6.01
2016

Fundamentalteoremet i algebra.

Alle polynomer med komplekse koeffisienter kan faktoriseres som et produkt av lineære faktorer.

$$x^2 - 1 = (x-1)(x+1)$$

$$x^2 + 1 = (x-i)(x+i)$$

$$x^2 + 9 = x^2 - 9 \cdot i^2 = x^2 - (3i)^2 = (x-3i)(x+3i)$$

$$x^2 - 2x + 5 = \underbrace{(x-1)^2}_{x^2-2x+1} - 1 + 5$$

$$= (x-1)^2 + 4$$

$$= (x-1)^2 - (2i)^2$$

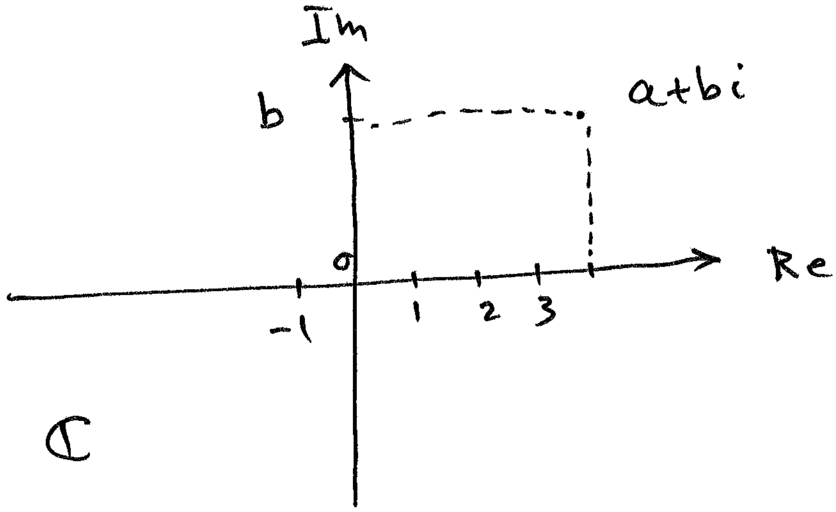
$$= (x-1+2i)(x-1-2i)$$

Fullføring av kvadrater

$$(x^2 + bx + c) = \left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 + c$$

(se 1.2 i boken til Lorentzen).

Det komplekse planet



$a+bi$ Kartesisk form.

$\mathbb{C}$

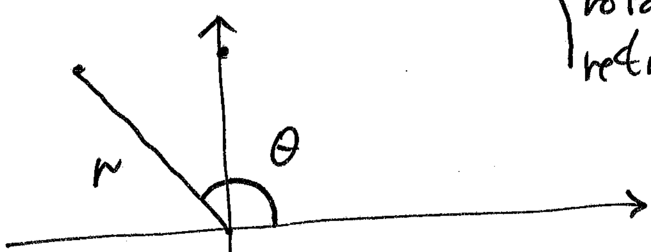
Komplekse tall $\leftrightarrow$ punkter i planet.

$\leftrightarrow$ vektorer (2-dim)

Addisjon i $\mathbb{C}$ svarer til vektoraddisjon

Polare koordinater

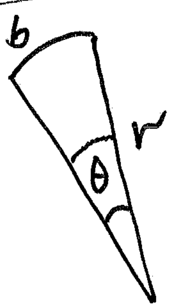
positiv rotasjons-retning



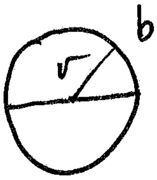
avstand $r \geq 0$

vinkel θ vilkårlig. reelt tall

Radianer



vinkel $\theta = \frac{b}{r}$



$b = 2\pi r$

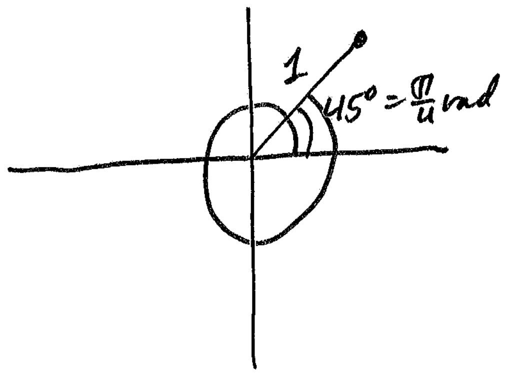
$360° = \frac{2\pi r}{r} = 2\pi$ radianer

$180° = \pi$ radianer

$90° = \frac{\pi}{2}$ rad $45° = \frac{\pi}{4}$ rad

$60° = \frac{\pi}{3}$ rad $30° = \frac{\pi}{6}$ rad   etc.

~ 1 (litt større)



r, θ beskriver samme punkt som $r, \theta + 2\pi \cdot n$ for heltall n.

Hvis $r = 0$, da ~~er punkt~~ beskriver r, θ origo for alle θ.

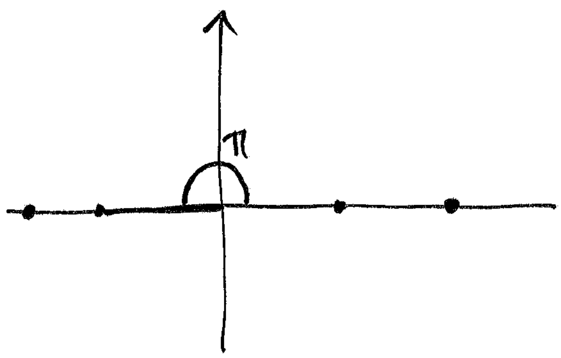
For å få en entydig vinkel må vi avgrense den til et omløp

$-\pi < \theta \leq \pi$

$0 \leq \theta < 2\pi$ etc

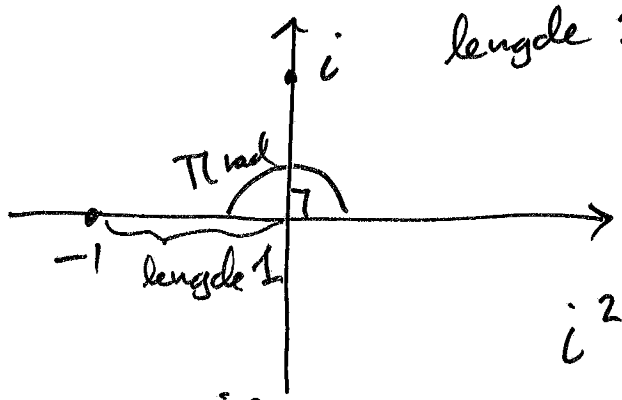
Multiklikasjon for komplekse tall er gitt ved:

* gange sammen lengdene
* legge sammen vinklene.



To negative tall:
Vinklene er π rad for begge.
sommen av vinklene er 2π

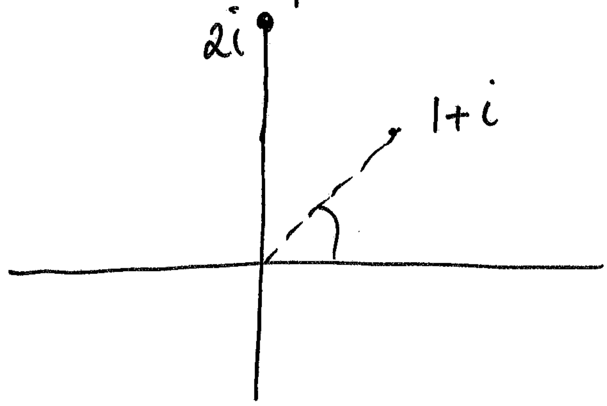
$(-2)(-3) = 2 \cdot 3 = 6$



lengde 1 , vinkel $\frac{\pi}{2}$ (90°)

i^2 lengde $1 \cdot 1$

vinkel $\frac{\pi}{2} + \frac{\pi}{2} = \pi$ rad.



$|1+i| = \sqrt{1^2+1^2} = \sqrt{2}$

vinkelen $\frac{\pi}{4}$ rad.

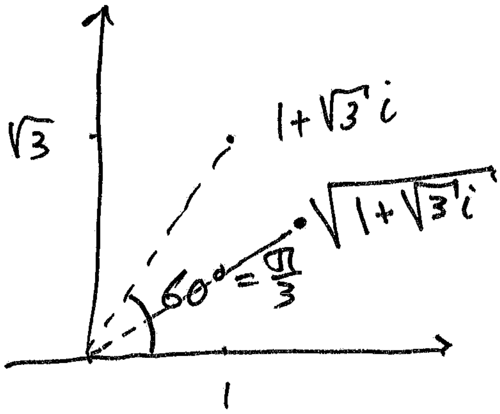
$(1+i)^2$

lengde $\sqrt{2} \cdot \sqrt{2} = 2$

vinkel $\frac{\pi}{4} + \frac{\pi}{4} = \frac{\pi}{2}$

$$(1+i)(1+i) = 1 \cdot 1 + i \cdot i + 1 \cdot i + i \cdot 1$$
$$= 1 + i^2 + 2i = \underline{2i}$$

Beskriv $\sqrt{1+\sqrt{3}i}$.



$|1+\sqrt{3}i| = \sqrt{1^2+(\sqrt{3})^2} = 2$

vinkelen $60° = \frac{\pi}{3}$ rad.

lengden til $\sqrt{1+\sqrt{3}i}$

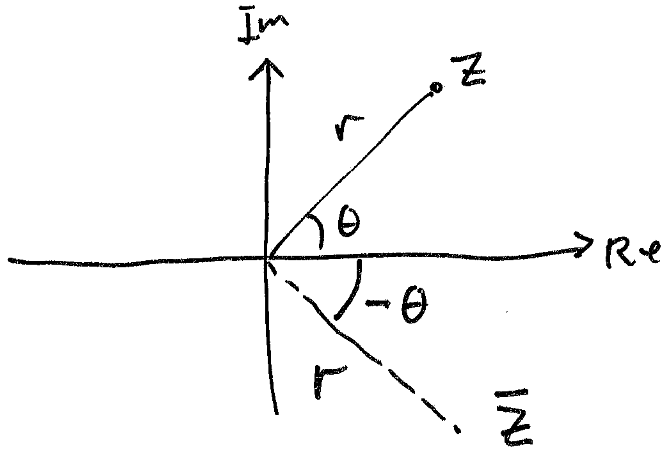
må være $\underline{\sqrt{2}}$

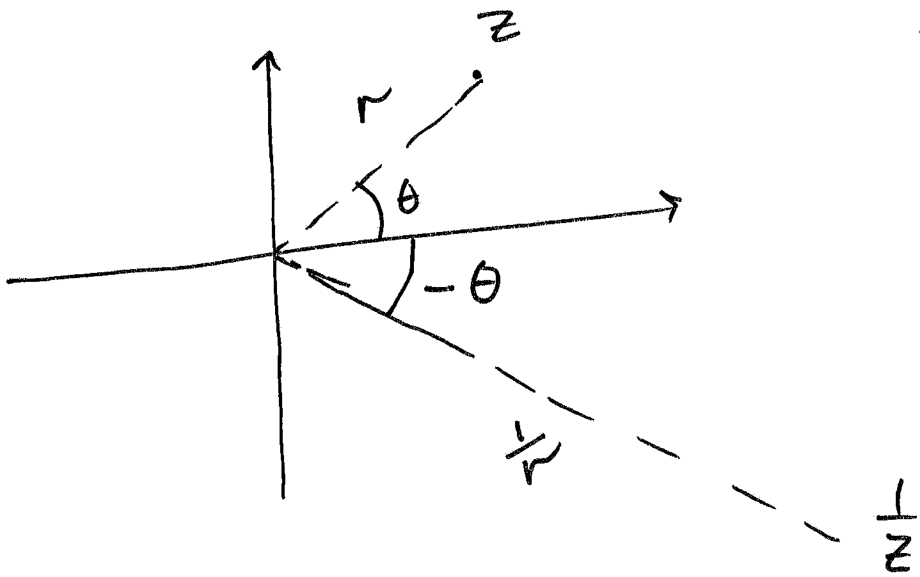
vinkelen er $\underline{\frac{\pi}{6}}$ (eller $\frac{\pi}{6} + \pi$)

$$\sqrt{1+\sqrt{3}i} = \sqrt{2}\left(\frac{\sqrt{3}}{2} + \frac{1}{2}i\right) = \underline{\sqrt{\frac{3}{2}} + \frac{1}{\sqrt{2}}i}$$

Geometrisk tolking av

$\bar{z}$ og $\frac{1}{z}$





$$\frac{1}{z} = \frac{\bar{z}}{z \cdot \bar{z}}$$