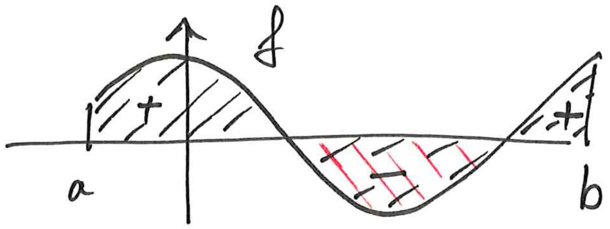
04.03.2020

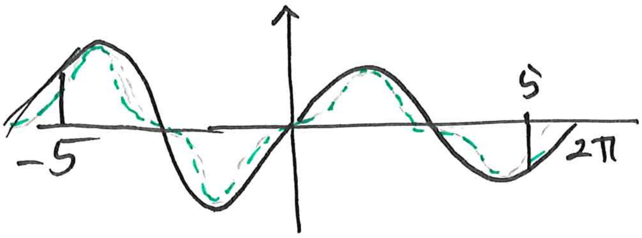
Bestemte integral $\int_a^b f(x)\,dx$ areal med fortegn mellom grafen til f(x) og x-aksen

(1)



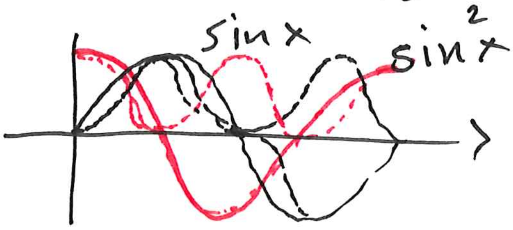
$$\int_{-5}^{5} \sin^3 x \, dx$$

$= 0$ siden det er like store areal under x-aksen som over x-aksen.



$$\int_0^{2\pi} \sin^2 x \, dx = \int_0^{2\pi} \cos^2 x \, dx$$

ser at de to integralene er like



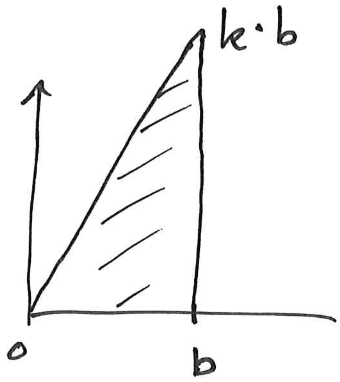
$$= \frac{1}{2}\int_0^{2\pi} (\underbrace{\sin^2 x + \cos^2 x}_{1})\, dx = \frac{2\pi \cdot 1}{2} = \underline{\pi}$$

$$\int_0^b k \cdot x \, dx$$

$$= \frac{1}{2} b \cdot k \cdot b = \underline{\frac{k}{2} b^2}$$

areal til entrekant med bredde b

høyde kb.



$k > 0$

$b > 0$

Fundamentalteoremet i kalkulus.

2 $F(x) = \int_a^x f(t)\,dt$ er en antiderivert til $f(x)$

Forutsetter at $f(x)$ er kontinuerlig

$F(a) = \int_a^a f(t)\,dt = 0$

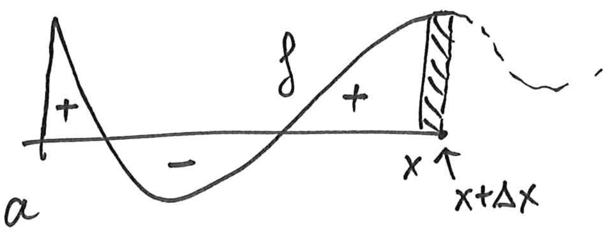
Hvis G er en antiderivert til $f(x)$

$$\int_a^x f(t)\,dt = G(x) - G(a)$$

$$\int_a^b f(x)\,dx = G(b) - G(a) = G(x)\Big|_a^b$$

hvor G er en antiderivert til $f(x)$

Forklaring.

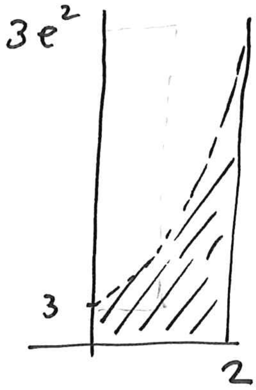


$$\lim_{\Delta x \to 0} \frac{F(x+\Delta x) - F(x)}{\Delta x} = f(x)$$

$$F'(x) = f(x)$$

oppgaver

3

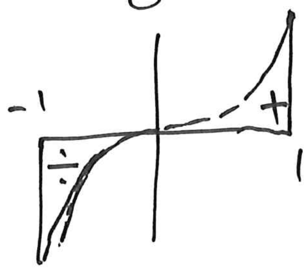


* $\int_0^2 3e^{+x}\,dx$

$3\int_0^2 e^x\,dx$

$= 3\left(e^x\Big|_0^2\right) = \underline{3(e^2-1)}$

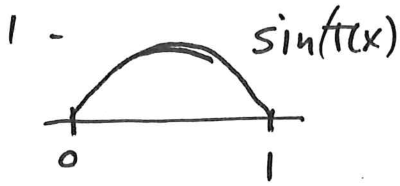
* $\int_{-1}^{1} x^3 + x^2\,dx = \underbrace{\int_{-1}^{1} x^3\,dx}_{0} + \int_{-1}^{1} x^2\,dx$



$= \int_{-1}^{1} x^2\,dx$

$= \frac{x^3}{3}\Big|_{-1}^{1} = \frac{1}{3}\left(1^3-(-1)^3\right)$

$= \underline{\underline{\frac{2}{3}}}$



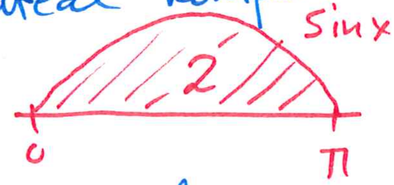
* $\int_0^1 \sin(\pi\cdot x)\,dx$

$= \frac{-\cos(\pi x)}{\pi}\Big|_0^1$

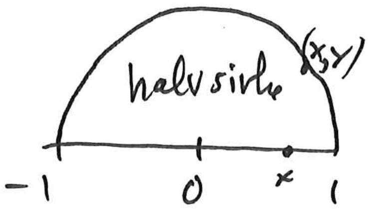
$= \frac{1}{\pi}\left(-\cos(\pi) + \cos(0)\right)$

$= \frac{1}{\pi}\left(-(-1)+1\right) = \underline{\underline{\frac{2}{\pi}}}$

lik følgende areal komprimert



med en faktor π.



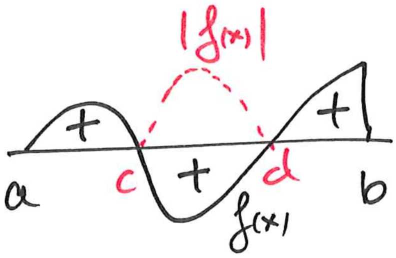
* Hva er $\int_{-1}^{1} \sqrt{1-x^2}\,dx = \frac{\pi}{2}$

$y = \sqrt{1-x^2} \Leftrightarrow y^2 = 1-x^2$

$\Leftrightarrow x^2+y^2 = 1$ så (x,y) har avstand 1 til origo.

Arealet avgrenset av grafen til $f(x)$ og x-aksen mellom $x = a$ og $x = b$ er

4 $$\int_a^b |f(x)|\, dx$$

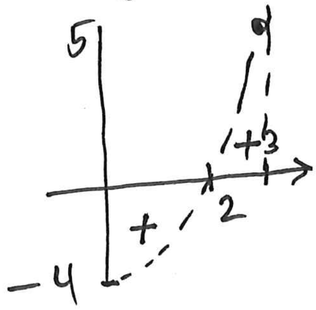


i geogebra er
$|f| = abs(f)$ ..

I figuren er

$$\int_a^b |f|\,dx = \int_a^c f\,dx + \int_d^b f\,dx - \int_c^d f\,dx$$

Eks Finn arealet avgrenset av $f(x) = x^2 - 4$ fra $x = a = 0$ til $x = b = 3$.



$f(2) = 0$

$$A = -\int_0^2 f\,dx + \int_2^3 f\,dx$$

En antiderivert til f er $F(x) = \frac{x^3}{3} - 4x$

$$A = -F(x)\Big|_0^2 + F(x)\Big|_2^3 = -\left(F(2) - F(0)\right) + F(3) - F(2)$$

$$= F(0) + F(3) - 2F(2)$$

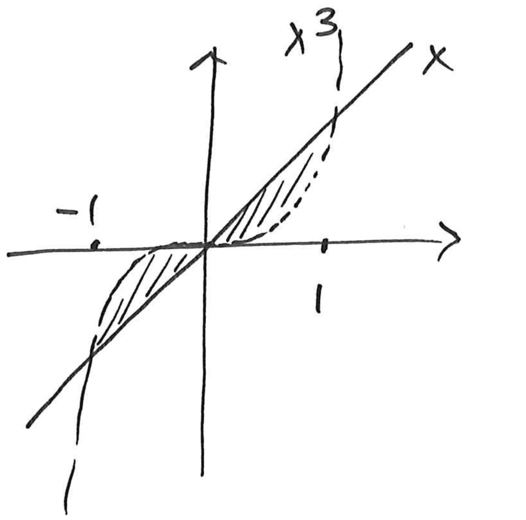
$$= 0 + \left(\frac{3^3}{3} - 4\cdot 3\right) - 2\left(\frac{2^3}{3} - 4\cdot 2\right)$$

$$= (9 - 12) - 2\cdot 8\left(\frac{1}{3} - 1\right) = \frac{2\cdot 2\cdot 8}{3} - 3$$

$$= \frac{1}{3}(32 - 9) = \underline{\underline{\frac{23}{3}}} = 7.66\ldots$$

(5) Finn arealet A avgrenset av grafene til

$f(x) = x^3$ og $g(x) = x$



Ved symmetri

$$A = 2\int_0^1 (x - x^3)\,dx$$

(to like arealer)

$$= 2\left[\frac{x^2}{2} - \frac{x^4}{4}\right]_0^1$$

$$= 2\left[\frac{1}{2} - \frac{1}{4} - 0\right] = \underline{\underline{\frac{1}{2}}}$$

Ex oppg. #3 (2012) $\qquad f(x) = x^3 - x^2 \quad g(x) = 2x$

a) Finn skjæringspunktene til f og g.

b) Finn arealet til regionen avgrenset av f og g.

a) $$x^3 - x^2 = 2x$$

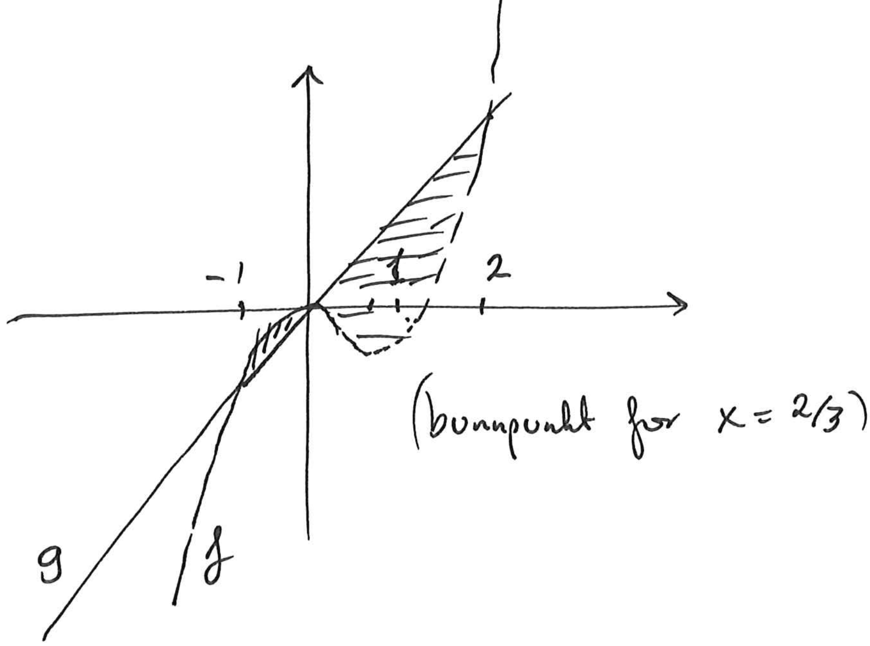
$$x^3 - x^2 - 2x = 0$$

$$x(x^2 - x - 2) = 0$$

$$x(x-2)(x+1) = 0$$

Nullpunktene er $\quad x = \underline{-1, 0, 2}$

6



$$A = \int_{-1}^{0} (f-g)\,dx + \int_{0}^{2} (g-f)\,dx$$

$$= \int_{-1}^{2} |f-g|\,dx$$

Finner en antiderivert H til $f-g = x^3 - x^2 - 2x$

$$H(x) = \frac{x^4}{4} - \frac{x^3}{3} - x^2 \quad , \quad -H = \frac{-x^4}{4} + \frac{x^3}{3} + x^2$$

$$A = H(x)\Big|_{-1}^{0} + (-H(x))\Big|_{0}^{2}$$

$$= 2\underbrace{H(0)}_{0} - H(-1) - H(2)$$

$$= (-H)(-1) + (-H)(2)$$

$$= \frac{-(-1)^4}{4} + \frac{(-1)^3}{3} + (-1)^2 + \frac{-2^4}{4} + \frac{2^3}{3} + 2^2$$

$$= \frac{-1}{4} - \frac{1}{3} + 1 - 4 + \frac{8}{3} + 4$$

$$= \frac{8}{3} - \frac{1}{3} - \frac{1}{4} + 1 = \frac{28 - 3 + 12}{12} = \underline{\underline{\frac{37}{12}}}$$

$$= 3 + \frac{1}{12}$$