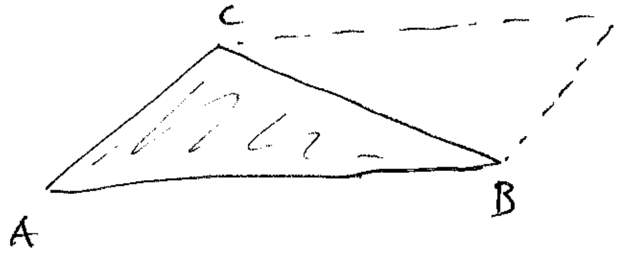
1. nov. 2011

(1) Finn arealet til trekanten ABC

$\vec{AB} = [1, 2, -1]$ $\vec{AC} = [2, -3, 1]$



$\frac{1}{2} |\vec{AB} \times \vec{AC}|$ er arealet til trekant.

$$\vec{AB} \times \vec{AC} = \begin{vmatrix} \vec{e}_1 & \vec{e}_2 & \vec{e}_3 \\ 1 & 2 & -1 \\ 2 & -3 & 1 \end{vmatrix}$$

$$= \vec{e}_1 \begin{vmatrix} 2 & -1 \\ -3 & 1 \end{vmatrix} - \vec{e}_2 \begin{vmatrix} 1 & -1 \\ 2 & 1 \end{vmatrix} + \vec{e}_3 \begin{vmatrix} 1 & 2 \\ 2 & -3 \end{vmatrix}$$

$$= \vec{e}_1 (2-3) - \vec{e}_2 (1-(-2)) + \vec{e}_3 (-3-4)$$

$$= -\vec{e}_1 - 3\vec{e}_2 + (-7)\vec{e}_3$$

$$= \underline{[-1, -3, -7]}$$

Arealet til ΔABC er $\frac{1}{2} |[-1, -3, -7]|$

$$= \frac{1}{2}\sqrt{1+9+49} = \underline{\frac{1}{2}\sqrt{59}}$$

(14.6)

② Trevektorproduktet er

$$\vec{V}_1 \cdot (\vec{V}_2 \times \vec{V}_3).$$

$$[x_1, y_1, z_1] \cdot \begin{vmatrix} \vec{e}_1 & \vec{e}_2 & \vec{e}_3 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix}$$

$$= [x_1, y_1, z_1] \cdot \left[\begin{vmatrix} y_2 & z_2 \\ y_3 & z_3 \end{vmatrix}, - \begin{vmatrix} x_2 & z_2 \\ x_3 & z_3 \end{vmatrix}, \begin{vmatrix} x_2 & y_2 \\ x_3 & y_3 \end{vmatrix} \right]$$

$$= x_1 \begin{vmatrix} y_2 & z_2 \\ y_3 & z_3 \end{vmatrix} - y_1 \begin{vmatrix} x_2 & z_2 \\ x_3 & z_3 \end{vmatrix} + z_1 \begin{vmatrix} x_2 & y_2 \\ x_3 & y_3 \end{vmatrix}$$

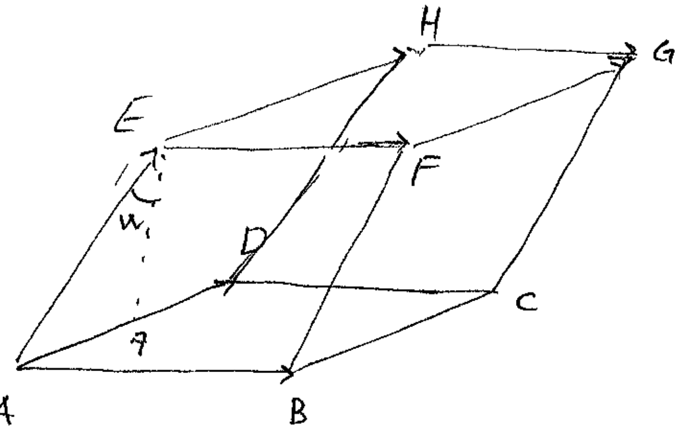
$$= \begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix}$$

Spørsmål: er $\vec{V}_1 \cdot (V_2 \times V_3)$ lik $(\vec{V}_1 \times \vec{V}_2) \cdot \vec{V}_3$?

ja.

Parallellepiped

③



Arealet til grunnflaten er $|\vec{AB} \times \vec{AD}|$

høyden er $|\vec{AE}| \cdot |\cos w|$

Volumet til parallellepipedet er

$$|\vec{AE}|\, |\vec{AB} \times \vec{AD}| \cdot |\cos w|$$

$$= \left| \vec{AE} \cdot (\vec{AB} \times \vec{AD}) \right| \quad \text{trippelproduktet.}$$

Eksempel. Et parallellepiped er utspent av $\vec{a} = [1, -1, -1]$, $\vec{b} = [0, 2, 3]$, $\vec{c} = [-1, 0, 1]$.

Hva er volumet? Det er absoluttverdien til trippelproduktet

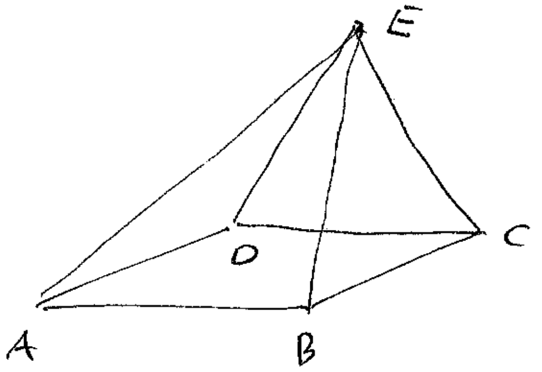
$$|\vec{a} \cdot (\vec{b} \times \vec{c})| = \left| \begin{vmatrix} 1 & -1 & -1 \\ 0 & 2 & 3 \\ -1 & 0 & 1 \end{vmatrix} \right|$$

$$= \left| 1 \begin{vmatrix} 2 & 3 \\ 0 & 1 \end{vmatrix} - (-1) \begin{vmatrix} 0 & 3 \\ -1 & 1 \end{vmatrix} + (-1) \begin{vmatrix} 0 & 2 \\ -1 & 0 \end{vmatrix} \right|$$

$$= |2 + 3 - 2| = \underline{\underline{3}}$$

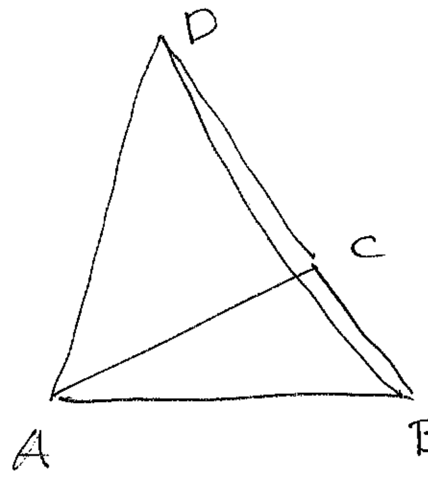
Pyramide med parallellogram grunnflate

(4)



har volum $\frac{1}{3}\left|(\vec{AB} \times \vec{AD}) \cdot \vec{AE}\right|$

Tetraeder



Volumet $\frac{1}{3}$ høyde · areal grunnflate

$$= \frac{1}{3}\left|\frac{1}{2}(\vec{AB} \times \vec{AC}) \cdot \vec{AD}\right|$$

$$= \frac{1}{6}\left|(\vec{AB} \times \vec{AC}) \cdot \vec{AD}\right|$$

Finn volumet til tetraedere ABCD

hvor $A = (1, 2, 3)$

(5) $B = (2, -1, 1)$

$C = (-1, 1, 2)$

$D = (1, 1, 1)$

$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = [2, -1, 1] - [1, 2, 3]$

$\overrightarrow{AB} = [1, -3, -2]$

$\overrightarrow{AC} = [-2, -1, -1]$

$\overrightarrow{AD} = [0, -1, -2]$

Volumet er $\frac{1}{6}\left| \begin{vmatrix} 0 & -1 & -2 \\ 1 & -3 & -2 \\ -2 & -1 & -1 \end{vmatrix} \right|$

$= \frac{1}{6}\left| 0 - (-1)\begin{vmatrix} 1 & -2 \\ -2 & -1 \end{vmatrix} + (-2)\begin{vmatrix} 1 & -3 \\ -2 & -1 \end{vmatrix} \right|$

$= \frac{1}{6}\left| -5 + 14 \right| = \frac{9}{6} = \frac{3 \cdot 3}{2 \cdot 3} = \frac{3}{2} \cdot \frac{3}{3} = \underline{\underline{\frac{3}{2}}}$