3 des. 2012

$$\left(\begin{array}{ll} 2 = 2 & 2 \ , \ -2 \\ \Downarrow & \\ 2^2 = 2^2 & 2^2 = 4 = (-2)^2 \end{array}\right.$$

1d) $\sqrt{5x+1} = x$ irrasjonale likninger

$$\Rightarrow \left(\sqrt{5x+1}\right)^2 = 5x+1 = x^2$$

$$x^2 - (5x+1) = 0$$

$$x^2 - 5x - 1 = 0$$

abc formelen: $x = \dfrac{5 \pm \sqrt{25 - 4(-1)}}{2 \cdot 1}$

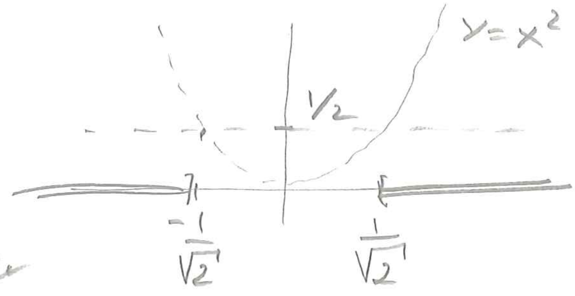
$$x = \frac{5 \pm \sqrt{29}}{2}$$

$x = \dfrac{5 - \sqrt{29}}{2} < 0$ $x = \dfrac{5 + \sqrt{29}}{2}$ (litt større enn 5)

Falsk ekte løsning

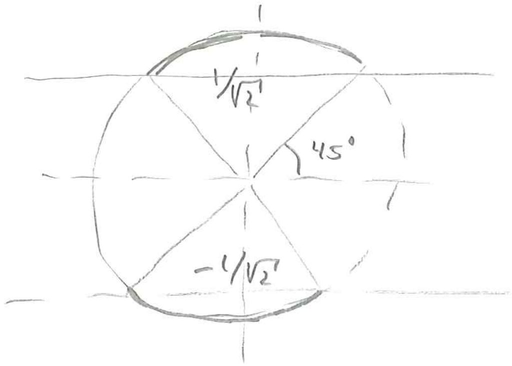
Løsningen er $\underline{\underline{x = \dfrac{5+\sqrt{29}}{2}}}$

e) $\sin^2 v \geq 1/2$



$\sin v \geq \frac{1}{\sqrt{2}}$ eller

$\sin v \leq \frac{-1}{\sqrt{2}}$



Vi ser at løsningen er

$v \in \langle 45°, 135° \rangle \cup \langle 225°, 315° \rangle$

(alt. Alle v slik at $45° \leq v \leq 135°$

eller $225° \leq v \leq 315°$)

1 g) Faktoriser $x^3 - 2x + 1$.

Prøver oss frem: $x = 1$ er en løsning

fordi $1^3 - 2 \cdot 1 + 1 = 0$ ✓

så $x - 1$ er en faktor i $x^3 - 2x + 1$

Polynomdivisjon

$$
\begin{array}{l}
x^3 - 2x + 1 : x - 1 = x^2 + x - 1 \\
\underline{x^3 - x^2} \\
x^2 - 2x + 1 \\
\underline{x^2 - x} \\
-x + 1 \\
\underline{-x + 1} \\
0
\end{array}
$$

$p(x)$ er delelig med $(x-a) \iff p(a) = 0$

$p(x) = (x-a) \cdot q(x) + r$

rest

sette $x = a$

$p(a) = 0 + r$ Resten er $p(a)$

Faktorisere $x^2 + x - 1$.

Røttene til $x^2 + x - 1$ er ved abc-formel

$$x = \frac{-1 \pm \sqrt{1 - (-4)}}{2} = \frac{-1 \pm \sqrt{5}}{2}$$

$$(x^2 + x - 1) = 1 \cdot \left(x + \frac{1}{2} - \frac{\sqrt{5}}{2}\right)\left(x + \frac{1}{2} + \frac{\sqrt{5}}{2}\right)$$

Så $x^3 - 2x + 1 = \underline{(x-1)\left(x + \frac{1}{2} - \frac{\sqrt{5}}{2}\right)\left(x + \frac{1}{2} + \frac{\sqrt{5}}{2}\right)}$

1) Løs ulikheten $\frac{2x-1}{x^2-x-2} \leq \frac{-1}{2} \Leftrightarrow$

$$\frac{2x-1}{x^2-x-2} + \frac{1}{2} \leq 0 \Leftrightarrow \frac{1}{2(x^2-x-2)}\left(2(2x-1) + x^2-x-2\right) \leq 0$$

$$\Leftrightarrow \frac{x^2+3x-4}{2(x^2-x-2)} \leq 0 \quad \overset{\text{Viser}}{\underset{\text{Faktoisere}}{\Leftrightarrow}} \quad \frac{(x+4)(x-1)}{2(x-2)(x+1)} \leq 0$$

2)

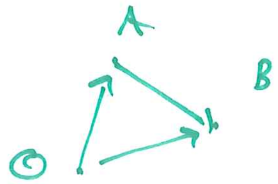
Fire punkt $A(1,0\,2)$ $C(0\,2\,3)$
$B(1,1,1)$ $D(2,4,6)$

a) Finn $\vec{AC}$, $\vec{AB}$ og lengden deres

b) Finn en likning for planet som inneholder A, B og C

c) Finn arealet til trekant ABC

d) Finn volumet til pyramiden ABCD.
Bestem korteste avstand fra D til planet som inneholder ABC.



$$\vec{AB} = \vec{OB} - \vec{OA}$$

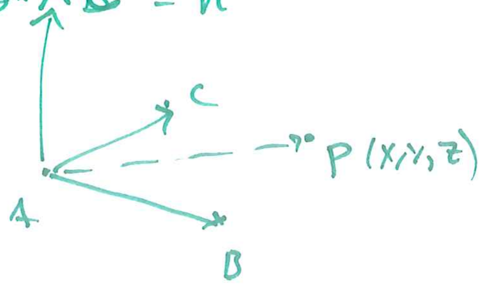
$$[1,1,1] - [1,0,2] = [0,1,-1]$$

$$|\vec{AB}| = \sqrt{0^2+1^2+(-1)^2} = \underline{\sqrt{2}}.$$

$$\vec{AB} \times \vec{AC} = \vec{n} \qquad (\vec{OP} - \vec{OA}) \cdot \vec{n} = 0$$

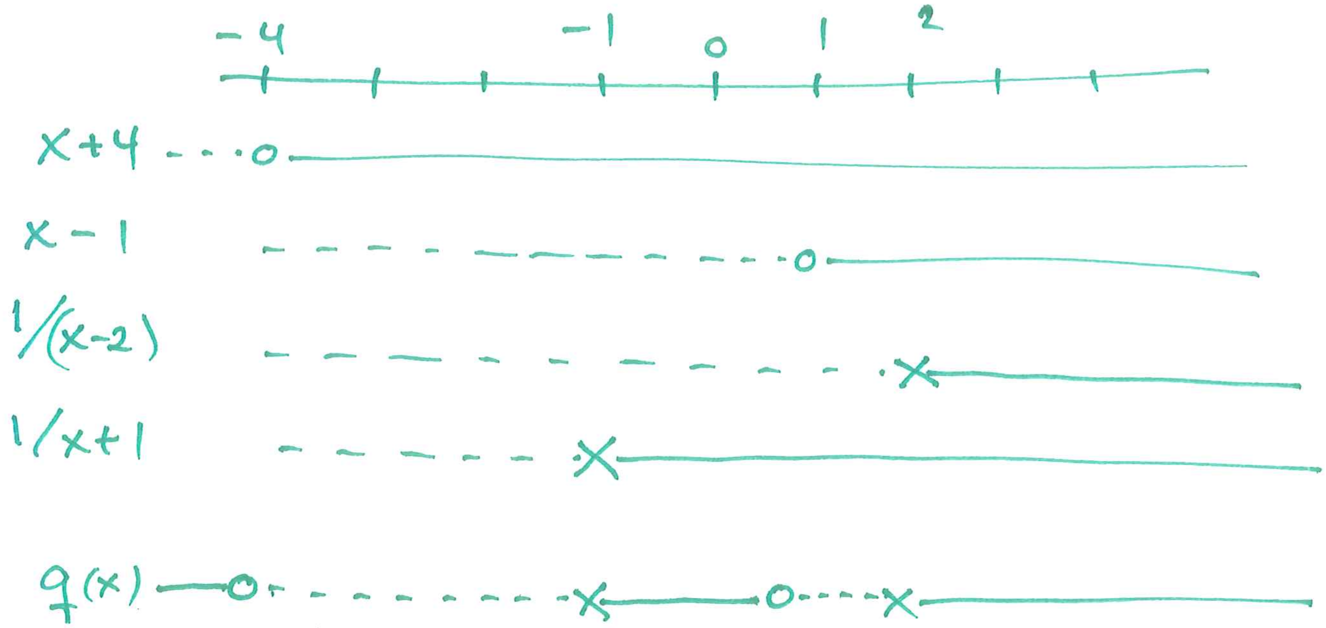
$$[x,y,z] \cdot \vec{n} = \vec{OA} \cdot \vec{n}$$

b



Oppgave 1 Fortsettelse på løsning

$$g(x) = \frac{(x+4)(x-1)}{2(x-2)(x+1)} \leq 0$$



Så $g(x) \leq 0$ for

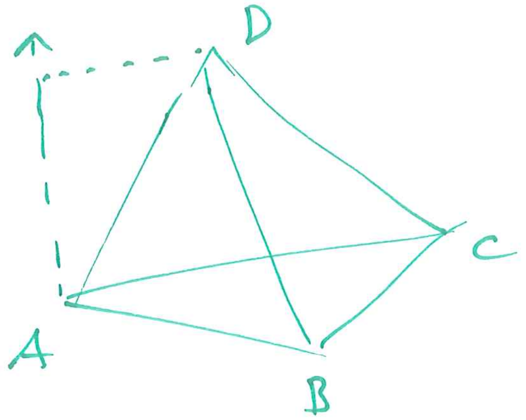
$\underline{-4 \leq x < -1}$ og for $\underline{1 \leq x < 2}$

Dette er løsningene til ulikheten.

c) Arealet til Δ ABC

$$\frac{1}{2}|\vec{AB}||\vec{AC}|\sin(\angle A) = \underline{\frac{1}{2}|\vec{AB}\times\vec{AC}|}$$

d) $\vec{n} = \vec{AB}\times\vec{AC}$



$$V = \frac{1}{2}\cdot\frac{1}{3}\left|\left(\vec{AB}\times\vec{AC}\cdot\vec{AD}\right)\right|$$

Høyden er lengden til komponenten til $\vec{AD}$ langs $\vec{n}$

$$h = \left|\frac{\vec{AD}\cdot\vec{n}}{|\vec{n}|}\right| .$$

(Alternativt: $V_{pyramide} = \dfrac{A_{grunnflate}\cdot h}{3}$)