Binomialkoeffisienter

Litt repetisjon:

$0! = 1$ $\binom{0}{0} = 1$ $\binom{n}{0} = 1$ $\binom{n}{1} = n$ $\binom{n}{n} = 1$

$$\binom{n}{r} = \frac{n!}{(n-r)!\cdot r!} \qquad r \geq 0,\ n \geq 0$$

$$\binom{n}{r} = \binom{n}{n-r}$$

Dette gir oss

$$\binom{n}{n-1} = n$$

fordi

$$\binom{n}{n-1} = \binom{n}{n-(n-1)} = \binom{n}{1} = n$$

Husk **første kvadratsetning**:

$(a+b)^2 = a^2 + 2ab + b^2$

Vi kan bruke binomialkoeffisienter til å bestemme polynomer av n'te grad:

$(a+b)^0 = 1 = \binom{0}{0}$

$(a+b)^1 = a+b = \binom{1}{0}a + \binom{1}{1}b$

$(a+b)^2 = a^2 + 2ab + b^2 = \binom{2}{0} a^2 + \binom{2}{1} ab + \binom{2}{2} b^2$

$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$= \binom{3}{0} a^3 + \binom{3}{1}a^2b + \binom{3}{2}ab^2 + \binom{3}{3}b^3$

Her er det et mønster!

Hva blir $(a+b)^5$?

$$(a+b)^5 = \binom{5}{0}a^5 + \binom{5}{1}a^4b + \binom{5}{2}a^3b^2 + \binom{5}{3}a^2b^3 + \binom{5}{4}ab^4 + \binom{5}{5}b^5$$

1 5 10 10 5 1

$$= a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$$

<u>Generell formel:</u>

$(a+b)^n$ =

$$\binom{n}{0}a^n + \binom{n}{1}a^{n-1}\cdot b + \binom{n}{2}a^{n-2}\cdot b^2 + \cdots + \binom{n}{n-1}a\cdot b^{n-1} + \binom{n}{n}b^n$$

$$= \sum_{r=0}^{n}\binom{n}{r}a^{n-r}\cdot b^r$$