LF til oppgaver 23.11.2018 Fauske

1 a) $x = -1(2x+1)$ $(x \neq -\frac{1}{2})$

$x = -2x - 1$

$3x = -1$

$\underline{\underline{x = -1/3}}$

b) $x^2 - x < x^3$

$0 < x^3 - x^2 + x$

$0 < x(x^2 - x + 1)$

Faktorisere $x^2 - x + 1$:

$x^2 - x + 1 = 0$ abc-formel:

$x = \frac{1 \pm \sqrt{1-4}}{2}$ ingen reelle røtter.

så $x^2 - x + 1 > 0$ for alle x

(lik 1 for $x = 0$)

Derfor er $0 < x(\underbrace{x^2 - x + 1}_{>0})$

når $\underline{\underline{x > 0}}$

c) For $x > 0$ så er $x + \frac{1}{x} \geq 2$

$\Leftrightarrow$ $x + \frac{1}{x} - 2 \geq 0$ $\Leftrightarrow$ $\frac{x^2 - 2x + 1}{x} \geq 0$

$\Leftrightarrow$ $\frac{(x-1)^2}{x} \geq 0$ $\Leftrightarrow$ $\underline{\underline{x > 0}}$

d) $6\cos^2 v = 3\cos v \qquad 0 \le v < 2\pi$

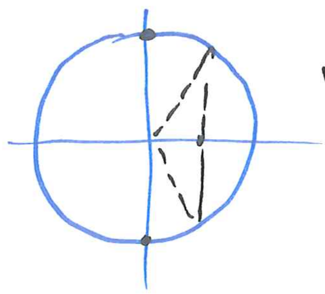
$\Leftrightarrow \quad 2\cos^2 v = \cos v$

$\Leftrightarrow \quad 2\cos^2 v - \cos v = 0$

$\Leftrightarrow \quad 2\cos v\left(\cos v - \frac{1}{2}\right) = 0$

$\Leftrightarrow \quad \cos v = 0$ eller $\cos v = \frac{1}{2}$

$v = \frac{\pi}{2}, \frac{3\pi}{2}$



$v = \pi/3 , 5\pi/3$

Løsningen er $v \in \left\{ \frac{\pi}{3}, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{3} \right\}$

(eller $v = \pi/3, \pi/2, 3\pi/2, 5\pi/3$)

e) $\frac{2}{1+2^x} = 1/6$

La $2^x = y$: $\frac{2}{1+y} = \frac{1}{6}$

$\Leftrightarrow \quad 12 = 1 + y \qquad (y \neq -1)$

$\Leftrightarrow \quad y = 11$

$y = 2^x = 11$

$\log 2^x = \log 11$

$x \log 2 = \log 11$

$$x = \frac{\log 11}{\log 2} \approx 3.45$$

f) $\sqrt[3]{3x+1} = x+1$

$\Leftrightarrow \left(\sqrt[3]{3x+1}\right)^3 = 3x+1 = (x+1)^3$

$\Leftrightarrow 3x+1 = x^3 + 3x^2 + 3x + 1$

$\Leftrightarrow 0 = x^3 + 3x^2$

$\Leftrightarrow 0 = x^2(x+3)$

Løsningene er $\underline{x = 0}$ og $\underline{x = -3}$

2 a) $$\frac{p(x)}{x^2-3} = 2x-3 + \frac{x-3}{x^2-3}$$

ganger med x^2-3 :

$$p(x) = (2x-3)(x^2-3) + x-3$$
$$= 2x^3 - 3x^2 - 6x + 9 + x - 3$$
$$p(x) = \underline{2x^3 - 3x^2 - 5x + 6}$$

b) $p(x) = x^7 - 1$ er delelig med $x-1$ fordi $p(1) = 1^7 - 1 = 0$.

Husk : $$1 + x + x^2 + \cdots + x^{n-1} = \frac{x^n - 1}{x-1}$$

La $n = 7$: $$\frac{x^7-1}{x-1} = \underline{1 + x + x^2 + x^3 + x^4 + x^5 + x^6}$$

c) $p(x) = x^3 - 3x^2 - 10x + 24$

Prøver oss frem $$p(2) = 8 - 12 - 20 + 24 = 0$$

Så $x-2$ deler $p(x)$.

2c)

$$x^3 - 3x^2 - 10x + 24 : x - 2 = x^2 - x - 12$$

$$\begin{array}{l} x^3 - 2x^2 \\ \hline \quad -x^2 - 10x + 24 \\ \quad -x^2 + 2x \\ \hline \qquad -12x + 24 \\ \qquad -12x + 24 \\ \hline \qquad\qquad 0 \end{array}$$

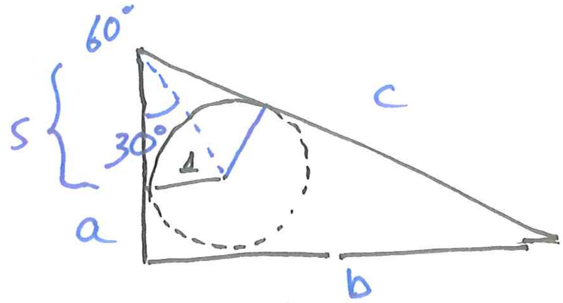
$$~~x~~ P(x) = \underbrace{(x^2 - x - 12)}(x - 2)$$

senere $\quad (x-4)(x+3)$

Polynomet faktoriseres som

$$P(x) = \underline{(x-4)(x-2)(x+3)}$$

3



$\tan(30°) = \frac{1}{s}$ så $\underline{s = \sqrt{3}}$

$\overset{\shortparallel}{1/\sqrt{3}}$

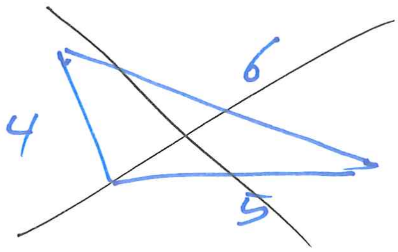
Så $a = 1 + s = \underline{\sqrt{3} + 1} = \underline{2{,}7320}$

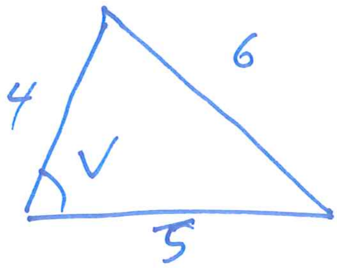
$c = 2 \cdot a = \underline{2(\sqrt{3}+1)} = \underline{5{,}4641}$

$\frac{b}{a} = \sqrt{3}$ så $b = \sqrt{3} \cdot a = \underline{3 + \sqrt{3}}$

$= \underline{4{,}7320}$

b)





Kosinussetningen

$$6^2 = 4^2 + 5^2 - 2 \cdot 4 \cdot 5 \cos(v)$$

$$36 = 41 - 40 \cdot \cos(v)$$

$$36 - 41 = -5 = -40 \cos v$$

så $\cos(v) = \frac{1}{8}$. $v \sim 82.8°$

$\sin^2 v + \cos^2 v = 1$ så $\sin^2 v = 1 - \frac{1}{8^2}$

$= \frac{63}{64}$

$\sin v = \sqrt{\frac{63}{64}} = \frac{\sqrt{63}}{8}$

Arealformelen $A = \frac{1}{2} \cdot 4 \cdot 5 \cdot \sin v = \underline{\frac{10}{8} \cdot \sqrt{63}} \sim \underline{9.92}$

4 xy-planet : $z = 0$

$A(3,-4,6)$

$B(-1,3,3)$

$$\vec{BA} = \vec{OA} - \vec{OB} = [3,-4,6] - [-1,3,3]$$

$$= [4,-7,3]$$

Parametrisere linjen gjennom B med retningsvektor $\vec{BA}$:

$$[x,y,z] = \underbrace{[-1,3,3]}_{\vec{OB}} + \underbrace{t[4,-7,3]}_{t\vec{BA}} .$$

Linja treffer planet når $z = 0$:

$$[x,y,z] = [-1+4t,\ 3-7t,\ \underbrace{3+3t}_{z \text{ er } 0 \text{ når } t=-1}]$$

$$\overset{t=-1}{=} [-5, 10, 0]$$

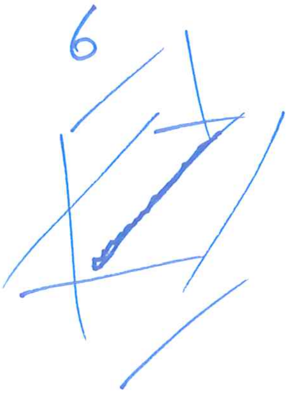
Linja gjennom A og B treffer xy-planet i punktet $(-5,10)$ (i $\mathbb{R}^2$)

(eventuelt $(-5,10,0)$ i $\mathbb{R}^3$)

5. $\vec{v} = \frac{[1,1,1]}{\sqrt{3}} = \frac{1}{\sqrt{3}}[1,1,1]$

$\vec{u} = \frac{[1,-2,1]}{\sqrt{6}} = \frac{1}{\sqrt{6}}[1,-2,1]$

6



$2x - y + 5z = 0$, $x + y = 4$

normalvektor normalvektor

$\vec{n} = [2,-1,5]$ $\vec{m} = [1,1,0]$

Linjen ligger i begge plana. Den er derfor normal til begge normalvektorene. Så linjen er parallell til kryssproduktet av normalvektorene.

$$\vec{n} \times \vec{m} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & -1 & 5 \\ 1 & 1 & 0 \end{vmatrix} = [-5, 5, 3]$$

Finner et felles punkt.

prøver med $z = 0$ $2x - y = 0$ så $y = 2x$

$x + y = x + 2x = 4$

$x = 4/3$, $y = 2x = 8/3$, $z = 0$

En parametrisering av snitt-linjen er

$$\underline{[x,y,z] = [4/3, 8/3, 0] + t[-5,5,3]}$$

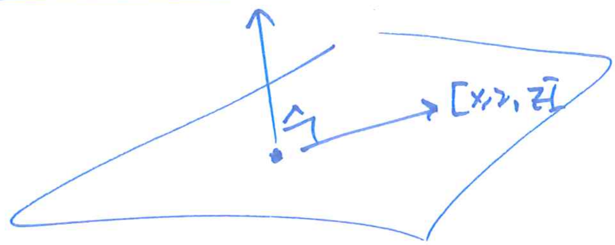
b) Planet inneholder $\left(\frac{4}{3}, \frac{8}{3}, 0\right)$

og er utspent av vektorene

$[-5,5,3]$ og $[1,0,-1]$.

Kryssproduktet av de to vektorene er en normalvektor til planet

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 0 & -1 \\ -5 & 5 & 3 \end{vmatrix} = \underline{[5, 2, 5]}$$



$$[5,2,5] \cdot [x,y,z] = [5,2,5]\left[\frac{4}{3}, \frac{8}{3}, 0\right]$$

$$\underline{5x+2y+5z} = \frac{20+16}{3} + 0 = \underline{12}$$

7

$\vec{a} = [1, 1, 0]$ $\quad |\vec{a}| = \sqrt{2}$

$\vec{b} = [0, 0, -1]$ $\quad |\vec{b}| = 1$

Vinkel mellom

1) $\vec{v}$ og $\vec{a}$ er $60°$:

2) $\vec{v}$ og $\vec{b}$ ——

$$\frac{\vec{a} \cdot \vec{v}}{|\vec{a}| \cdot |\vec{v}|} = \cos(60°) = 1/2$$

$$\frac{\vec{b} \cdot \vec{v}}{|\vec{b}||\vec{v}|} = \cos(60°) = 1/2$$

La $V = [x, y, z]$

$|V| = 1 \iff$

$x^2 + y^2 + z^2 = 1$

Vi får da:

$$x + y = \frac{1}{2} \cdot \sqrt{2} = \frac{1}{\sqrt{2}}$$

$$-z = \frac{1}{2}$$

Vi setter inn for $z = -1/2$:

$$x^2 + y^2 = 1 - \left(\frac{1}{2}\right)^2 = \frac{3}{4}$$

$$x + y = \frac{1}{\sqrt{2}} \quad , \quad x = \frac{1}{\sqrt{2}} - y \quad \text{setter inn}$$

$$\left(\frac{1}{\sqrt{2}} - y\right)^2 + y^2 = 2y^2 - \sqrt{2}\,y + \frac{1}{2} = \frac{3}{4}$$

$$2y^2 - \sqrt{2}\,y - \frac{1}{4} = 0$$

$$y = \frac{\sqrt{2} \pm \sqrt{2 - 2 \cdot 4(-1/4)}}{2 \cdot 2} = \frac{\sqrt{2} \pm \sqrt{4}}{4} = \underline{\frac{\sqrt{2} \pm 2}{4}}$$

$$x = \frac{1}{\sqrt{2}} - y = \frac{\sqrt{2} \mp 2}{4}$$

Løsningene er $\underline{\vec{v} = \left[\frac{\sqrt{2}+2}{4}, \frac{\sqrt{2}-2}{4}, \frac{-1}{2}\right]}$

og $\underline{\vec{v} = \left[\frac{\sqrt{2}-2}{4}, \frac{\sqrt{2}+2}{4}, \frac{-1}{2}\right]}$

8 a) $12+14+16+\cdots+120$

$=2(6+7+8+\cdots+60)$

$=2\left(\sum_{i=1}^{60} i - \sum_{i=1}^{5} i\right)$

$=2\left[\frac{60\cdot 61}{2} - \frac{5\cdot 6}{2}\right]$

$= 60\cdot 61 - 5\cdot 6$

$= 3600+60 - 30$

$= \underline{\underline{3630}}$

b) $1+3+9+27+\cdots \leq 10\,000$

$$\sum_{i=1}^{n} x^{i-1} = \frac{x^n-1}{x-1}$$

$x=3$ $\quad 1+3+\cdots+3^{n-1} = \frac{3^n-1}{2}$

n er antall ledd

ønsker største n s.a $\frac{3^n-1}{2} \leq 10000$

$3^n \leq 20001$

økende

$3^n = 20\,0001$

når $n = \frac{\log 20\,001}{\log 3}$

$= 9.0145$

Det største antall ledd er $\underline{9 \text{ ledd}}$

c) $k^2 + k^3 + k^4 + \cdots = 2$

$k^2(1 + k + k^2 + \cdots)$

$\frac{k^2}{1-k} = 2$ når $|k| < 1$

$k^2 = 2(1-k) = 2 - 2k$

$k^2 + 2k - 2 = 0$

Fullfører kvadratet $(k+1)^2 - 3 = 0$

$k + 1 = \pm\sqrt{3}$

$k = -1 \pm \sqrt{3}$

$k = -1 + \sqrt{3} \sim 0.732$

$k = -1 - \sqrt{3} \sim -2.73 < -1$

så rekken konvergerer ikke.

Løsningen er $\underline{\underline{k = \sqrt{3} - 1}}$

9 $\quad 0.43214321\ldots$

$$= 4321 \cdot (0.00010001\ldots$$

$$= 4321 \cdot \sum_{i=1}^{\infty} \left(\frac{1}{10000}\right)^i$$

$$= 4321 \cdot \frac{1/10000}{1 - 1/10000}$$

$$= 4321 \cdot \frac{1}{10000 - 1}$$

$$= \frac{4321}{9999}$$