28.01.2015

(1)

# Derivasjon

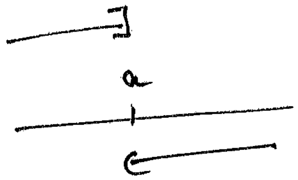
Definisjon av den deriverte

$$\frac{df}{dx}(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{\Delta x \to 0} \frac{\Delta f}{\Delta x}$$

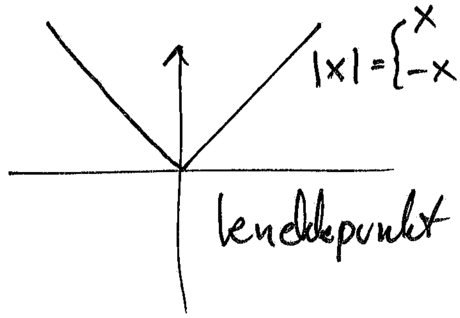
(Leibniz notasjon)

$$\Delta f = f(x+h) - f(x).$$

Den deriverte til $f(x)$ trenger ikke eksistere i alle punkt i def. mengden til $f(x)$.




ikke deriverbar i $x=a$.




$$|x| = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$$

$|x|$ er ikke deriverbar i 0: $\frac{|0+h|-|0|}{h} = \frac{|h|}{h}$

$\lim_{h\to 0^+} \frac{|h|}{h} = 1$ og $\lim_{h\to 0^-} \frac{|h|}{h} = -1$. $\lim_{h\to 0} \frac{|h|}{h}$ eksisterer ikkje.

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Resultat: Hvis $f(x)$ er deriverbar i $a$ så er $f(x)$ kontinuerlig i $a$.

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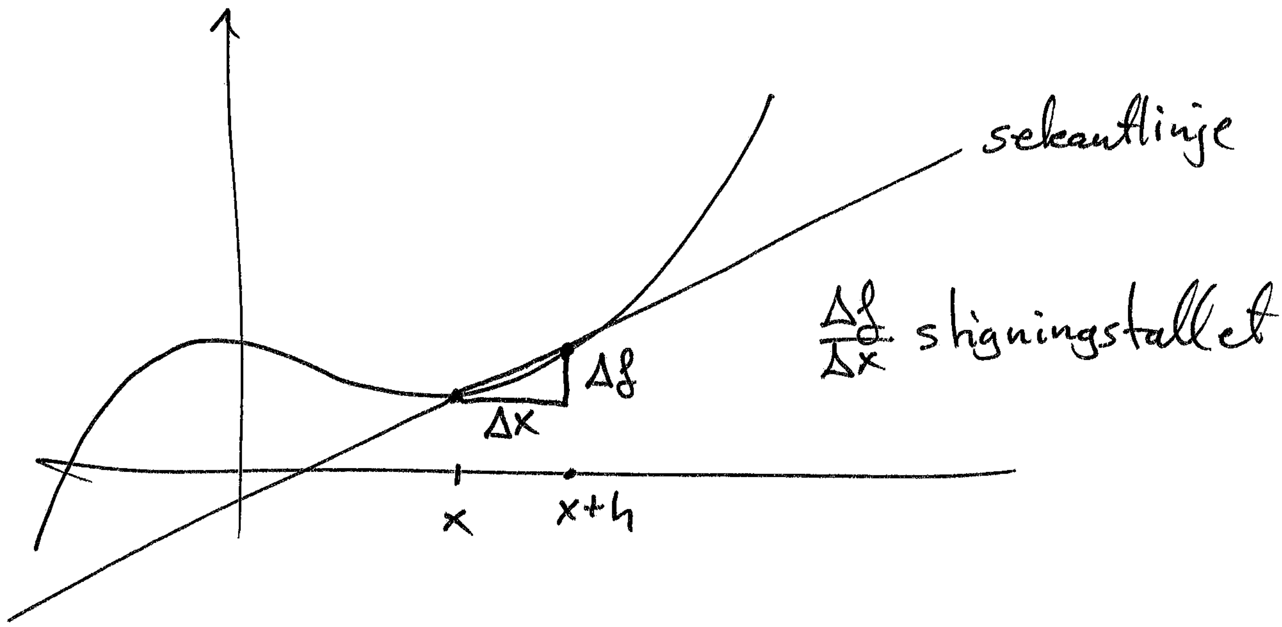
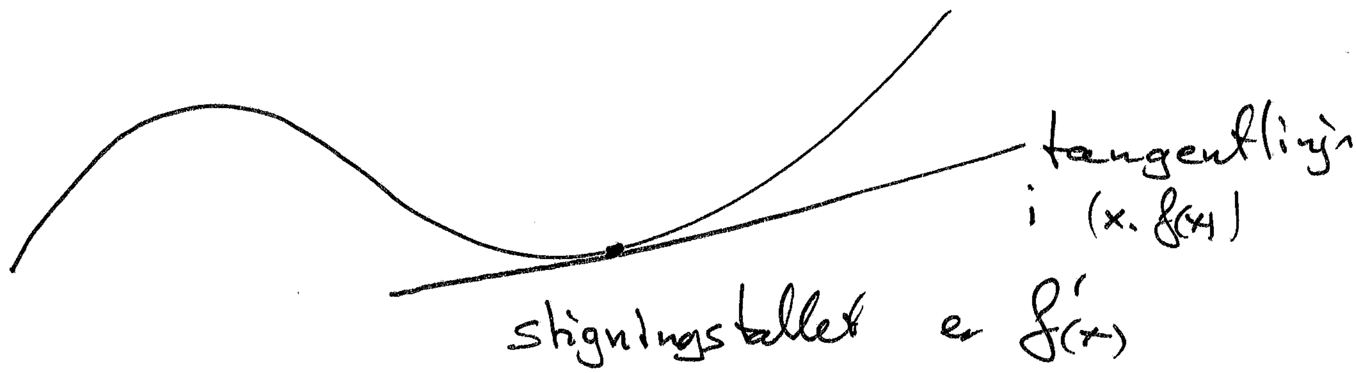
Den deriverte til noen funksjoner:

$$\frac{d(ax+b)}{dx} = (ax+b)' = a$$

$$\left( \lim_{h\to 0} \frac{a(x+h)+b-(ax+b)}{h} = \lim_{h\to 0} \frac{a\cdot h}{h} = a \right)$$

$\frac{d}{dx}(2\pi) = 0$ etc. Den deriverte til en konstant funksjon er identisk lik 0.


sekantlinje
$\frac{\Delta f}{\Delta x}$ stigningstallet
$\Delta f$
$\Delta x$
$x$
$x+h$


tangentlinje
i $(x, f(x))$
stigningstallet er $f'(x)$

(2) $\frac{d}{dx} x^2 = 2x$

$\frac{d}{dx} x^3 = 3x^2$ : $\lim_{h\to 0} \frac{(x+h)^3 - x^3}{h} = \lim_{h\to 0} \frac{x^3 + 3x^2 \cdot h + 3xh^2 + h^3 - x^3}{h}$

$\vdots$ $= \lim_{h\to 0} 3x^2 + 3x \cdot h + h^2 = \underline{3x^2}$

$\frac{d}{dx} x^n = n \cdot x^{n-1}$.

$n = 1, 2, 3 \ldots$

↳ Vi beviser dette ved å benytte den utvida konjugatsetningen : $b^n - a^n = (b-a)(b^{n-1} + b^{n-2} \cdot a + \ldots + b \cdot a^{n-2} + a^{n-1})$

$$\frac{d}{dx} x^n = \lim_{h\to 0} \frac{(x+h)^n - x^n}{h} = \lim_{h\to 0} \frac{\overbrace{(x+h-x)}^{h}((x+h)^{n-1} + (x+h)^{n-2} \cdot x + \cdots + (x+h) \cdot x^{n-2} + x^{n-1})}{h}$$

$$= \lim_{h\to 0} (x+h)^{n-1} + (x+h)^{n-2} \cdot x + \cdots + (x+h)x^{n-2} + x^{n-1}$$

(n ledd)

$$= \underline{n \cdot x^{n-1}}$$

For eksempel $\frac{d}{dx} x^{98} = 98 \cdot x^{97}$.

$\frac{d}{dx} \sqrt{x} = \frac{1}{2\sqrt{x}}$ $\qquad \frac{d}{dx} x^{1/2} = \frac{1}{2} x^{(1/2)-1} = \frac{1}{2x^{1/2}}$

Fra definisjonen:

$$\lim_{h\to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} = \lim_{h\to 0} \frac{(\sqrt{x+h} - \sqrt{x})(\sqrt{x+h} + \sqrt{x})}{h \cdot (\sqrt{x+h} + \sqrt{x})}$$

$$= \lim_{h\to 0} \frac{(x+h) - x}{h(\sqrt{x+h} + \sqrt{x})} = \lim_{h\to 0} \frac{1}{\sqrt{x+h} + \sqrt{x}} = \underline{\frac{1}{2\sqrt{x}}} \qquad x > 0$$

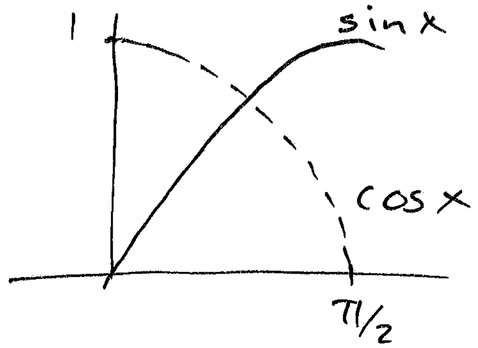
$$\frac{d}{dx} \sqrt[n]{x} = \frac{d}{dx} x^{1/n} = \frac{1}{n} \cdot x^{\frac{1}{n} - 1} = \frac{1}{n} x^{\frac{1-n}{n}} = \frac{1}{n(\sqrt[n]{x})^{n-1}}$$

$$\left( \frac{1}{n} (x^{\frac{1}{n}})^{1-n} = \frac{1}{n} ((x^{\frac{1}{n}})^{n-1})^{-1} = \frac{1}{n (x^{1/n})^{n-1}} \right)$$

bevis er tilsvarende beviset for $\frac{d}{dx} x^n = n x^{n-1}$, n naturlig.

$$\frac{d}{dx}\sin x = \cos x$$

$$\frac{d}{dx}\cos x = -\sin x$$

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$$\frac{d}{dx}\sin x = \lim_{h\to 0} \frac{\sin(x+h) - \sin(x)}{h}$$ addisjonsformelen

$$= \lim_{h\to 0} \frac{\sin(x)\cdot\cos(h) + \sin(h)\cdot\cos(x) - \sin(x)}{h}$$

$$= \sin(x) \underbrace{\lim_{h\to 0} \frac{\cos(h) - 1}{h}}_{0} + \cos(x) \underbrace{\lim_{h\to 0} \frac{\sin(h)}{h}}_{1}$$

$$= \cos(x)$$

(her har vi benyttet grensesetningene

$$\lim_{x\to a} k\cdot g(x) = k\cdot \lim_{x\to a} g(x)$$ k konstant )

Tilsvarende bevises $\frac{d}{dx}\cos x = -\sin x$.

Hva er $\frac{d}{dx}a^x$ ?

$$\lim_{h\to 0} \frac{a^{x+h} - a^x}{h} = \lim_{h\to 0} \frac{a^x\cdot a^h - a^x}{h}$$

$$= \lim_{h\to 0} a^x\cdot\frac{a^h - 1}{h} = a^x\cdot\underbrace{\left(\lim_{h\to 0}\frac{a^h-1}{h}\right)}_{\text{grensen eksisterer for } a>0.}$$

Det finnes et tall $e = 2{,}71828\ldots$ slik at grensen $\lim_{h\to 0} \frac{e^h - 1}{h} = 1$. e kalles Eulers tall.

$$\frac{d}{dx}(e^x) = e^x.$$

Tangentlinjen til $f(x)$ for $x = a$ er linjen med stigningstall $f'(a)$ som går gjennom $(a, f(a))$.

(4) $$y = f'(a)(x-a) + f(a).$$

Stigningstallet til sekanten til $f$ gjennom $(a, f(a))$ og $(a+h, f(a+h))$ er

$$\frac{f(a+h) - f(a)}{h}.$$

"Nummerisk deriverte" $\dfrac{f(a+h) - f(a-h)}{2h}$

konvergerer mye raskere mot $f'(a) = \frac{df}{dx}(a)$

enn $\dfrac{f(a+h) - f(a)}{h}$.

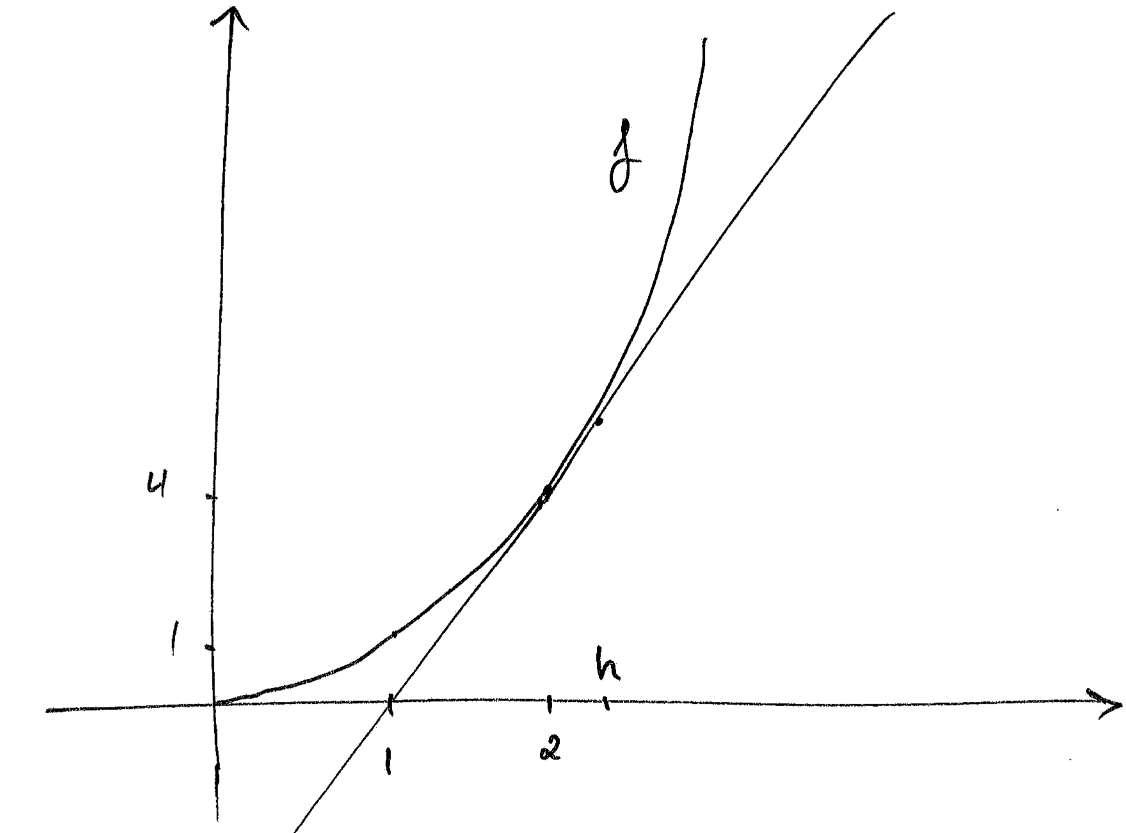
Illustrere dette ved bruk av Geogebra.

Tangentlinjen i $(a, f(a))$ er den lineære tilnærmingen til $f$ rundt $x = a$

$$f(a+h) \sim f(a) + f'(a)(\cdot h)$$ $(h = x-a)$ liten.

$$f(x) \sim f(a) + f'(a)(x-a)$$

(eksempler side 6)




tangentlinje til $y = x^2$ i $(2,4)$

$y'_{(x)} = 2x$

i $x=2$ er stigningstallet $4$

Tangentlinjen er gitt ved

$$y = 4(x-2) + 4 = \underline{4x - 4}$$

1. ordens tilnærming til $y_{(x)}$ nær $x=2$ er gitt ved tangentlinjen

$$y \sim 4(x-1).$$

Høyere ordens deriverte

15 $\frac{d}{dx} f(x)$ er en funksjon. Vi kan derfor gjenta derivasjonen.

$$\frac{d}{dx}\left(\frac{d}{dx} f\right) = \frac{d^2}{dx^2} f = (f'(x))' = f''(x)$$

n ganger: $\frac{d^n}{dx^n} f(x) = f^{(n)}(x)$ ← må bruke parenteser.

$f = x^3$ $\quad f' = 3x^2$ $\quad f'' = 3(x^2)' = 3 \cdot 2 \cdot x = 6x$

$f''' = 6$ $\quad f^{(n)} \equiv 0 \quad n \geq 4.$

$f = x^n$ $\quad f' = n x^{n-1}$ $\quad f^{(2)} = n(x^{n-1})' = n(n-1)x^{n-2}$ $(n \geq 2)$

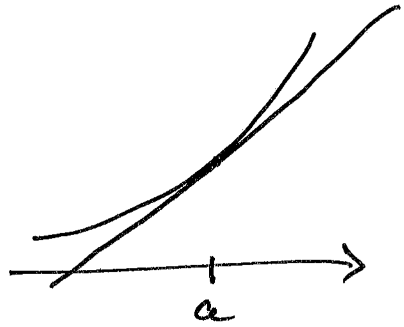
$$f^{(n)} = n(n-1)(n-2) \cdots 2 \cdot 1 = n!$$

Hva er $f^{(m)}$ for $m < n$?

$f^{(m)} \equiv 0$ når $m > n$.

29.01

Tilnærmer $f(x)$ med tangentlinjen (for $x=a$) for $x$ nær $a$.




Dette kalles den lineære tilnærmingen til $f(x)$ rundt $a$.

$$f(x) \sim f'(a)(x-a) + f(a).$$

$$\left( \begin{aligned} \Delta f &= f(x) - f(a) \\ &\sim f'(a)(x-a) \\ &= f'(a)\,\Delta x \end{aligned} \right)$$

(6)

$$\sqrt{x} \sim \sqrt{a} + \frac{1}{2\sqrt{a}}(x-a)$$

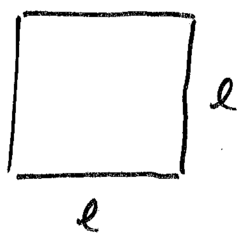
Eksempel $x = 24$ $\quad a = 25 = 5^2$

$$\sqrt{24} \sim \sqrt{25} + \frac{1}{2\sqrt{25}}(24-25)$$

$$5 + \frac{1}{10}(-1) \quad \text{så}$$

$$\sqrt{24} \sim 4.9 \qquad (\sqrt{24} = 4.898979\ldots)$$


Anta lengden har en usikkerhet på 1% $\quad \left|\frac{\Delta l}{l}\right| < 1\%$




Areal $A = l^2$

$$\Delta A = 2l \cdot \Delta l$$

$$\frac{\Delta A}{A} = \frac{2l\,\Delta l}{l^2} = 2\frac{\Delta l}{l}$$

$$\left|\frac{\Delta A}{A}\right| < 2\%$$

4.02.2015.

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# Derivasjonsreglene

$$\left.\begin{array}{l}(f+g)' = f' + g' \\ c \text{ konstant} \quad (cf)' = c\cdot f'\end{array}\right\}\ \text{derivasjon er lineær}$$

$$(f\cdot g)' = f'\cdot g + f\cdot g' \quad \text{(produktregelen)}$$

$$\left(f(u(x))\right)' = f'(u(x))\cdot u'(x) \quad \text{(kjerneregelen)}$$

$$\frac{d\, f\circ u}{dx} = \frac{df}{du}\cdot\frac{du}{dx}$$

Eksempler: * $(3\sin x - \pi\cdot\cos x + 2)'$

$$= (3\cdot\sin x)' + (-\pi\cos x)' + (2)'$$
$$= 3(\sin x)' - \pi(\cos x)' + (2)'$$
$$= 3\cos x - \pi(-\sin x) + 0 = \underline{3\cos x + \pi\sin x}$$

* $f(x) = (x-1)^4(x+2)^7$ mye arbeid å gange ut! Ungår det.

$$f'(x) = \underbrace{\left((x-1)^4\right)'}_{4(x-1)^3\cdot(x-1)'}(x+2)^7 + (x-1)^4\cdot\underbrace{\left((x+2)^7\right)'}_{7(x+2)^6(x+2)'}$$
$$= 4(x-1)^3(x+2)^7 + 7(x-1)^4\cdot(x+2)^6$$
$$= (x-1)^3(x+2)^6\left[4(x+2) + 7(x-1)\right]$$
$$= \underline{(x-1)^3(x+2)^6(11x+1)}$$

* $x^2\cdot\sin x$

$$(x^2\cdot\sin x)' = (x^2)'\sin x + x^2(\sin x)'$$
$$= \underline{2x\sin x + x^2\cos x}$$

$$f(x) = \cos^3(2x-3) = \left(\cos(2x-3)\right)^3$$

$$f'(x) = 3\cos^2(2x-3) \cdot \left(\cos(2x-3)\right)'$$

$$= 3\cos^2(2x-3) \cdot \left(-\sin(2x-3) \cdot \underbrace{(2x-3)'}_{\underbrace{2(x)'-(3)'}_{2}}\right)$$

$$f'(x) \underline{= -6\cos^2(2x-3) \cdot \sin(2x-3)}$$

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produktregel for $x^n$.

$$(x^7 \cdot x^3)' = (x^{10})' = 10x^9$$

1) $(x^7)' \cdot (x^3)' = 7 \cdot x^6 \cdot 3 \cdot x^2 = 21 \cdot x^8$ $\times$

2) $(x^7)' \cdot x^3 + x^7 \cdot (x^3)' = 7x^6 \cdot x^3 + x^7 \cdot 3x^2$

$= 7x^9 + 3x^9 = \underline{10x^9}$ ✓

Kjerneregelen

$$(x^3)^5 = x^{15}$$

$$(x^{15})' = 15x^{14}$$

$$\left((x^3)^5\right)' = 5(x^3)^4 \cdot (x^3)'$$

$= 5 \cdot x^{12} \cdot 3 \cdot x^2 = 15\underline{x^{14}}$ ✓

* $f(x) = \sin^4(x) = (\sin(x))^4$

② $g(x) = \sin(x^4)$

$$f'(x) = 4(\sin(x))^3 \cdot (\sin x)'$$
$$= \underline{4\sin^3(x) \cdot \cos x}$$

$$g'(x) = \underline{\cos(x^4) \cdot 4x^3}$$

$h(u) = u^4$

$f(x) = h(\sin x)$
$g(x) = \sin(h(x))$

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* $f(x) = \sqrt{2x}$

$$f(x) = \sqrt{2} \cdot \sqrt{x}$$
$$f'(x) = \sqrt{2} \cdot (\sqrt{x})' = \sqrt{2}(x^{1/2})'$$
$$= \sqrt{2} \cdot \frac{1}{2} x^{-1/2} = \frac{\sqrt{2}}{2} \frac{1}{\sqrt{x}}$$
$$= \frac{1}{\sqrt{2}\sqrt{x}} = \frac{1}{\sqrt{2x}}.$$

Vi observerer at

$f(x) \cdot f'(x) = 1 \quad (x>0)$

* $f(x) = e^{(e^{-x^2})}$

Vi anvender kjerneregelen 2ganger:

$$f'(x) = e^{(e^{-x^2})} \cdot (e^{-x^2})' = e^{(e^{-x^2})}(e^{-x^2}) \cdot (-x^2)'$$
$$= \underline{-2x\, e^{e^{-x^2}} \cdot e^{-x^2}}.$$

* Linear substitusjon (kjernen er en lineær funksjon)

$$\underline{\frac{d}{dx} f(ax+b)} = \underline{a f'(ax+b)}$$

Minner om definisjon av den deriverte

(3) $f'(x) = \frac{df}{dx} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{\Delta f}{\Delta x}$.

$\Delta f \sim f'(x) \cdot \Delta x$ lineær tilnærming.

$$(f+g)' = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) + g(x+\Delta x) - (f(x) + g(x))}{\Delta x} = \lim_{\Delta x \to} \frac{\Delta(f+g)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\Delta f + \Delta g}{\Delta x} = \lim_{\Delta x \to 0} \frac{\Delta f}{\Delta x} + \frac{\Delta g}{\Delta x} =$$

$$\lim_{\Delta x \to 0} \frac{\Delta f}{\Delta x} + \lim_{\Delta x \to 0} \frac{\Delta g}{\Delta x} = f'(x) + g'(x)$$

$(c \cdot f)'$ tilsvarende.

produktregelen : $(f \cdot g)(x+\Delta x) = f(x+\Delta x) \cdot g(x+\Delta x)$

$$(f(x) + \Delta f)(g(x) + \Delta g) = f(x) \cdot g(x) + \Delta f \cdot g(x) + f(x)\,\Delta g + \Delta f \cdot \Delta g$$

$$\frac{f(x+\Delta x) \cdot g(x+\Delta x) - f(x) \cdot g(x)}{\Delta x} = \frac{(f(x) + \Delta f)(g(x) + \Delta g) - f(x)\,g(x)}{\Delta x}$$

$$= \frac{\Delta f}{\Delta x} \cdot g(x) + f(x) \cdot \frac{\Delta g}{\Delta x} + \frac{\Delta f \cdot \Delta g}{\Delta x}.$$

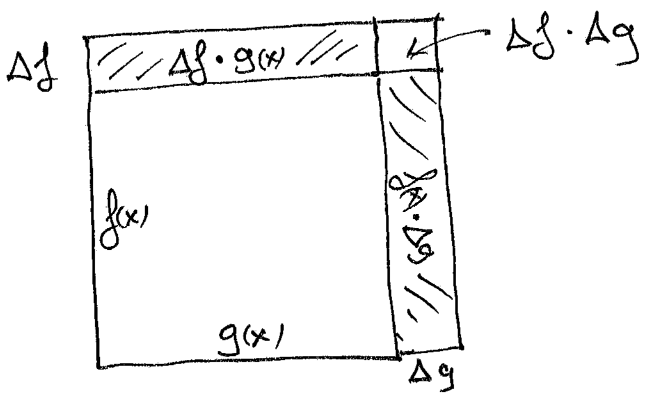
Tar vi grensen $\Delta x \to 0$ får vi

$$(f \cdot g)' = f' \cdot g + f \cdot g'$$

$$\left( \lim_{\Delta x \to 0} \frac{\Delta f}{\Delta x} \cdot \Delta g = \lim_{\Delta x \to 0} \frac{\Delta f}{\Delta x} \cdot \lim_{\Delta x \to 0} \Delta g = f'(x) \cdot 0 = 0 \right)$$

(antar $f(x), g(x), \Delta f, \Delta g > 0$ )

geometrisk :



Kjerneregelen:

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$$\Delta(f\,u)(x) = f(\underbrace{u(x+\Delta x)}_{\sim u(x)+u'(x)\Delta x}) - f(u(x))$$

$$f(u(x)+u'(x)\Delta x) - f(u(x))$$

$$\sim f(u(x)) + f'(u(x))\cdot(u'(x)\Delta x) - f(u(x))$$

$$= \underline{\underline{f'(u(x))\cdot u'(x)}}\ \Delta x.$$

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$\frac{1}{x}$ Bruker def. til å derivere $\frac{1}{x}$

$$\lim_{\Delta x\to 0}\frac{\frac{1}{x+\Delta x}-\frac{1}{x}}{\Delta x} = \lim_{\Delta x\to 0}\frac{\left(\frac{x-(x+\Delta x)}{(x+\Delta x)\cdot x}\right)}{\Delta x}$$

$$= \lim_{\Delta x\to 0}\frac{-\Delta x}{\Delta x}\cdot\frac{1}{(x+\Delta x)\cdot x} = \frac{-1}{x^2}.$$

Legg merke til: $\left(\frac{1}{x}\right)' = (x^{-1}) = (-1)\cdot x^{-1-1} = \frac{-1}{x^2}.$

(følger formelen $x^r = r x^{r-1}$)

$$(x^{-7}) = (x^7)^{-1} = \frac{1}{x^7}.$$

Kjerneregelen: $(x^{-7})' = \frac{-1}{(x^7)^2}\cdot(7x^6) = \underline{-7x^{-8}}.$

Tilsvarende $(x^n)' = n\cdot x^{n-1}$ for alle heltall $n$.

Vi viser: $(x^{p/q})' = \frac{p}{q}\cdot x^{\frac{p}{q}-1}$

$$(x^{p/q})' = ((x^p)^{1/q})' = \frac{1}{q}(x^p)^{\frac{1}{q}-1}\cdot(x^p)'$$

$$= \frac{1}{q}x^{\frac{p}{q}-p}\cdot p x^{p-1} = \frac{p}{q}x^{\frac{p}{q}-p+p-1} = \underline{\frac{p}{q}x^{\frac{p}{q}-1}}$$

(5) $x^r \overset{\text{def}}{=} \lim_{\frac{p}{q} \to r} x^{p/q}$

Derivasjonsformelen $\boxed{\frac{d}{dx} x^r = r \cdot x^{r-1}}$

er gyldig for alle reelle tall $r$
(når $x^r$ og $x^{r-1}$ er definert)

(det følger ved å forsikre seg om at grensen $\frac{p}{q} \to r$
ovenfor respekterer derivasjon.)

$$\left(x^{2/3}\right)' = \frac{2}{3} x^{2/3 - 1} = \underline{\frac{2}{3} x^{-1/3}}$$

$$\left(x^{\pi}\right)' = \pi x^{\pi - 1}.$$

Kvotientregelen: $\left(\frac{f}{g}\right)' = \frac{f' \cdot g - g' \cdot f}{g^2}$.

Utledes:

$$\frac{f}{g} = f \cdot \frac{1}{g} = f \cdot (g)^{-1}$$

$$\left(\frac{f}{g}\right)' \overset{\text{prod. regel}}{=} f' \cdot (g)^{-1} + f \underbrace{\left((g)^{-1}\right)'}_{\frac{-1}{g^2} \cdot g'}$$

$$= \frac{f'}{g} - \frac{f \cdot g'}{g^2}$$

$$= \underline{\underline{\frac{f' \cdot g - f g'}{g^2}}}$$

Eksempel $\dfrac{e^x}{\sqrt{2x-1}}$

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Kvotientregelen: $\dfrac{(e^x)'\sqrt{2x-1} - e^x(\sqrt{2x-1})'}{2x-1}$

$$= \frac{e^x\sqrt{2x-1} - e^x\left(\frac{1}{2}(2x-1)^{-1/2}\cdot 2\right)}{2x-1}$$

$$= \frac{e^x\left(\sqrt{2x-1} - 1/\sqrt{2x-1}\right)}{(2x-1)} = e^x\left(\frac{1}{\sqrt{2x-1}} - \frac{1}{(2x-1)^{3/2}}\right)$$

Alternativt: $\dfrac{e^x}{\sqrt{2x-1}} = e^x\cdot(2x-1)^{-1/2}$

Benytter produktregelen: $\left(e^x\cdot(2x-1)^{-1/2}\right)'$

$$= (e^x)'(2x-1)^{-1/2} + e^x\left((2x-1)^{-1/2}\right)'$$

$$= e^x(2x-1)^{-1/2} + e^x\left(-\frac{1}{2}(2x-1)^{-\frac{1}{2}-1}\overbrace{(2x-1)'}^{2}\right)$$

$$= \underline{e^x\left((2x-1)^{-1/2} - (2x-1)^{-3/2}\right)}.$$

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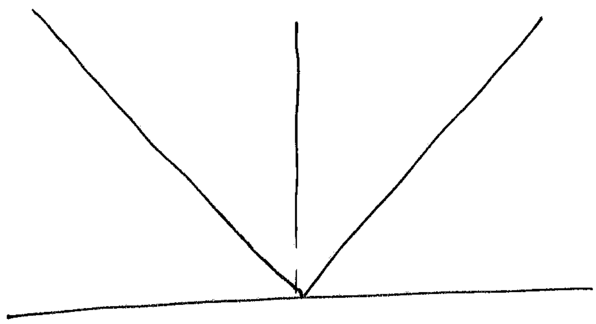
Hva er $\dfrac{d}{dx}|x|$?



$$|x| = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$$

$$\frac{d}{dx}|x| = \begin{cases} 1 & x > 0 \\ -1 & x < 0 \end{cases}$$

den deriverte eksisterer ikke i 0.

$$\lim_{h\to 0}\frac{|h|-|0|}{h} = \lim_{h\to 0}\frac{|h|}{h} = \lim_{h\to 0}\begin{cases} 1 & h>0 \\ -1 & h<0 \end{cases}$$ eksisterer ikke

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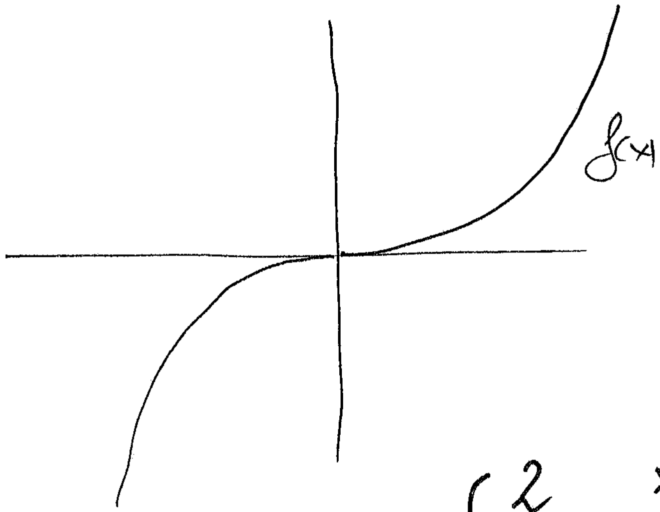
$$f(x) = \begin{cases} x^2 & x \geq 0 \\ -x^2 & x < 0 \end{cases}$$




$$f'(x) = \begin{cases} 2x & x > 0 \\ 0 & x = 0 \\ -2x & x < 0 \end{cases}$$

$$\lim_{h \to 0} \frac{f(h) - f(0)}{h} = f'(0)$$

$$= \lim_{h \to 0} \begin{cases} h & h > 0 \\ -h & h < 0 \end{cases} = 0.$$

$$f''(x) = \begin{cases} 2 & x > 0 \\ -2 & x < 0 \end{cases}$$

eksisterer ikkje når $x = 0$.

Kvotientregel eksempel:

$$(\tan x)' = \left(\frac{\sin x}{\cos x}\right)' = \frac{(\sin x)' \cos x - \sin x (\cos x)'}{(\cos x)^2}$$

$$= \frac{\cos^2 x - \sin x(-\sin x)}{(\cos x)^2} = \frac{\cos^2 x + \sin^2 x}{\cos^2 x}$$

$$= \underline{\frac{1}{\cos^2 x}} \quad (= \sec^2 x) = \frac{\cos^2 x}{\cos^2 x} + \frac{\sin^2 x}{\cos^2 x}$$

$$= \underline{1 + \tan^2 x}$$

$$\underline{(\tan x)' = \frac{1}{\cos^2 x} = 1 + \tan^2 x}$$

# L'Hopitals regel.

(8) Hvis $\lim_{x\to a} f(x) = 0 = \lim_{x\to a} g(x)$ og $\lim_{x\to a} \frac{f'(x)}{g'(x)}$ eksisterer, da eksistere $\lim_{x\to a} \frac{f(x)}{g(x)}$

og $$\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}.$$

Eksempler: * $\lim_{x\to 0} \frac{e^x-1}{3x}$ type $\frac{0}{0}$

$$\lim_{x\to 0} \frac{(e^x-1)'}{(3x)'} = \lim_{x\to 0} \frac{e^x}{3} = \frac{1}{3}$$

så ved L'Hopital $\lim_{x\to 0} \frac{e^x-1}{3x} = \frac{1}{3}$.

* $\lim_{x\to 0} \frac{x^4-2x^7}{5x^4-8x^9}$ type $\frac{0}{0}$.

$$\lim_{x\to 0} \frac{x^4(1-2x^3)}{5x^4(1-\frac{8}{5}x^5)} = \lim_{x\to 0} \frac{1}{5}\left(\frac{1-2x^3}{1-\frac{8}{5}x^5}\right) = \frac{1}{5}$$

(Klønete å bruke L'H her!)

* $\lim_{x\to 0} \frac{1-\cos x}{\sin(x^2)}$ type $\frac{0}{0}$

L'H : $\lim_{x\to 0} \frac{\sin x}{\cos(x^2)\cdot 2x}$ type $\frac{0}{0}$

L'H (to ganger) $\lim_{x\to 0} \frac{\cos x}{-\sin(x^2)\cdot(2x)^2 + 2\cos(x^2)} = \underline{\frac{1}{2}}$.