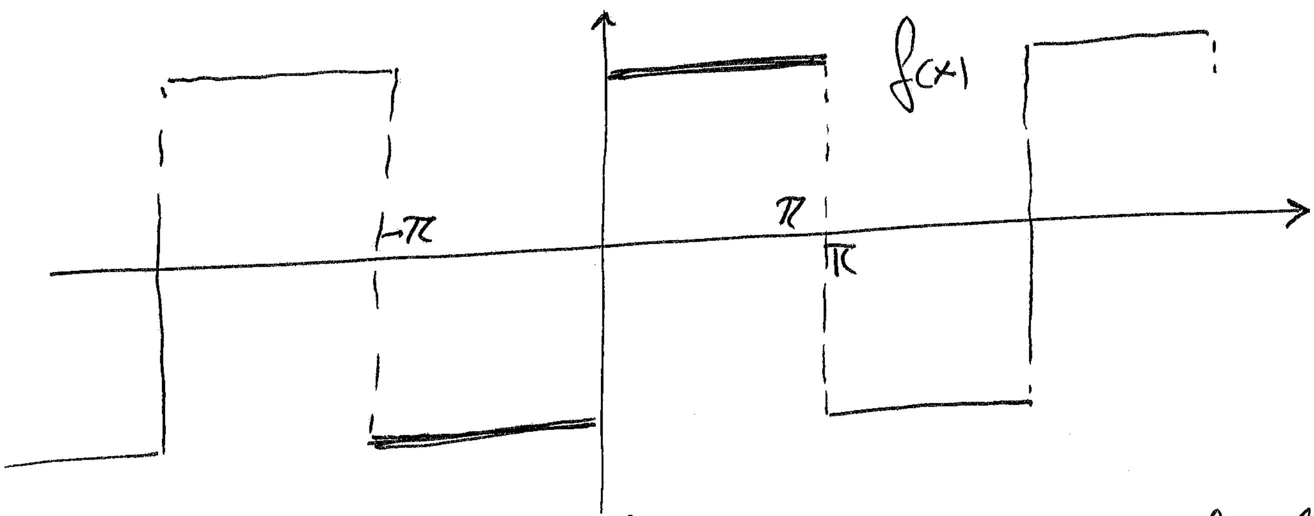
Fourier rekker

$$f(x) = \underbrace{a_0 + \sum_{n=1}^{\infty} a_n \cos(nx)}_{\text{jevn}} + \underbrace{\sum_{n=1}^{\infty} b_n \sin(nx)}_{\text{odde funksjon}}$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos(nx)\,dx$$

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\,dx$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin(nx)\,dx \;.$$



$f(x)$ er en odde funksjon så $a_n = 0$ for alle n.

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx = \frac{2}{\pi}\int_0^{\pi} 1\sin(nx)\,dx$$

(siden $\int_{-\pi}^{0} \cdots dx = \int_0^{\pi} \cdots dx$)

$$b_n = \frac{2}{\pi}\cdot \frac{-\cos(nx)}{n}\Big|_0^{\pi} = \frac{2}{\pi}\cdot\frac{1}{n}\Big[-\overbrace{\cos(\pi n)}^{(-1)^n} + \overbrace{\cos(0)}^{1}\Big]$$

$$b_n = \frac{2}{\pi\cdot n}\left(1-(-1)^n\right) = \begin{cases} 0 & n \text{ jevn} \\ \frac{4}{\pi}\cdot\frac{1}{n} & n \text{ odde.}\end{cases}$$

(2) Fourier rekken til

$$f(x) = \begin{cases} 1 & 0<x<\pi \\ -1 & -\pi<x<0 \end{cases} \qquad 2\pi\text{- periodisk}$$

er $\sum_{m=0}^{\infty} \frac{4}{\pi} \frac{\sin((2m+1)x)}{2m+1}$

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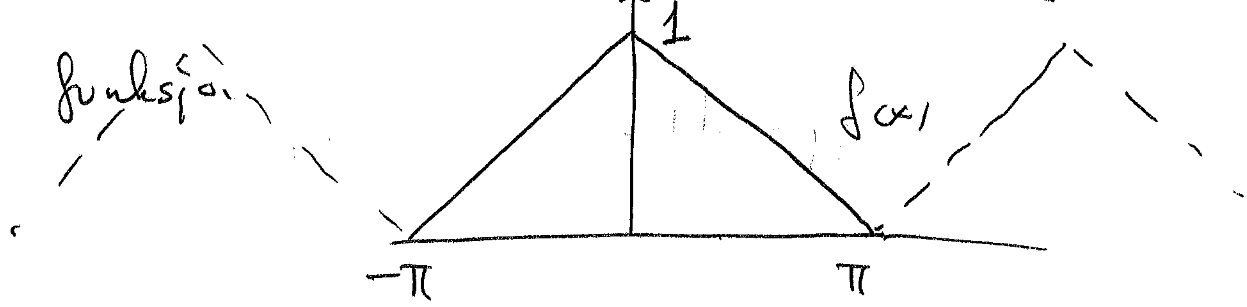
jevn funksjon



$$f(x) = 1 - \frac{x}{\pi} \quad \text{for} \quad 0 \leq x < \pi$$

$b_n = 0$ for alle n.

$$\Big(b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} \underbrace{f(x)\cdot \sin(nx)}_{\text{odde funksjon}} dx = 0 \Big)$$

$a_0 = 1/2$ (gjennomsnittsverdi)

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} \underbrace{f(x)\cdot\cos(nx)}_{\text{jevn}} dx = \frac{2}{\pi}\int_0^{\pi} f(x)\cos(nx)dx$$

$$= \frac{2}{\pi}\int_0^{\pi}\left(1-\frac{x}{\pi}\right)\cos nx\, dx$$

$$= \frac{2}{\pi}\left[\int_0^{\pi}\cos nx\,dx - \frac{1}{\pi}\int_0^{\pi} x\cdot\cos(nx)\,dx\right]$$

$$= \frac{2}{\pi}\left[\underbrace{\frac{\sin(nx)}{n}\Big|_0^{\pi}}_{0} - \frac{1}{\pi}\left[x\,\frac{\sin(nx)}{n}\Big|_0^{\pi} - \int_0^{\pi} 1\cdot\frac{\sin(nx)}{n}dx\right]\right]$$

$$= \frac{+2}{\pi^2}\left[\int_0^{\pi}\frac{\sin(nx)}{n}dx\right] = \frac{2}{\pi^2}\left[\frac{-\cos(nx)}{n^2}\Big|_0^{\pi}\right]$$

$$= \frac{2}{\pi^2\cdot n^2}\left[-\overbrace{\cos(n\pi)}^{(-1)^n} + 1\right] = \frac{4}{\pi^2 n^2}\cdot\begin{cases} 1 & n \text{ odde} \\ 0 & n \text{ jevn}\end{cases}$$

③ Fourier rekken til funksjonen

er $\frac{1}{2} + \frac{4}{\pi^2} \sum_{m=0}^{\infty} \frac{1}{(2m+1)^2} \cos((2m+1)x)$

Funksjonen er lik 0 når $x = \pi$.

(Auta Fourier rekken for $x=\pi$ er lik $f(\pi) = 0$).

$$0 = \frac{1}{2} + \frac{4}{\pi^2} \sum_{m=0}^{\infty} \frac{1}{(2m+1)^2} \underbrace{\cos((2m+1)\cdot\pi)}_{\cos(2m\pi+\pi) = \cos(\pi) = -1}$$

$$0 = \frac{1}{2} + \frac{4}{\pi^2} \sum_{m=0}^{\infty} \frac{-1}{(2m+1)^2}$$

Så $\sum_{m=0}^{\infty} \frac{1}{(2m+1)^2} = \frac{1}{2} \cdot \frac{\pi^2}{4} = \underline{\underline{\frac{\pi^2}{8}}}$

$$1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \cdots = \frac{\pi^2}{8}$$

$$\left(1 + \frac{1}{3^2} + \frac{1}{5^2} + \cdots\right)\left(1 + \frac{1}{2^2} + \frac{1}{(2^2)^2} + \frac{1}{(2^3)^2} + \cdots\right)$$

$$= 1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \cdots = \sum_{n=1}^{\infty} \frac{1}{n^2}.$$

Rekka $1 + \frac{1}{4} + \frac{1}{4^2} + \frac{1}{4^3} + \frac{1}{4^4} + \cdots = \frac{1}{1-1/4} = \frac{4}{4-1} = \underline{\frac{4}{3}}$

$$1 + x + x^2 + x^3 + \cdots = \frac{1}{1-x}$$

Derfor er $\sum_{n=1}^{\infty} \frac{1}{n^2} = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \cdots = \frac{\pi^2}{8} \cdot \frac{4}{3} = \underline{\underline{\frac{\pi^2}{6}}}$

Til slutt gikk vi gjennom to oppgaver (repetisjon)

1. Avgjør om rekken konvergerer og finn summen hvis den konvergerer $\sum_{n=2}^{\infty} 15^n / 5^{2n-1}$
2. Avgjør om rekken $\sum_{n=1}^{\infty} \cos(\pi n)/n$ konvergerer betinget/absolutt.