PRØVE 05. desember – Løsningsforslag

Oppgave 1

a) $f'(x) = 6x^2 + 10x - \frac{3}{x^2}$

b) $f'(x) = 10(1-3x)^9 \cdot (-3) = -30(1-3x)^9$

c) $f'(x) = \frac{-1(1-2x) - (2-x)\cdot(-2)}{(1-2x)^2} = \frac{(2x-1)+4-2x}{(1-2x)^2} = \frac{3}{(1-2x)^2}$

Oppgave 2

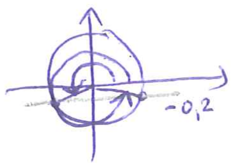
a) $125^{\frac{2}{3}} \cdot 2a \cdot \frac{1}{a^3} = \left(\sqrt[3]{125}\right)^2 \cdot \frac{2a}{a^3} = 50a^{-2}$

b) $\frac{x(x+1) - 2(x-1) - 2(1)}{x^2-1} = \frac{x^2+x-2x+2-2}{x^2-1} = \frac{x^2-x}{x^2-1} = \frac{x\cancel{(x-1)}}{(x+1)\cancel{(x-1)}}$

$= \frac{x}{x+1}$

Oppgave 3

a) $\sin x = -\frac{1}{5} = -0{,}2$



$x = \sin^{-1}(-0{,}2) = -11{,}5^\circ$

$x = 180^\circ + 11{,}5^\circ = \underline{\underline{191{,}5^\circ}}$

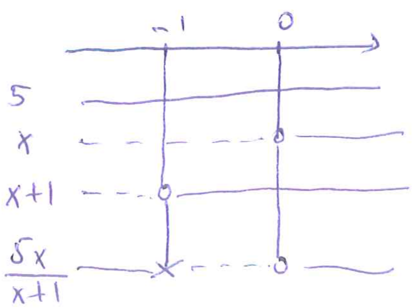
eller

$x = 360^\circ - 11{,}5^\circ = \underline{\underline{348{,}5^\circ}}$

b) $\frac{2x-3}{x+1} + \frac{3(x+1)}{x+1} \leq 0 \qquad x \neq -1$

$\frac{2x-3+3x+3}{x+1} \leq 0$

$\frac{5x}{x+1} \leq 0$



Løsning:

$x \in \langle -1, 0]$

c) $x - 1 = 2\sqrt{1-x} \quad |^2$

$(x-1)^2 = 4(1-x)$

$x^2 - 2x + 1 = 4 - 4x$

$x^2 + 2x - 3 = 0$

$(x+3)(x-1) = 0$

$x = -3$ eller $x = 1$

PRØVE

For $x = -3$

V.S: $x - 1 = -3 - 1 = -4$

H.S: $2\sqrt{1-x} =$
$= 2\sqrt{1+3} = 4$

V.S $\neq$ H.S

For $x = 1$

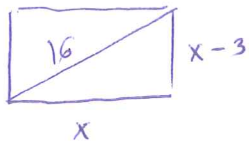
V.S: $x - 1 = 1 - 1 = 0$

H.S: $2\sqrt{1-x} = 2\sqrt{1-1} =$
$= 2 \cdot 0 = 0$

V.S = H.S.

Løsning: $\underline{\underline{x = 1}}$

Oppgave 4



$x^2 + (x-3)^2 = 16^2$

$x^2 + x^2 - 6x + 9 = 256$

$2x^2 - 6x - 247 = 0$

$a = 2,\ b = -6,\ c = -247$

$\underline{x_1 = 12{,}7}$, $\underline{x_2 = -9{,}7}$

Løsning:

$x = \underline{\underline{12{,}7}}$

$x - 3 = 12{,}7 - 3 = \underline{\underline{9{,}7}}$

Den ene siden = 12,7 m,
den andre 9,7 m

Oppgave 5

$f(x) = \dfrac{2x^2 - x}{x+1}$ $\qquad x + 1 \neq 0 \Rightarrow x \neq -1$

a) $D_f = \mathbb{R} \setminus \{-1\}$

b) $f(x) = 0 \iff 2x^2 - x = 0 \iff x(2x - 1) = 0$

$x = 0$ eller $x = \frac{1}{2}$

Nullpunkter: $(0, 0)$ og $(\frac{1}{2}, 0)$

c) Vertikal asymptote:

Nevner: $x+1=0$ for $x=-1$

Teller: $2x^2-x$ for $x=-1$

$2\cdot(-1)^2-(-1) = 2+1 = 3 \neq 0$

Det betyr at $x=-1$ er en vertikal asymptote.

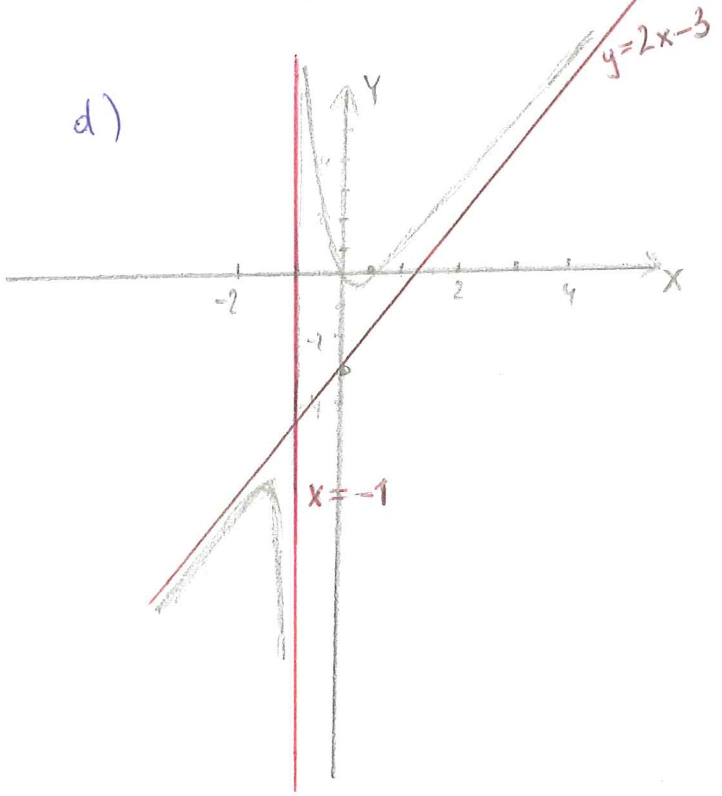
Skrå asymptote

$2x^2-x : x+1 = 2x-3 + \frac{3}{x+1}$

$f(x) \approx 2x-3$ når $x \to \pm\infty$.

$y = 2x-3$ er en skrå asymptote.

d)



Oppgave 6

Nullpunkt for $x=-2$

$P(-2)=0$

$a(-2)^2+b(-2)+c=0$

$4a-2b+c=0$

Toppunkt $(0, 12)$

$P'(x) = 2ax+b$

$2ax+b=0$

$P'(0) = b \Rightarrow b=0$

$P(0)=12$

$\Downarrow$

$a\cdot0^2+b\cdot0+c=12$

$\Downarrow$

$c=12$

$4a+12=0 \Rightarrow a=-3$

Svar:

$a=-3$, $b=0$, $c=12$

Oppgave 7

a) $g'(x) = -3x^2$

$g'(-1) = -3(-1)^2 = \underline{\underline{-3}}$

b) $g(-1) = -(-1)^3 = 1$

$y - y_1 = a(x - x_1)$

$y - 1 = -3(x - (-1))$

$y - 1 = -3(x + 1)$

$\underline{\underline{y = -3x - 2}}$

Oppgave 8

a) $x^2 - 1 = (x+1)(x-1)$

$$\left.\begin{aligned} P(1) &= 1^4 - 4 \cdot 1^2 + 3 = 1 - 4 + 3 = 0 \\ P(-1) &= (-1)^4 - 4 \cdot (-1)^2 + 3 = 1 - 4 + 3 = 0 \end{aligned}\right\} \Rightarrow P(x) \text{ er delelig med } x^2 - 1$$

b) $x^4 - 4x^2 + 3 : x^2 - 1 = x^2 - 3$

$-(x^4 - x^2)$

$-3x^2 + 3$

$-(-3x^2 + 3)$

0

$x^4 - 4x^2 + 3 = (x^2 - 1)(x^2 - 3) = (x+1)(x-1)(x - \sqrt{3})(x + \sqrt{3})$

Nullpunkter for $x = 1$, $x = -1$, $x = \sqrt{3}$, $x = -\sqrt{3}$.

dvs: $(1,0)$, $(-1,0)$, $(\sqrt{3},0)$, $(-\sqrt{3},0)$

c) $x^4 - 4x^2 + 3 : x - a =$

$$x^4 - 4x^2 + 3 : x - a = x^3 + ax^2 + a^2x - 4x + a^3$$

$$\begin{aligned}
&-(x^4 - ax^3) \\
&\overline{\quad ax^3 - 4x^2 + 3} \\
&-(ax^3 - a^2x^2) \\
&\overline{\quad a^2x^2 - 4x^2 + 3} \\
&-(a^2x^2 - a^3x) \\
&\overline{\quad -4x^2 + a^3x + 3} \\
&-(-4x^2 + 4xa) \\
&\overline{\quad a^3x - 4xa + 3} \\
&-(a^3x - a^4) \\
&\overline{\quad -4xa + a^4 + 3} \\
&-(-4xa + 4a^2) \\
&\overline{\quad a^4 - 4a^2 + 3}
\end{aligned}$$

$$\Rightarrow \quad a^4 - 4a^2 + 3 = 3$$

$$a^4 - 4a^2 = 0$$

$$a^2(a^2 - 4) = 0$$

$$a^2(a-2)(a+2) = 0$$

$$\underline{\underline{a = 0 \text{ eller } a = 2 \text{ eller } a = -2}}$$

Oppgave 9

$$(1 - \sin^2 v)(1 + \tan^2 v) \overset{(?)}{=} 1$$

$$\cos^2 v \left(1 + \frac{\sin^2 v}{\cos^2 v}\right) = \cos^2 v + \cancel{\cos^2 v} \cdot \frac{\sin^2 v}{\cancel{\cos^2 v}} =$$

$$= \cos^2 v + \sin^2 v = 1.$$

Oppgave 10

$$\left.\begin{aligned}
&\lim_{x \to -1^-} f(x) = \lim_{x \to -1^-} \left(\frac{1}{2}x^2 + \frac{3}{2}\right) = \frac{1}{2} + \frac{3}{2} = 2 \\
&\lim_{x \to -1^+} f(x) = \lim_{x \to -1^+} \left(-\frac{2}{x}\right) = 2
\end{aligned}\right\} \Rightarrow \lim_{x \to -1} f(x) = 2$$

$$f(-1) = 2 \longrightarrow \lim_{x \to -1} f(x) = f(-1)$$

$f(x)$ er kontinuerlig i $x = -1$.