x - \frac{\pi}{2} \le \frac{\pi}{4}$.



$f(x)$ har et minimumspunkt i $(\frac{3}{4}, +1)$.

$f(x)$ er konkav opp : $(0, \frac{1}{2})$

$f(x)$ er konkav ned : $(\frac{1}{2}, \frac{3}{4}]$.

② $f(x) = x - \sin 2x + 1$ $\quad -2 \leq x \leq 2$

Finn ekstremalverdiene (ekstremal punktene)

Bestem konkavitet.

Lag en skisse av grafen.

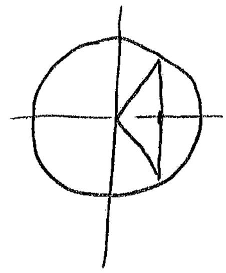
$$f'(x) = (x - \sin 2x + 1)'$$
$$= 1 - \cos(2x) \cdot (2x)' + 0$$
$$= 1 - 2 \cdot \cos 2x.$$

$$f''(x) = +4\sin(2x)$$

Kritiske punkt: endepunkt og x slik at $f'(x) = 0$.

$f'(x) = 0 = 1 - 2\cos 2x$.

$\cos(2x) = \frac{1}{2}$



$2x = \frac{\pi}{3} + 2\pi \cdot n$

$2x = \frac{-\pi}{3} + 2\pi \cdot n$ $\quad$ n heltall.

Siden $-4 \leq 2x \leq 4$, er $2x = \frac{-\pi}{3}$ og $\frac{\pi}{3}$.

$x = \frac{-\pi}{6}$ og $x = \frac{\pi}{6}$.

$f\left(\frac{\pi}{6}\right) = \frac{\pi}{6} - \sin\left(\frac{\pi}{3}\right) + 1 = \underbrace{\left(\frac{\pi}{6} - \frac{\sqrt{3}}{2}\right)}_{-0.342\ldots} + 1 = 0.657$

$f\left(-\frac{\pi}{6}\right) = -\left(\frac{\pi}{6} - \sin\frac{\pi}{3}\right) + 1 = -\left(\frac{\pi}{6} - \frac{\sqrt{3}}{2}\right) + 1 = 1,342$

$\left(-\frac{\pi}{6}, f\left(-\frac{\pi}{6}\right)\right)$ toppunkt $\quad$ $\left(\frac{\pi}{6}, f\left(\frac{\pi}{6}\right)\right)$ bunnpunkt.

$f''\left(-\frac{\pi}{6}\right) = 4\sin\left(-\frac{\pi}{3}\right) < 0$ $\quad$ $f''\left(\frac{\pi}{6}\right) = 4\sin\left(\frac{\pi}{3}\right) > 0$

⌒ konkav ned $\quad$ ◡ konkav opp

Endepunktene: $(-2, f(-2)) = (-2, -2-\sin(-4)+1)$

(3)

$= (-2, -1+\sin(4))$

$(2, f(2)) = (2, 2-\sin 4+1)$

$= (2, 3-\sin(4))$

Globalt minimumspunkt $(-2, -1+\sin(4)) = \underline{(-2, -1.76)}$

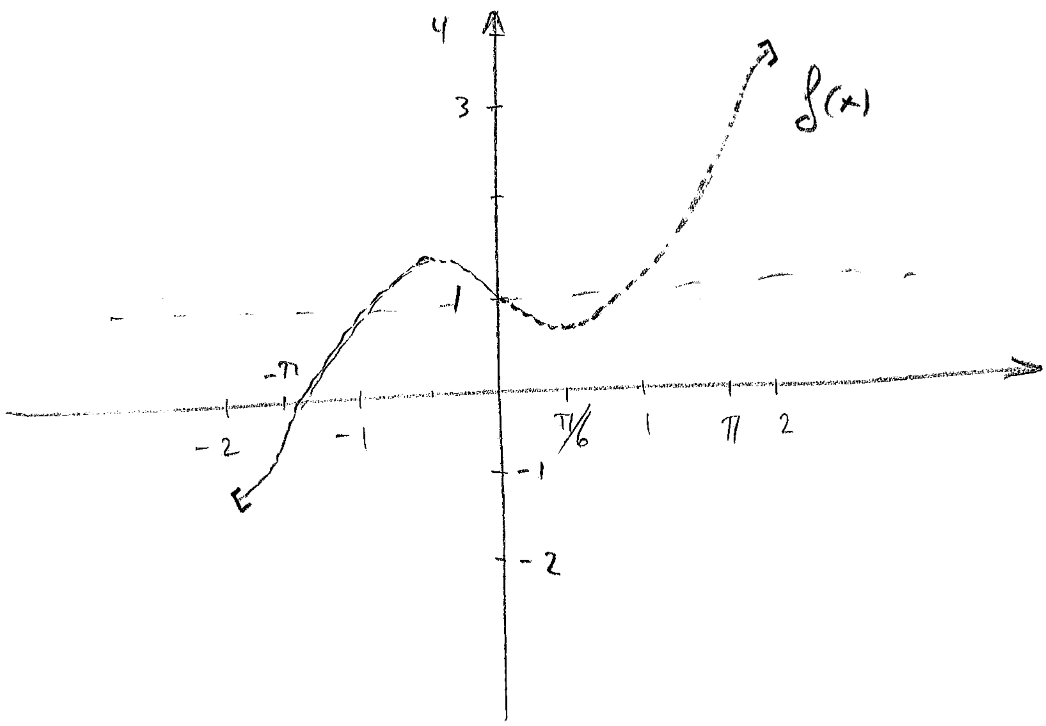
maksimumspunkt $(2, 3-\sin(4)) = \underline{(2, 3.76)}$

$f(x)$ er konkav op når $f''(x) = 4\sin(2x) > 0$:

$0 < x < \pi/2$, $-2 \leq x \leq -\frac{\pi}{2}$

$f(x)$ er konkav ned når $f''(x) = 4\sin(2x) < 0$

$-\frac{\pi}{2} < x < 0$, $\frac{\pi}{2} < x \leq 2$



$$y(x) = \sin(ax)$$

$$y'(x) = \cos(ax)(ax)' = a\cdot\cos(ax)$$

$$y''(x) = a(\cos(ax))' = a(-\sin(ax)\cdot(ax)')$$

$$= -a^2\cdot\sin(ax)$$

$$y''(x) = -a^2\cdot y(x),$$

$$y'' + a^2\cdot y = 0.$$

$z(x) = \cos(ax)$, $\quad z'(x) = -\sin(ax)\cdot(ax)'$

$$= -a\sin(ax)$$

$$z''(x) = -a^2 z(x).$$

Resultat: Løsninger til <u>differensial likningen</u>

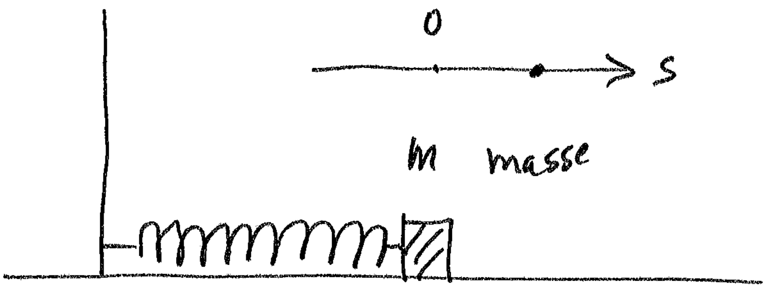
$$y'' + a^2 y = 0 \quad \text{er på formen}$$

$$\underline{y(x) = A\sin(ax) + B\cos(ax)}.$$

A, B konstanter.

(Merk at dette er lik $c\cdot\sin(a(x-d))$ for konstanter c og d.)

Harmonisk oscilator (svingninger?)



Friksjonsfritt.

Antar at kraften fra fjæren er proporsjonal til forflytting fra likevektsposisjon ($s=0$).

$F = -ks$ k fjærstivheten.

Newtons 2. lov.   Masse × akselerasjon = kraft.

akselerasjonen er $s''(t)$

$$m \cdot s''(t) = -k \cdot s \qquad (k>0)$$

$$m \cdot s''(t) + k \cdot s = 0 \qquad \text{deler med}$$

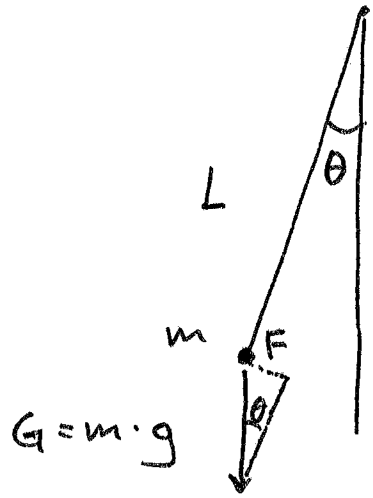
$$s''(t) + \frac{k}{m} \cdot s = 0$$

Løsningene er derfor på formen

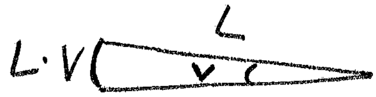
$$\underline{s(t) = A \sin\left(\sqrt{\tfrac{k}{m}}\,(t-c)\right)} \qquad A, c \text{ konstanter.}$$

Pendelen

(θ theta)



$$F = G \cdot \sin\theta$$
$$= m \cdot g \sin\theta$$



$$\left(\frac{d^2}{dt^2}(L \cdot v) = L \frac{d^2}{dt^2} v \right)$$

$$\text{masse} \cdot \text{akselerasjon} = -mg\sin\theta$$

$$m \cdot L \cdot \frac{d^2\theta}{dt^2} = -mg\sin\theta$$

$$\underline{\theta''(t) + \frac{g}{L}\sin\theta = 0}$$ Vanskelig!

Husk at $\lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1$.

En god tilnærming for $\sin\theta$, når θ er liten, er θ. ($\sin\theta \sim \theta$)

Tilnærmet diff. likning (for θ liten)

$$\theta''(t) + \frac{g}{L} \cdot \theta = 0.$$

$$\theta(t) = A\sin\left(\sqrt{\tfrac{g}{L}}\, t\right) + B\cos\left(\sqrt{\tfrac{g}{L}} \cdot t\right)$$

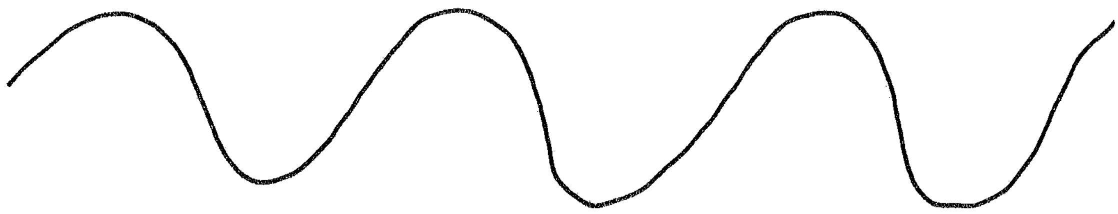
(Tids) Perioden for en svingning er $T = \frac{2\pi}{\sqrt{g/L}} = \underline{2\pi\sqrt{\frac{L}{g}}}$

θ_0 største vinkelutslag. Eksakt: $T = 2\pi\sqrt{\frac{L}{g}}\left(1 + \frac{1}{16}\theta_0^2 + \frac{11}{3072}\theta_0^4 + \dots\right)$

$\sim$

Bølgelengde λ (lambda) lengden på en svingning

Frekvens ν (nu) antall svingninger / tid



Farten til bølgen $v = \lambda \cdot \nu$.

$\sin\left(2\pi \frac{x}{\lambda}\right)$ λ perioden (posisjon)

$\sin\left(2\pi \cdot \nu \cdot t\right)$ $\frac{1}{\nu}$ perioden (Tid)

$\left(\sin(kx) \quad \text{periode} : p = \frac{2\pi}{k} \right)$