LF øving til 22.08.2016

$$z \cdot w = (3-4i)(1+3i)$$

$$= 3\cdot 1 + 3(3i) + (-4i)\cdot 1 + (-4i)(3i)$$

$$= 3 + 9i - 4i - 12\cdot i^2$$

$$= 3 - 12(-1) + (9-4)i$$

$$= \underline{15 + 5i}$$

$$z^{-1} = \frac{\bar{z}}{|z|^2}$$

$$z = a + bi$$

$$\bar{z} = a - bi$$

$$|z| = \sqrt{a^2+b^2}$$

$$z \cdot \bar{z} = (a+bi)(a-ib) = a^2 - b^2 i^2$$

$$= a^2 + b^2 = |z|^2$$

$$w/z = w \cdot z^{-1} = (1+3i) \cdot \frac{(3+4i)}{3^2+4^2}$$

$$= \frac{(1+3i)(3+4i)}{25} = \frac{3 + 12(i^2) + 9i + 4i}{25}$$

$$= \frac{3-12 + (9+4)i}{25}$$

$$= \underline{\underline{\frac{-9+13i}{25}}}$$

2. $2i\,z = 4$ dele med $2i$

(gange med $(2i)^{-1} = \frac{1}{2i}$)

$$\frac{1}{2i} = \left(\frac{1}{2}\right)\cdot\frac{1}{i} = \frac{1}{2}(-i)$$

$$= \frac{\overline{2i}}{|2i|^2} = \frac{-2i}{2^2} = \underline{\frac{-i}{2}}$$

$$(2i)^{-1}\cdot 2i\,z = (2i)^{-1}4$$

$$z = \frac{-i}{2}\cdot 4 = \underline{-2i}$$

$$(1+i)z - (1.3+i) = 0$$

$$(1+i)z = (1.3+i)$$

$$z = (1+i)^{-1}(1.3+i)$$

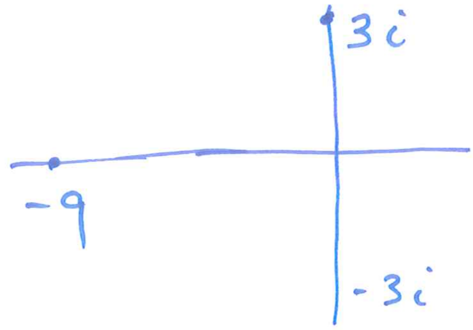
$$= \frac{(1-i)}{2}(1.3+i)$$

$$= \frac{1}{2}\left(1\cdot 1.3 + i - 1.3i - i^2\right)$$

$$= \frac{1}{2}(2.3 - 0.3i)$$

$$= \underline{1.15 - 0.15i}$$

3



$$z^2 + 9 = 0$$

$$z^2 = -9$$

$$z = \pm\sqrt{-9}$$

$$\left(= \pm\sqrt{9} \cdot \sqrt{-1} \right)$$

$$= \pm\sqrt{9} \cdot i = \underline{\pm 3i}$$

$$z^2 + z + 1 = 0$$

(abc formelen $\quad az^2 + bz + c = 0$

Løsningene (røttene) er

$$z = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$)

abc-formelen gir røttene:

$$z = \frac{-1 \pm \sqrt{1^2 - 4 \cdot 1 \cdot 1}}{2 \cdot 1} = \frac{-1 \pm \sqrt{3}\, i}{2}$$

$$\underline{z = \frac{-1 + \sqrt{3}\, i}{2}} \qquad \underline{z = \frac{-1 - \sqrt{3}\, i}{2}}$$

$\longrightarrow$

Dei neste to sidene repeterer fullføring av kvadratet og benytter teknikken til å utlede abc-formelen

Fullføring av kvadrater

$$x^2 + x + 1 = 0$$

$$(x+d)^2 = x^2 + 2d\,x + d^2$$

velger d slik at

$$x^2 + 2d\,x = x^2 + x \,.$$

så $2d = 1$, $d = \frac{1}{2}$.

$$\left(x+\tfrac{1}{2}\right)^2 = x^2 + x + \left(\tfrac{1}{2}\right)^2 .$$

Vi forenkler likningen

$$x^2 + x + 1 = 0$$

$$\left(x+\tfrac{1}{2}\right)^2 - \left(\tfrac{1}{2}\right)^2 + 1 = 0$$

$$\left(x+\tfrac{1}{2}\right)^2 = \frac{-3}{4}$$

$$x + \tfrac{1}{2} = \pm\sqrt{\frac{-3}{4}} = \pm\frac{\sqrt{3}\,i}{2}$$

$$x = \frac{-1}{2} \pm \frac{\sqrt{3}}{2}\,i$$

Utleding av abc-formel.

Fullfører kvadratet og forenkler likningen

$$ax^2 + bx + c \; (=0)$$

$$= a\left(x^2 + \frac{b}{a}\cdot x + \frac{c}{a}\right)$$

$$= a\left[\left(x+\frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2 + \frac{c}{a}\right]$$

$$= a\left[\left(x+\frac{b}{2a}\right)^2 + \frac{4ac}{4a\cdot a} - \frac{b^2}{4a^2}\right]$$

$$= a\left[\left(x+\frac{b}{2a}\right)^2 + \frac{4ac-b^2}{4a^2}\right]$$

Fullføring av kvadratet

2gradslikningen (a kanseleres)

$$\cancel{a}\quad \left(x+\frac{b}{2a}\right)^2 = -\left(\frac{4ac-b^2}{4a^2}\right) = \frac{b^2-4ac}{4a^2}$$

$$x+\frac{b}{2a} = \pm\sqrt{\frac{b^2-4ac}{4a^2}}$$

$$\left(\sqrt{4a^2} = 2\sqrt{a^2} = 2|a|\right)$$

$$x = \frac{-b}{2a} \pm \frac{\sqrt{b^2-4ac}}{2a}$$

3 Faktoriseringen til

$$ax^2+bx+c$$

er $a(x-r_1)(x-r_2)$ for to røtter r_1 og r_2

$a(x-r)^2$ for én dobbel rot r.

Vi får derfor

$$z^2+9=(z-3i)(z+3i) \quad \text{og}$$

$$z^2+z+1 = \left(z+\tfrac{1}{2}-\tfrac{\sqrt{3}}{2}i\right)\left(z+\tfrac{1}{2}+\tfrac{\sqrt{3}}{2}i\right).$$

4

$z = 3-4i$ $\qquad$ $w = 5+12i$

$|z| = \sqrt{3^2+(-4)^2} = 5$

$|w| = \sqrt{5^2+12^2}$

$= \sqrt{25+144}$

$= \sqrt{169}$

$|w| = 13$

$\left((10+2)^2 = 10^2 + 2(2\cdot 10) + 2^2 = 144\right)$

$(10+3)^2 \ldots$

$z\cdot w = 3\cdot 5+(-4)\cdot 12 i^2 + i(3\cdot 12 - 4\cdot 5)$

$= (15+48) + i(36-20)$

$z\cdot w = 63 + 16i$

$|z\cdot w| = \sqrt{(63)^2 + (16)^2}$

$= \sqrt{(60+3)^2 + (2^4)^2}$ $\qquad$ $\left((2^4)^2 = 2^8\right)$

$= \sqrt{3600 + \underbrace{360 + \underbrace{9 + 256}_{265}}_{625}}$

$= \sqrt{4225}$ $= \sqrt{65^2}$

$= \underline{\underline{65}}$

$\left(65^2 = (60+5)^2 \quad 3600 + 600 + 25 = 4225\right)$

Så $|z|\cdot|w| = |z\cdot w|$ i dette tilfellet.