1, 2 og 3

a) $M = \begin{bmatrix} 3 & 4 \\ 5 & -6 \end{bmatrix}$

Vi utfører radoperasjon og overfører matrisen til en ekvivalent matrise på redusert trappeform. Vi gjør samtidig radoperasjonene på $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \mathbb{1}_2$ slik at vi finner inversmatrisen M^{-1} (hvis den da finnes).

$$\left[\begin{array}{cc|cc} 3 & 4 & 1 & 0 \\ 5 & -6 & 0 & 1 \end{array}\right] \cdot \frac{1}{3} \quad -5 \sim \left[\begin{array}{cc|cc} 1 & 4/3 & 1/3 & 0 \\ 0 & \underbrace{-6-\frac{20}{3}}_{-38/3} & -5/3 & 1 \end{array}\right] \cdot \frac{-3}{38}$$

$$\sim \left[\begin{array}{cc|cc} 1 & 4/3 & \frac{1}{3} & 0 \\ 0 & 1 & \frac{+5}{38} & \frac{-3}{38} \end{array}\right] \cdot \frac{-4}{3} \sim \left[\begin{array}{cc|cc} 1 & 0 & \frac{38-20}{38\cdot 3} & \frac{4}{38} \\ 0 & 1 & \frac{+5}{38} & \frac{-3}{38} \end{array}\right]$$

$$\sim \left[\begin{array}{cc|cc} 1 & 0 & \frac{6}{38} & \frac{4}{38} \\ 0 & 1 & \frac{5}{38} & \frac{-3}{38} \end{array}\right]$$

M på redusert trappeform er $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \mathbb{1}_2$ så M er inverterbar.

Inversmatrisen er $M^{-1} = \frac{1}{38}\begin{bmatrix} 6 & 4 \\ 5 & -3 \end{bmatrix}$

determinanten er $\dfrac{1}{\frac{1}{3} \cdot \frac{-3}{38}} = -38.$

Alternativt: $\det M = 3 \cdot (-6) - 5 \cdot 4 = -38.$

$$M^{-1} = \frac{1}{\det M}\begin{bmatrix} -6 & -4 \\ -5 & 3 \end{bmatrix} \ldots$$

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Løsningen til likningssystemet

$$M\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} a \\ b \end{bmatrix} \quad \text{er}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = M^{-1}\begin{bmatrix} a \\ b \end{bmatrix} = \frac{1}{38}\begin{bmatrix} 6 & 4 \\ 5 & -3 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix}$$

$$\underline{\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{38}\begin{bmatrix} 6a+4b \\ 5a-3b \end{bmatrix}}$$

b) $A = \begin{bmatrix} 1 & 1-i \\ 2i & -i \end{bmatrix}$ som i a):

$$\left[\begin{array}{cc|cc} 1 & 1-i & 1 & 0 \\ 2i & -i & 0 & 1 \end{array}\right] \cdot(-2i) \sim \left[\begin{array}{cc|cc} 1 & 1-i & 1 & 0 \\ 0 & -3i-2 & -2i & 1 \end{array}\right] \cdot\frac{-2+3i}{13}$$

(invers elementet til $-3i-2$ er $\dfrac{-2+3i}{13}$)

$$\sim \left[\begin{array}{cc|cc} 1 & 1-i & 1 & 0 \\ 0 & 1 & \frac{6+4i}{13} & \frac{-2+3i}{13} \end{array}\right] \cdot(-(1-i)) = (i-1)$$

$$\sim \left[\begin{array}{cc|cc} 1 & 0 & \frac{(i-1)(6+4i)}{13}+1 & \frac{(-2+3i)(i-1)}{13} \\ 0 & 1 & \frac{6+4i}{13} & \frac{-2+3i}{13} \end{array}\right]$$

$$\sim \left[\begin{array}{cc|cc} 1 & 0 & \frac{-10+2i}{13}+\frac{13}{13} & \frac{-1-5i}{13} \\ 0 & 1 & \frac{6+4i}{13} & \frac{-2+3i}{13} \end{array}\right]$$

$$\sim \left[\begin{array}{cc|cc} 1 & 0 & \frac{3+2i}{13} & \frac{-1-5i}{13} \\ 0 & 1 & \frac{6+4i}{13} & \frac{-2+3i}{13} \end{array}\right]$$

A på redusert trappeform er I_2.

Invers matrisen til A er

$$A^{-1} = \frac{1}{13}\begin{bmatrix} 3+2i & -1-5i \\ 6+4i & -2+3i \end{bmatrix}$$

Determinanten er $\det A = \underline{\underline{-3i-2}}$

Løsningen til likningssystemet

$$A\begin{bmatrix} z \\ w \end{bmatrix} = \begin{bmatrix} 1 \\ i \end{bmatrix}$$ er

$$\begin{bmatrix} z \\ w \end{bmatrix} = A^{-1}\begin{bmatrix} 1 \\ i \end{bmatrix} = \frac{1}{13}\begin{bmatrix} 3+2i + i(-1-5i) \\ 6+4i + i(-2+3i) \end{bmatrix}$$

$$\begin{bmatrix} z \\ w \end{bmatrix} = \underline{\underline{\frac{1}{13}\begin{bmatrix} 8+i \\ 3+2i \end{bmatrix}}}$$

c) $C = \begin{bmatrix} 8 & 4 & 16 \\ 6 & 17 & 0 \\ 5 & -15 & 25 \end{bmatrix}$

$$\left[\begin{array}{ccc|ccc} 8 & 4 & 16 & 1 & 0 & 0 \\ 6 & 17 & 0 & 0 & 1 & 0 \\ 5 & -15 & 25 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} \cdot \frac{1}{4} \\ \\ \cdot \frac{1}{5} \end{array}$$

$$\sim \left[\begin{array}{ccc|ccc} 2 & 1 & 4 & \frac{1}{4} & 0 & 0 \\ 6 & 17 & 0 & 0 & 1 & 0 \\ 1 & -3 & 5 & 0 & 0 & \frac{1}{5} \end{array}\right] \begin{array}{l} -2 \\ -6 \end{array}$$

$$\sim \left[\begin{array}{ccc|ccc} 0 & 7 & 4-10 & \frac{1}{4} & 0 & -2/5 \\ 0 & 17+18 & -30 & 0 & 1 & -6/5 \\ 1 & -3 & 5 & 0 & 0 & \frac{1}{5} \end{array}\right] \text{bytte}$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & -3 & 5 & 0 & 0 & 1/5 \\ 0 & 35 & -30 & 0 & 1 & -6/5 \\ 0 & 7 & -6 & 1/4 & 0 & -2/5 \end{array}\right] \begin{array}{l} \\ \cdot 1/5 \\ -1 \end{array}$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & -3 & 5 & 0 & 0 & 1/5 \\ 0 & 7 & -6 & 0 & 1/5 & -6/25 \\ 0 & 0 & 0 & 1/4 & -1/5 & -4/25 \end{array}\right]$$ trappeform.

Matrisen er ikke inverterbar og determinanten er lik 0.

Dropper å utføre rad. op. på matrisen til venstre.

$$\begin{bmatrix} 1 & -3 & 5 \\ 0 & 7 & -6 \\ 0 & 0 & 0 \end{bmatrix} \begin{array}{l} \\ \cdot \frac{1}{7} \end{array} \; 3 \sim \begin{bmatrix} 1 & 0 & 17/7 \\ 0 & 1 & -6/7 \\ 0 & 0 & 0 \end{bmatrix}$$

2c)

$$\begin{bmatrix} 8 & 4 & 16 \\ 6 & 17 & 0 \\ 5 & -15 & 25 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ a \\ -1 \end{bmatrix}$$

Totalmatrise

$$\left[\begin{array}{ccc|c} 8 & 4 & 16 & 2 \\ 6 & 17 & 0 & a \\ 5 & -15 & 25 & -1 \end{array}\right] \begin{array}{l} \cdot \frac{1}{2} \\ \\ \cdot \frac{1}{5} \end{array} \sim \left[\begin{array}{ccc|c} 4 & 2 & 8 & 1 \\ 6 & 17 & 0 & a \\ 1 & -3 & 5 & -1/5 \end{array}\right] \begin{array}{l} -4 \\ -6 \\ \\ \end{array}$$

$$\sim \left[\begin{array}{ccc|c} 0 & 14 & -12 & 9/5 \\ 0 & 35 & -30 & a + 6/5 \\ 1 & -3 & 5 & -1/5 \end{array}\right] \begin{array}{l} \cdot \frac{1}{2} \\ \cdot \frac{1}{5} \\ \\ \end{array}$$

$$\sim \left[\begin{array}{ccc|c} 1 & -3 & 5 & -1/5 \\ 0 & 7 & -6 & \frac{a}{5} + \frac{6}{25} \\ 0 & 7 & -6 & 9/10 \end{array}\right] \begin{array}{l} \\ \\ -1 \end{array}$$

$$\sim \left[\begin{array}{ccc|c} 1 & -3 & 5 & -1/5 \\ 0 & 7 & -6 & \frac{a}{5} + \frac{6}{25} \\ 0 & 0 & 0 & -\frac{a}{5} + \frac{33}{50} \end{array}\right]$$

Ingen løsning hvis $\quad -\frac{a}{5} + \frac{33}{50} = \frac{-10a + 33}{50} \neq 0$

$$\underline{a \neq 33/10 = 3.3}$$

Hvis $a = 3.3$ $\quad$ (så $\frac{a}{5} + \frac{6}{25} = \frac{33+12}{50} = \frac{45}{50} = \frac{9}{10}$)

så har vi følgende løsning parametrisert av z:

$$\underline{y = \frac{1}{7}\left(\frac{9}{10} + 6z\right)}, \qquad x = -\frac{1}{5} + 3y - 5z = \underline{\frac{13}{70} - \frac{17}{7}z}$$

d) $D = \begin{bmatrix} 12 & 4 & 19 & 2 & 0 \\ 3 & 1 & 5 & 0 & -2 \end{bmatrix}$

Matrisen er ikke kvadratisk og har derfor ingen determinant eller inversmatrise.

Vi overfører til redusert trappeform (likningssystemet i 3d)

$$\left[\begin{array}{ccccc|c} 12 & 4 & 19 & 2 & 0 & 2 \\ 3 & 1 & 5 & 0 & -2 & 0 \end{array}\right] \begin{array}{l} \text{Bytter} \\ \cdot(-4) \end{array}$$

$$\sim \left[\begin{array}{ccccc|c} 3 & 1 & 5 & 0 & -2 & 0 \\ 0 & 0 & -1 & 2 & 8 & 2 \end{array}\right] \; 5$$

$$\sim \left[\begin{array}{ccccc|c} 3 & 1 & 0 & 10 & 38 & 10 \\ 0 & 0 & -1 & 2 & 8 & 2 \end{array}\right] \begin{array}{l} \cdot 1/3 \\ \cdot(-1) \end{array}$$

$$\sim \left[\begin{array}{ccccc|c} 1 & 1/3 & 0 & 10/3 & 38/3 & 10/3 \\ 0 & 0 & 1 & -2 & -8 & -2 \end{array}\right]$$

matrisen D på redusert trappeform.

Løsningene til likningssystemet er

$$x_1 = 10/3 - x_2/3 - 10x_4/3 - 38x_5/3$$

$$x_3 = -2 + 2x_4 + 8x_5$$

x_2, x_4, x_5 frie variabler.